21, rue d'Artois, F-75008 PARIS
http : //www.cigre.org

International Colloquium on
Lightning and Power Systems

BOLOGNA 2016

Evaluation of the Lightning Performance of an Overhead Multiconductor Transmission Line System

M. BRIGNONE, F. DELFINO, R. PROCOPIO, M. ROSSI
Department of Naval, Electrical, and ICT Engineering,
University of Genoa, Italy
I. BENDATO, R. MOSCA
Department of Mechanical, Energy, Management and Transportation Engineering
University of Genoa, Italy
F. RACHIDI
EMC Laboratory, Swiss Federal Institute of Technology, Lausanne, Switzerland

SUMMARY

The lightning performance of an overhead Multiconductor Transmission Line (MTL) system is typically represented by means of curves reporting how many lightning faults per year the system may experience as a function of its insulation level [1]. In other words, it expresses the probability that the lines are subject to an overvoltage greater than their critical flashover voltage (CFO). Such curves can be obtained by means of a statistical approach (typically a Monte Carlo method is applied): first of all, a large number of lightning events is randomly generated, each one characterized by a specified point of impact and a channel-base current peak following a specified distribution (log-normal for the current and uniform for the point of impact). Then, the adoption of the electro-geometrical (or similar) model allows to determine whether each event originates a direct or an indirect strike [2]. The power system simulation provides the corresponding maximum overvoltage on the overall system. This latter step, especially in the case of indirect strikes, is a delicate task from a computational point of view, since the lightning overvoltage calculation requires the evaluation of the electromagnetic fields and the solution of the field-to-line coupling equations. Therefore, it is necessary to have at disposal an efficient algorithm that can evaluate the lightning-induced overvoltages in an accurate manner, with a reasonable computational effort. Moreover, a smart application of the Monte Carlo method is required in order to limit the number of calls to the coupling simulator [3].
In the paper, the attention will be focused on this latter aspect: defining a methodology that allows to determine the optimal sample size (number of runs) as the best trade-off between the experimental cost/time and the accuracy of the expected results. The methodology proposed allows to control the experimental error (Mean Square Pure Error – MSPE for short)) which affects the Monte Carlo model output, i.e. the uncertainty in the final result due to the chosen number of runs. This is addressed by examining the curves which describe the evolution of the MSPE of the mean and of the MSPE of the standard deviation varying the replicated number of runs. These curves, with their typical "knee shape", after a first phase of fluctuation, become stationary with the increasing of the replicated number of runs, approaching zero for a number of runs tending to infinity.
When the two curves are in the stationary phase, the number of runs is sufficient to obtain a stable (and known) experimental error. So the number of runs can be chosen as the first value of the stationary phase or the following ones depending on the desired experimental error.

Finally, depending on the desired accuracy level, one can define the corresponding confidence/prevision/error intervals.
The whole procedure is tested in a set of different cases that will account for both termination effect and for the finite soil conductivity.

KEYWORDS

Lightning performance, Monte Carlo method, Mean Square Pure Error

1. THE STATISTICAL APPROACH FOR THE LIGHTNING PERFORMANCE EVALUATION

The procedure for the evaluation of the lightning performance of power networks has been described in [2]-[3] and is briefly recalled in what follows.

1. A large number of lightning events n_{tot} is randomly generated. For a given *Oxyz* reference system, each event is characterized by the point of impact $P_F = (x_F, y_F, 0)$ and the channel-base current amplitude I_0. According to [4-5], the current is assumed to follow a log-normal probability density function, while the point of impact coordinates are uniformly distributed into a striking area containing the power system of interest and all the possible lightning events that can cause critical flashovers [2-3].
2. The application of the electrogeometric model (EGM) [6] allows to choose whether the selected event is a direct or an indirect strike.
3. Such overvoltage is compared with the line CFO and a counter *n* is increased of one unit if the overvoltage is greater than the CFO.
4. The ratio between the final value of the counter *n* and n_{tot} is computed in order to evaluate the probability to have a dangerous overvoltage.

The effectiveness and computational efficiency of the above-described procedure strongly depends on the choice of the number of lightning events. From the effectiveness standpoint, one should choose a sufficiently large value for n_{tot} in order to correctly reproduce the Probability Density Function (PDF) of the resulting overvoltage. However, if such value becomes too big, it results in an unnecessary high number of simulations which, especially for the case of indirect lightning events, becomes computationally cumbersome, as each run requires the solution of the field-to-line coupling problem. So, one has to define a methodology that allows to find out an optimal value for n_{tot} and to evaluate an upper bound for the error which is committed as a function of the number of generated lightning events.

2. OVERVIEW ON THE MSPE METHOD

Complex systems modeled with Monte Carlo simulation present, generally, one important challenge: the trade-off between computational effort vs correct representation of the real system behavior. A high number of iterations allows to correctly reproduce the PDF of the input variables and, as a consequence, to accurately represent the real system but with a high computational effort. On the other hand, an insufficient number of iterations involves a low computational effort but the simulation results might be meaningless or biased.
In this section, the application of a methodology which allows to identify the minimum number of runs necessary to obtain correct outputs from the Monte Carlo model is presented.
It is based on the evaluation and graphical representation of the experimental error evolution as a function of the number of runs.

Experimental error is generally distributed as a normal distribution with a zero mean and a variance σ^2, which can be evaluated, according to Cochran's theorem [7], through the measurement of the MSPE, which is shown to be an unbiased estimator of such distribution.
In the proposed methodology, both the variance of the mean response ($MSPE^{MEAN}$) and of the standard deviation ($MSPE^{STDEV}$) must be monitored [8].
The analysis of these two parameters makes it possible to choose the optimal number of runs needed to obtain unbiased results from the simulator.
In particular, the MSPE method can be divided in the following phases:

1. set a number K>2 of simulation runs, carried out in parallel, in which the independent model variables are maintained the same;
2. establish, for each run, a number N >>1 of replications so that one can construct the matrix Y, whose generic entry y_{ij} is the simulator output at run $j \in \{1,\ldots, K\}$ and replication $i \in \{1, \ldots, N\}$;
3. calculate, for each $n = 1,\ldots, N$, and $j = 1,\ldots, K$ the means matrix and the standard deviation matrix

$$\bar{y}_{nj} = \frac{1}{n}\sum_{i=1}^{n} y_{ij} \tag{1}$$

$$\sigma_{nj} = \begin{cases} 0 & n = 1 \\ \sqrt{\dfrac{1}{n-1}\sum_{i=1}^{n}\left(y_{ij} - \bar{y}_{nj}\right)^2} & n \geq 2 \end{cases} \tag{2}$$

4. calculate the N means of means and of standard deviations (for any $n = 1,\ldots, N$)

$$\bar{y}_{n} = \frac{1}{K}\sum_{j=1}^{K} \bar{y}_{nj} \tag{3}$$

$$\bar{\sigma}_{n} = \frac{1}{K}\sum_{j=1}^{K} \sigma_{nj} \tag{4}$$

5. calculate N values of $MSPE^{MEAN}$ and of $MSPE^{STDEV}$:

$$MSPE_n^{MEAN} = \frac{1}{K-1}\sum_{j=1}^{K}\left(\bar{y}_{nj} - \bar{y}_{n}\right)^2 \tag{5}$$

$$MSPE_n^{STDEV} = \frac{1}{K-1}\sum_{j=1}^{K}\left(\sigma_{nj} - \bar{\sigma}_{n}\right)^2 \tag{6}$$

It can be shown that [7]:

$$\lim_{n\to\infty} MSPE_n^{MEAN} = \lim_{n\to\infty} MSPE_n^{STDEV} = 0 \tag{7}$$

This means that when the sample is broad enough to provide an exhaustive description of the population, the two MSPEs would crash on the x-axis. The number of runs needed to obtain unbiased results is identified by checking the experimental error evolution, both in terms of magnitude and stabilization.

Once N is set to a value such that the MSPEs curves' stabilization is reached and the pure error level is low, the results reliability is guaranteed. The knowledge of the MSPE values is also useful to obtain important inferences on the results behavior.
For instance, it allows to calculate the confidence interval to a significance level $1-\alpha$:

$$\bar{y}_n - t_{\alpha/2,n-1}\left(\frac{\bar{\sigma}_n}{\sqrt{n}}\right) \leq \bar{y}_n \leq \bar{y}_n + t_{\alpha/2,n-1}\left(\frac{\bar{\sigma}_n}{\sqrt{n}}\right) \tag{8}$$

where $t_{\alpha/2,n-1}$ is the t-student distribution with $n-1$ degrees of freedom.

The confidence interval defines the bounds for the mean response and this information is essential to understand the accuracy and precision of the results proposed by the model.

3. NUMERICAL SIMULATIONS

The proposed section presents the application of the MSPE method in order to find out a suitable number n_{tot} of lightning events to be generated for the evaluation of the lightning performance of a distribution line and of the upper bound on the committed error.
Let us consider the situation depicted in Fig. 1, where a Multiconductor Transmission Line (MTL) consisting of *M* straight and parallel conductors of length *L* and diameter a_i, each of them placed at a height h_i (i=1,…,M) over a lossy ground (in the following σ_g will denote its conductivity and ε_g its dielectric constant). In the reference frame depicted in Fig. 1, a reference system Oxyz is placed with the x-axis parallel to the direction of the line conductors, so that it is possible to define the position y_i of each conductor.

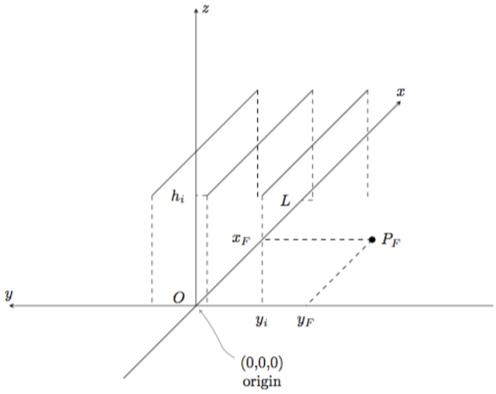

Fig. 1: Geometry of the problem. $P_F(x_F,y_F,0)$ is the location of the lightning strike

Let us suppose that a lightning strike occurs in a point at the ground level with coordinates $P_F(x_F,y_F,0)$. As explained earlier, the application of the electrogeometric model (EGM) [6] allows to choose whether the selected event is a direct or indirect strike. Now, the direct lightning events can be simulated basically with the injection of a current source which can be represented using Heidler's function [9] in parallel with a suitable resistance accounting for the lightning channel. Otherwise, in case of an indirect strike, one has to resort to the equations describing the field-to line-coupling problem that can be solved by means of a dedicated software (see e.g. LIOV [10-11]).
In the next simulations, the following MTL system has been considered, consisting of M=3 conductors of length L=1 km, whose geometry is summarized in Table I.

TABLE I
GEOMETRY OF THE MTL SYSTEM

	Cond. 1	*Cond. 2*	*Cond. 3*
height from ground	8.0 m	8.0 m	8.6 m
distance from *y* axis	−1.2 m	1.2 m	0.0 m
conductor diameter	0.64 cm	0.64 cm	0.64 cm

Moreover, both a lossy (σ_g=0.001 S/m, ε_{rg}=10) and a perfectly conducting ground (PEC) have been considered. As far as the terminations are concerned, two simple configurations have been analyzed: (*i*) each conductor is terminated at both ends with a resistance equal to the characteristic impedance of the conductor and ground (in the following labeled as Case 1) and (*ii*) each conductor is terminated at both ends with a resistance equal to twice the value of the previous case (labeled as Case 2). Moreover, both first and subsequent strokes have been considered, characterized by a log-normal pdf for the peak current with a mean of 31.1 kA for first strokes and 12.3 kA for subsequent, and standard deviations of 0.48 kA for first and 0.53 kA for subsequent strokes [4].

The MSPE matrix Y is built-up by considering as its *ij*-entry the probability that the obtained overvoltage is greater than the specified CFO ($n_{tot}=i$ for any $j=1,\ldots,K$). The MSPE runs number obtained with N=10000, K=5 and α=0.05, as well as the resulting probability and the corresponding inferior and superior confidence interval limits are presented in Table II, considering 4 different CFO values. The exam of the table allows to conclude that the maximum number of necessary replications is at maximum 8500 and that the upper bound on the error is about 2%.

TABLE II

MSPE RESULTS IN THE FOUR DIFFERENT CONFIGURATIONS FOR FOUR CFO VALUES

	CFO (kV)							
	50				100			
configuration	runs number	probability	confiedence interval inferior limit	confiedence interval superior limit	runs number	probability	confiedence interval inferior limit	confiedence interval superior limit
Lossy Case 1	1500	20,96%	18,81%	23,10%	1500	8,94%	7,96%	9,91%
PEC Case 1	4000	6,58%	5,14%	8,00%	8000	3,39%	2,80%	3,98%
lossy Case 2	3000	18,56%	17,21%	19,90%	6000	8,21%	7,81%	8,62%
PEC Case 2	1000	6,71%	5,46%	7,96%	8000	3,62%	3,12%	4,12%

	CFO (kV)							
	150				200			
configuration	runs number	probability	confiedence interval inferior limit	confiedence interval superior limit	runs number	probability	confiedence interval inferior limit	confiedence interval superior limit
Lossy Case 1	1500	5,22%	4,40%	6,04%	1500	3,95%	3,22%	4,67%
PEC Case 1	6000	3,00%	2,45%	3,58%	8000	3,00%	2,53%	3,48%
Lossy Case 2	3500	5,26%	5,01%	5,61%	2000	3,88%	3,60%	4,16%
PEC Case 2	8000	3,20%	2,61%	3,78%	8500	3,17%	2,62%	3,71%

Fig. 2 shows, as an example, the $MSPE^{MEAN}$ and $MSPE^{STEDV}$ curves for a PEC, Case 1 configuration with a CFO equals to 50 kV. From this figure, it can be seen that 4000 number of runs allows to stabilize the curves with a low pure error level. Therefore, 4000 can be chosen as the number of runs which guarantees the results reliability for this kind of configuration and CFO.

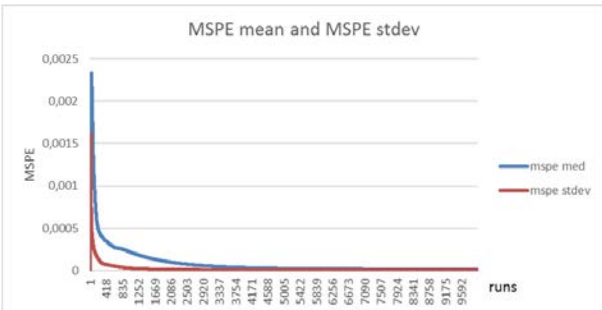

Fig. 2: $MSPE^{MEAN}$ and $MSPE^{STEDV}$ curves

In Fig. 3 the comparison of the probability computed by means of Monte-Carlo with N=100000 and the probability obtained with the runs number (N_{MSPE}) provided by the MSPE algorithm is presented. As can be seen there is a good agreement as the lightning performance obtained with 100000 replications always falls in the confidence interval predicted by the MSPE method.

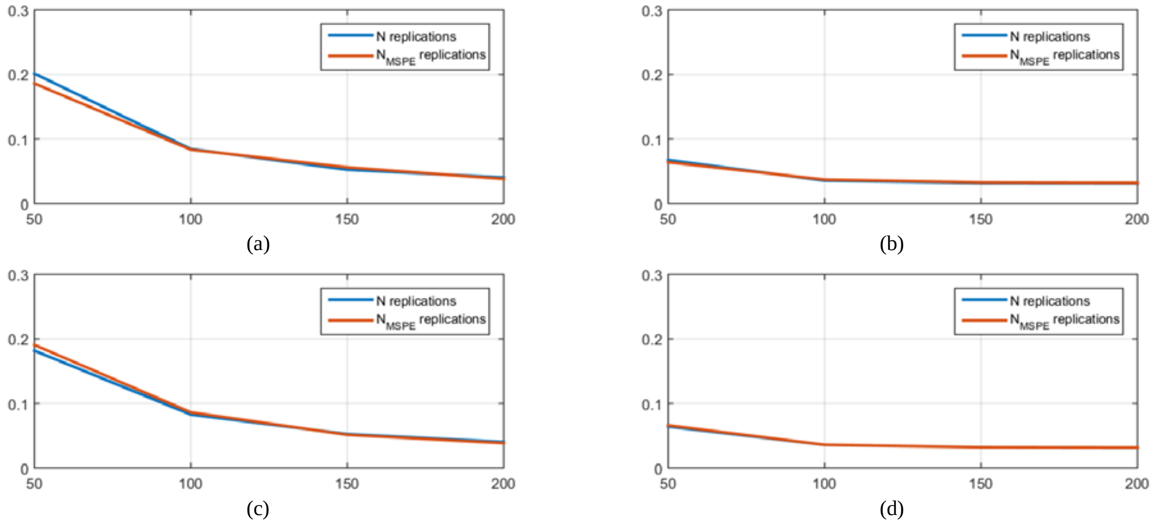

Fig. 3: Probability to have a flashover greater than the CFO computed with N=100000 replications vs N_{MSPE} replications. More precisely the four panels are related to the presented configurations as follows: (a) is the lossy Case 1, (b) is the PEC Case 1, (c) is the lossy Case 2 and (d) is the PEC Case 2.

4. CONCLUSIONS

The lightning performance of a distribution network is typically obtained by means of a statistical approach that generates a large number of lightning and evaluates the corresponding overvoltage. As for indirect lightning strikes, the overvoltage computation is a time-consuming task, a smart application of the Monte Carlo method is required, in order to limit the number of calls to the field-to-line coupling simulator.

In this contribution, a methodology that allows to determine the optimal number of runs as the best trade-off between the experimental cost/time and the accuracy of the expected results was proposed. The methodology is based on the evaluation of $MSPE^{MEAN}$ and $MSPE^{STDEV}$ curves. These curves represent the system stochasticity and their knowledge allows to keep under control the uncertainty level which affects the final result associated with the chosen number of runs. Moreover, the knowledge of these two values, allows to identify the confidence interval where the system response will be contained, with a desired accuracy level.

The whole procedure was tested in a set of different cases that account for both termination effects and for the finite soil conductivity.

Finally, the obtained results were compared with those which could be achieved with a much higher number of runs (at least one order of magnitude more) showing a very good fit.

BIBLIOGRAPHY

[1] Guide for improving the lightning performance of electric power overhead distribution lines, IEEE Std. 1410, IEEE Working Group on the Lightning Performance of Distribution Lines, 2004.

[2] Borghetti, C. A. Nucci, and M. Paolone, “An improved procedure for the assessment of overhead line indirect lightning performance and its comparison with the IEEE Std. 1410 method,” IEEE Trans. Power Del., vol. 22, no. 1, pp. 684–692, January 2007.

[3] Borghetti, C. A. Nucci, and M. Paolone, “Indirect-Lightning Performance of Overhead Distribution Networks With Complex Topology,” IEEE Trans. Power Del., vol. 24, no. 4, pp. 2206–2213, October 2009.
[4] K. Berger, R. B. Anderson, and H. Kröninger, “Parameters of lightning flashes,” Electra, no. 41, pp. 23–37, July 1975.
[5] Cigrè Working Group 01 of SC33, Guide to Procedures for the Lightning Performance to Transmission Lines, Cigrè Oct. 1991, Ref. no 63.
[6] C. A. Nucci, “A survey on cigré and IEEE procedures for the estimation of the lightning performance of overhead transmission and distribution lines,” in Electromagnetic Compatibility (APEMC), 2010 Asia-Pacific Symposium, pp. 1124–1133, April 2010.
[7] D. C. Montgomery, Design and Analysis of Experiments, JohnWiley and Sons, 1997.
[8] L. Cassettari, R. Mosca, R. Revetria. “Monte Carlo simulation models evolving in replicated runs: a methodology to choose the optimal experimental sample size”, Mathematical Problems in Engineering, Vol. 2012, 17 pages, doi:10.1155/2012/463873
[9] F. Heidler, “Analytische Blitzstromfunktion zur EMP Berechnung,” in *18th Int. Conf. Lightning Protection*, Munich, Germany, 1985.
[10] C. A. Nucci, M. Paolone, and F. Rachidi, “A new finite difference time domain scheme for the evaluation of lightning induced overvoltage on multiconductor overhead lines,” in Proc. Int. Conf. Power Syst. Trans., Rio de Janeiro, Brazil, Jun. 24–28 2001, vol. 2, no. 5, pp. 596–602.
[11] A. Borghetti, J.A. Gutierrez, C. A. Nucci, M. Paolone, E. Petrache and F. Rachidi, “Lightning-induced voltages on complex distribution systems: models, advanced software tools and experimental validation,” Journal of Electrostatics, vol. 60, pp. 163–174, 2004.