Hybrid Cyclicity: Combining The Benefits Of Cyclic And Non-Cyclic Timetables
The main product of a Train Operating Company is a train timetable.
What is a timetable?
A railway timetable is defined as a set of arrival and departure times of every train from each of its stopping stations.
Two types of timetables exist: Cyclic and Non-Cyclic.
The cyclic timetable originates from the Periodic Event Scheduling Problem (PESP), which was first defined by Serafini and Ukovich (1989).
The cyclic timetable originates from the Periodic Event Scheduling Problem (PESP), which was first defined by Serafini and Ukovich (1989).

A set of events is scheduled in an equally spaced intervals, e.g. STRC - approx. every 365 days.
A special subset of cyclicity is the **clockfaced timetables**:

Event every xx:15.
A special subset of cyclicity is the clockfaced timetables:

Event every xx:15. Especially popular within:
Issue: The demand is not uniformly distributed.
Issue: The demand is not uniformly distributed.
Issue: The demand is not uniformly distributed.
Passengers find the regularity of a timetable easier to be memorized (Wardman et al. (2004), Johnson et al. (2006)).
Therefore one is not superior to the other.
Therefore one is not superior to the other.

Why not both?
What we want to combine and how:

Figure: Ursus Wehrli

Regularity: Taken care of by the design
What we want to combine and how:

Figure: Ursus Wehrli

Regularity: Taken care of by the design

Flexibility: Passenger satisfaction, maximized by solving the Passenger Centric Train Timetabling Problem
\[ S_{i}^{tp} = -VOT \cdot \left( \sum_{\ell \in L_p} r_{\ell i}^{pl} + \beta_W \cdot w_{ti}^{tp} + \beta_T \cdot (|L_p| - 1) + \beta_E \cdot \delta_{ip}^{t} + \beta_L \cdot \gamma_{ip}^{t} \right), \quad \forall i \in I, \forall t \in T_i, \forall p \in P_i, \]

- \( r_{\ell i}^{pl} \) – running time/in-vehicle time
- \( w_{ti}^{tp} \) – waiting time
- \(|L_p| - 1\) – number of transfers
- \( \delta_{ip}^{t} \) – early schedule passenger delay
- \( \gamma_{ip}^{t} \) – late schedule passenger delay
- \( VOT \) – value of time
- \( \beta_W, \beta_T, \beta_E, \beta_L \) – estimates from literature
What are the combinations?
$\theta$ Shifted Cyclic Timetable

For a cycle of 60 minutes:
- $\theta = 0$ is equivalent to the cyclic timetable
- $\theta = 30$ is the maximum deviation without overlapping trains
- We test values between 0 and 30 in 3 minute intervals

$\Delta_{\theta}^{\ell} \in (-\theta, \theta)$
\( \theta \) Shifted Cyclic Timetable

For a cycle of 60 minutes:

- \( \theta = 0 \) is equivalent to the cyclic timetable
- \( \theta = 30 \) is the maximum deviation without overlapping trains
- We test values between 0 and 30 in 3 minute intervals

\( \Delta^\ell_v \in (-\theta, \theta) \)
\[ \theta \text{ Shifted Cyclic Timetable} \]

\[ \Delta^\ell_v \in \langle -\theta, \theta \rangle \]

For a cycle of 60 minutes:
- \( \theta = 0 \) is equivalent to the cyclic timetable
- \( \theta = 30 \) is the maximum deviation without overlapping trains
- We test values between 0 and 30 in 3 minute intervals
Partially Cyclic Timetable

\( \eta = \max(|V^c|) \cdot \frac{\xi}{100} \)
\( \eta \) trains per line have a cyclic departure time, the rest is free

For a cycle of 60 minutes:
- \( \xi = 0 \) is equivalent to the cyclic timetable
- \( \xi = 100 \) is equivalent to the non-cyclic timetable
- We test values between 0 and 100 in 10% intervals
## Hybrid Cyclic Timetable

A cycle can have:
- no train
- a cyclic train
- a cyclic train and one or more non-cyclic ones

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Model

\[
\text{max satisfaction} \\
\text{satisfaction function} \\
\text{at most one path per passenger} \\
\text{link trains with paths} \\
\text{cyclicity} \\
\text{train scheduling} \\
\text{train capacity} \\
\text{schedule delay} \\
\text{waiting time}
\]
Methodology: Simulated Annealing
Case Study
Israel 2008

- OD Matrix for an average working day (Sunday to Thursday) in Israel during 2008
- 47 Stations
- 2162 ODs
- 34 (unidirectional) lines
- 380 trains
- Min. transfer – 4 mins
- VOT – 21.12 NIS per hour
- 126 036/193 886 Passengers
Table: Computational results of the existing timetables for the 2008 demand

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<th>IR 13/14 as Strictly Cyclic</th>
<th>IR 13/14 cyclic</th>
<th>non-cyclic</th>
<th>perfect service</th>
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<td>-476 774</td>
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Table: Computational results of the existing timetables for the 2014 demand

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Figure: *Breakdown of the passenger satisfaction for various timetables under the 2014 demand*
Passenger Satisfaction [mNIS]

-4.4
-4.6
-4.8
-5
-5.2
-5.4
-5.6

0 5 10 15 20 25 30

θ [min]

--- non-cyclic

- - θ shifted

-- cyclic
Passenger Satisfaction [mNIS]

\[ \xi \text{ [\%]} \]

- non-cyclic
- \( \xi \) partial
- cyclic
Conclusion
Case Study

- Difference in Pax. Sat. between cyclic and non-cyclic timetable: 18.5%
- $\theta$ Shifted Timetable can reduce the difference to a half
- $\xi$ Partially Cyclic can diminish the difference already at $\xi = 60$ with a train ratio 3:1
- Hybrid Cyclic finds the same ratio, provides good level of service

General

- As the demand is time dependent, purely cyclic timetable is not a good option
- Hybrid cyclic timetable can diminish the impact of the cyclicity constraints
Future Work
• Elastic Demand
• Need of an opt-out
• Maximize Profit
• Adapt Pricing Scheme
The regularity of a habit is generally in proportion to its absurdity.

(Marcel Proust)
References

