A new mathematical formulation to integrate supply and demand within a choice-based optimization framework

Meritxell Pacheco
Shadi Sharif Azadeh, Michel Bierlaire

Transport and Mobility Laboratory (TRANSP-OR)
École Polytechnique Fédérale de Lausanne

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Outline

Motivation

Customer behavioral models

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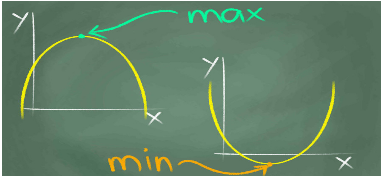
Demand

Motivation

Operations research



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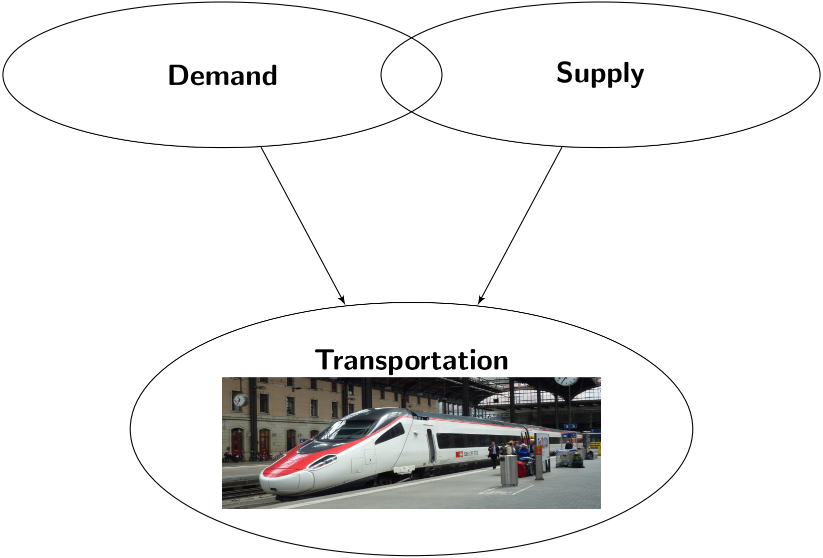
Supply

Motivation



Demand vs. supply

Customer behavioral models

- Given the configuration of the system ⇒ predict the demand
- Maximize satisfaction
- **Here:** discrete choice models

Operations Research

- Given the demand ⇒ configure the system
- Minimize costs
- **Here:** MILP

Discrete choice models in optimization problems

- Integrated choice model ⇒ source of nonconvexity
- Many techniques to convexify and linearize. **Here:** different approach
  - Nonconvex representation of choice probabilities
  - Include a wide class of discrete choice models

Utilities

Demand and supply

- Population of N individuals
- Set of products $\mathcal{C}$ in the market
    - artificial "opt-out" product
- $\mathcal{C}_n \subseteq \mathcal{C}$ subset of available products to individual n

Utility

U_{in} associated score with alternative i by individual n: $U_{in} = V_{in} + \varepsilon_{in}$

- V_{in}: deterministic part
- ε_{in}: error term

Behavioral assumption: n chooses i if U_{in} is the highest in $\mathcal{C}_n$

Probabilistic model

Choice variable

$$w_{in} = \begin{cases} 1 & \text{if } n \text{ chooses } i \\ 0 & \text{otherwise} \end{cases} \quad \forall n, \forall i \in \mathcal{C}.$$

Probabilistic model

$$\Pr(w_{in} = 1) = \Pr(U_{in} \geq U_{jn}, \forall j \in \mathcal{C}_n)$$

$$D_i = \sum_{n=1}^{N} \Pr(w_{in} = 1)$$

Simulation

Non linearity

D_i is in general non linear

Example:

$$\Pr(w_{in} = 1) = \frac{y_{in} e^{V_{in}}}{\sum_{j \in \mathcal{C}} y_{jn} e^{V_{jn}}} \text{ (logit model)}$$

Simulation

- Assume a distribution for ε_{in}
- Generate R draws $\xi_{in1} \ldots \xi_{inR}$
- r behavioral scenario
- The choice problem becomes **deterministic**

Demand model

$$U_{inr} = V_{in} + \xi_{inr} = \sum_k \beta_k x_{ink} + f(z_{in}) + \xi_{inr}$$

$\Rightarrow U_{inr}$ is not a random variable

Endogenous part of V_{in}

- Linear in the variables x_{ink}
- Decision variables (involved in the optimization problem)
- Assumption for the integration in a MILP

Exogenous part of V_{in}

- Depends on other variables z_{in}
- f not necessarily linear

Mixed Integer Linear Problem

Availability of alternatives

- Availability at operator level
- Availability at scenario level (*e.g.* demand exceeding capacity)

Preference of alternatives

- Take into account only the available alternatives
- Choose the alternative with highest utility

Choice

Choice at scenario level: w_{inr}

$$D_i = \frac{1}{R} \sum_{n=1}^{N} \sum_{r=1}^{R} w_{inr}$$

Demand based revenues maximization (I)

Application

- Operator selling services to a market, each offered service:
  - Price
  - Capacity (number of customers)
- Demand is price elastic and heterogenous
- **Goal:** best strategy in terms of capacity allocation and pricing

Maximization of revenues

- p_{in} price that individual n has to pay to access service i

$$\max R_i = \max \frac{1}{R} \sum_{n=1}^{N} p_{in} \sum_{r=1}^{R} w_{inr}$$

- p_{in} endogenous variable $\Rightarrow R_i$ non linear

Demand based revenues maximization (II)

Pricing

- Linearization of the price
- Discretization ⇒ price levels

Capacity allocation

- Capacity for each alternative i
  - We assume it given
  - It could be a decision variable
- Who has access?
- Provide a priority list to the model

Perspective

Conclusions

- High dimensionality of the problem (*INR*)
- Any assumption can be made for the ε_{in}

Ongoing research

- **Proof of concept**: case study from the literature (mixed logit)
- Define different scenarios to test the formulation
- More accurate values (*e.g.* price levels)

Future work

- Decomposition techniques to speed up the computational results
  - By **customer**: capacity!
  - By **scenario**: only considered together in the objective function
- Introduce new features (*e.g.* N as a group of individuals), capacity?

Questions?