Sparse regularization methods in ultrafast ultrasound imaging

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Ultrafast ultrasound imaging
  Principle
  Spatial-based approaches
  Fourier-based approaches

Sparse regularization for ultrasound imaging
  General framework
  The measurement model
  Sparsifying model and reconstruction algorithm

Applications of the proposed approach
  Sparse regularization for image quality enhancement
  Compressed beamforming

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Ultrafast ultrasound imaging

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Ultrafast Ultrasound Imaging

Principle

Ultrafast ultrasound imaging
Ultrafast Ultrasound Imaging

Principle

Ultrafast ultrasound imaging

(a) Transmit

Image reconstruction - Beamforming
▶ Infer the reflectivity $s(x, z)$ from the received echoes $r(x_i, t)$

State of the art approaches
▶ Spatial-based approaches [Montaldo et al., 2009]
▶ Fourier-based approaches [Lu et al., 1997] [Garcia et al., 2013] [Bernard et al., 2014]
Ultrafast Ultrasound Imaging

Principle

Ultrafast ultrasound imaging

(a) Transmit

(b) Receive (Source: David et al., 2015)
Ultrafast Ultrasound Imaging

Principle

Ultrafast ultrasound imaging

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(b) Receive (Source: David et al., 2015)

Image reconstruction - Beamforming
Ultrafast Ultrasound Imaging

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Ultrafast ultrasound imaging

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(b) Receive (Source: David et al., 2015)

Image reconstruction - Beamforming

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Ultrafast ultrasound imaging

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Image reconstruction - Beamforming

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- State of the art approaches
Ultrafast Ultrasound Imaging

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Ultrafast Ultrasound Imaging

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Ultrafast ultrasound imaging

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Ultrafast Ultrasound Imaging
Spatial-based approaches

Delay-and-sum beamforming

(a)
Ultrafast Ultrasound Imaging
Spatial-based approaches

Delay-and-sum beamforming

(a) 

(b)
Ultrafast Ultrasound Imaging
Spatial-based approaches

Delay-and-sum beamforming

(a)  
(b)  
(c)
Ultrafast Ultrasound Imaging
Fourier-based approaches

Principle

- Backscattered echoes
- Fourier spectrum
- Remapped Fourier spectrum
Ultrafast Ultrasound Imaging
Fourier-based approaches

Principle

▶ Based on a remapping of the Fourier spectrum of the echoes in the space of the desired image
Ultrafast Ultrasound Imaging
Fourier-based approaches

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(a) Backscattered echoes
Ultrafast Ultrasound Imaging
Fourier-based approaches

Principle

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(a) Backscattered echoes
(b) Fourier spectrum

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Ultrafast Ultrasound Imaging
Fourier-based approaches

**Principle**

- Based on a remapping of the Fourier spectrum of the echoes in the space of the desired image

(a) Backscattered echoes  
(b) Fourier spectrum  
(c) Remapped Fourier spectrum
Image reconstruction is an ill-posed problem

- The backscattered echoes do not necessarily carry all the information necessary to reconstruct images
Main drawbacks of the classical approaches

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Interpolation on discrete grids creates artifacts
Ultrafast Ultrasound Imaging
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- Interpolation of delays
- Interpolation of remapped Fourier spectrum
Ultrafast Ultrasound Imaging
Main drawbacks of the classical approaches

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Interpolation on discrete grids creates artifacts
- Interpolation of delays
- Interpolation of remapped Fourier spectrum

Classical methods are \textit{filtered-backprojection} solutions of the inverse problem
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Sparse regularization for ultrasound imaging

General framework

Sparse regularization as an alternative to filtered-backprojection
Sparse regularization for ultrasound imaging

General framework

Sparse regularization as an alternative to filtered-backprojection

- Use sparse regularization methods to solve the inverse problem posed by the image reconstruction
Sparse regularization for ultrasound imaging

General framework

Sparse regularization as an alternative to filtered-backprojection

- Use sparse regularization methods to solve the inverse problem posed by the image reconstruction
- Two pillars:
Sparse regularization for ultrasound imaging

General framework

Sparse regularization as an alternative to filtered-backprojection

- Use sparse regularization methods to solve the inverse problem posed by the image reconstruction
- Two pillars:
  - The ability to express acquisition as a linear inverse problem \( r = Hs + n \) with \( r \in \mathbb{R}^M \) the element raw data and \( s \in \mathbb{R}^N \) the desired image
Sparse regularization as an alternative to filtered-backprojection

- Use sparse regularization methods to solve the inverse problem posed by the image reconstruction
- Two pillars:
  - The ability to express acquisition as a linear inverse problem $r = Hs + n$ with $r \in \mathbb{R}^M$ the element raw data and $s \in \mathbb{R}^N$ the desired image
  - The sparsity of ultrasound images in a given model $\Psi$
Sparse regularization for ultrasound imaging

General framework

Sparse regularization as an alternative to filtered-backprojection

▶ Use sparse regularization methods to solve the inverse problem posed by the image reconstruction

▶ Two pillars:
  ▶ The ability to express acquisition as a linear inverse problem
    \[ r = Hs + n \]
    with \( r \in \mathbb{R}^M \) the element raw data and \( s \in \mathbb{R}^N \) the desired image
  ▶ The sparsity of ultrasound images in a given model \( \Psi \)

Image reconstruction An analysis-based problem is solved
\((\ell_1\text{-minimization})\)

\[
\min_{\tilde{s} \in \mathbb{C}^N} \| \Psi H \tilde{s} \|_1 \quad \text{subject to} \quad \| r - H \tilde{s} \|_2 \leq \epsilon
\]
Sparse regularization for ultrasound imaging

Measurement model

Spatial-based approach

\[ r(x_i, t) = \int_{(x, z) \in \Omega(x_i, t)} s(x, z) \, dx \, dz \]

- In the continuous domain:

- Discretization of the integral leads to

\[ r = H S + n \]

Fourier-based approach

[Besson et al., 2016]
Sparse regularization for ultrasound imaging

Measurement model

Spatial-based approach

- In the continuous domain:

\[ r(x_i, t) = \int \int_{(x,z) \in \Omega(x_i, t)} s(x, z) \, dx \, dz \]

\[ \Omega(x_i, t) \] propagation delay curve
Sparse regularization for ultrasound imaging
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Sparse regularization for ultrasound imaging

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**Fourier-based approach** [Besson et al., 2016]
Sparse regularization for ultrasound imaging

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\( \Omega(x_i, t) \) propagation delay curve

▶ Discretization of the integral leads to \( r = H_S s + n \)

Fourier-based approach [Besson et al., 2016]

\[ r = F^H_R G_F S s + n \]
Sparse regularization for ultrasound imaging
Sparsifying model and reconstruction algorithm

Sparsifying model

Several models already studied in the literature: Wave Atom frame, Fourier basis, Dirac basis, Wavelet-based models

Sparsity averaging model ($\Psi$) used

$\Psi$\footnote{Carrillo et al., 2013}: Concatenation of wavelet bases:

$$\Psi = \sqrt{q} [\Psi_1, \ldots, \Psi_q]$$

In the study:

$q = 8$, Daubechies wavelet as mother function

Image reconstruction problem

Analysis-based problem:

$$\min_{\bar{s} \in \mathbb{C}^N} \|\Psi H \bar{s}\|_1$$
subject to

$$\|r - H \bar{s}\|_2 \leq \epsilon$$

with $\Psi$ the SA model and $H$ Spatial-based or Fourier-based reconstruction model

Alternating Direction of Minimizers (ADMM) used to solve the problem
Sparse regularization for ultrasound imaging
Sparsifying model and reconstruction algorithm

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Sparse regularization for ultrasound imaging

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Image reconstruction problem

- Analysis-based problem:
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Problem & Objective
Applications of the proposed approach
Sparse regularization for image quality enhancement

Problem & Objective

- Image reconstruction poses an ill-posed problem
- Discretization of the measurement models leads to inaccuracies [Carrillo et al., 2015]
- Use sparse regularization to enhance image quality
Applications of the proposed approach
Sparse regularization for image quality enhancement

Problem & Objective

▶ Image reconstruction poses an ill-posed problem
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Numerical study of the contrast

▶ 8-mm anechoic inclusion positioned at 4 cm embedded in a medium with high density of scatterers
▶ Insonification with 1 plane wave
▶ Reconstruction with classical approaches and sparse regularization
Applications of the proposed approach
Sparse regularization for image quality enhancement

B-mode images of the anechoic inclusion

(a) Classical
CR = 5.59 dB

(b) Sparse spatial
CR = 9.10 dB

(c) Sparse Fourier
CR = 9.62 dB
Applications of the proposed approach
Compressed beamforming

Problem & Objective
Applications of the proposed approach
Compressed beamforming

Problem & Objective

- Acquisition with few transducer elements
Applications of the proposed approach
Compressed beamforming

Problem & Objective

- Acquisition with few transducer elements
- Creation of a new measurement vector: \( r_u = P r \) with \( P \in \mathbb{R}^{R \times M} \) and \( R \ll M \)

\[ \min_{\bar{s} \in \mathbb{C}^N} \| \Psi H \bar{s} \|_1 \]
\[ \text{subject to} \]
\[ \| r_u - H u \bar{s} \|_2 \leq \epsilon \]

\[ H u = P H \in \mathbb{R}^{P \times N} \]
Applications of the proposed approach
Compressed beamforming

Problem & Objective

▶ Acquisition with few transducer elements
▶ Creation of a new measurement vector: \( r_u = Pr \) with \( P \in \mathbb{R}^{R \times M} \) and \( R \ll M \)
▶ Retrieve high quality images from this undersampled measurement vector [Besson et al., 2016]

\[
\min_{\bar{s} \in \mathbb{C}^N} \| \Psi^H \bar{s} \|_1 \quad \text{subject to} \quad \| r_u - H_u \bar{s} \|_2 \leq \epsilon
\]

with \( H_u = PH \in \mathbb{R}^{P \times N} \)
Applications of the proposed approach
Compressed beamforming

Experiments

- Acquisition of an in-vivo carotid with a Verasonics US system
- 1 PW insonification with a 128 transducer-elements probe
- Random selection of 30 transducer elements in receive
- Reconstruction with the sparse regularization approach
Applications of the proposed approach
Compressed beamforming

B-mode images obtained with 30 transducer elements

(a) Fourier  (b) Spatial  (c) Reference
Applications of the proposed approach
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Peak-signal-to-noise-ratio against number of transducers
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Conclusion & Perspectives

Framework for sparse regularization in US imaging

- Measurement model for Fourier-based and spatial-based approaches
- Sparsity Averaging model as a sparsifying model
- Image reconstruction algorithm

Applications

- Image quality enhancement (contrast)
- Compressed beamforming

Perspectives

- Extension to 3D imaging
- More robust reconstruction algorithms which take into account spatially variable point spread functions
Conclusion & Perspectives

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Thank you!