# Sparse regularization methods in ultrafast ultrasound imaging

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# Outline

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Ultrafast ultrasound imaging
- Principle
- Spatial-based approaches
- Fourier-based approaches

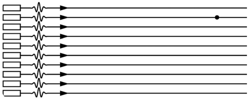
Sparse regularization for ultrasound imaging
- General framework
- The measurement model
- Sparsifying model and reconstruction algorithm

Applications of the proposed approach
- Sparse regularization for image quality enhancement
- Compressed beamforming

Conclusion

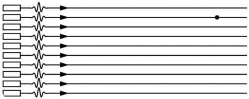
Ultrafast ultrasound imaging

## Ultrafast ultrasound imaging



(a) Transmit

### Ultrafast ultrasound imaging



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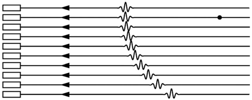

(b) Receive (Source: David *et al.*, 2015)

# Ultrafast Ultrasound Imaging
## Principle

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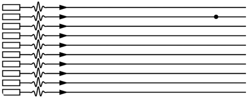

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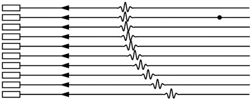

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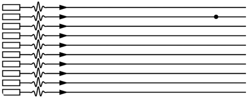

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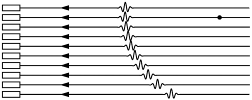

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- Infer the reflectivity $s(x, z)$ from the received echoes $r(x_i, t)$

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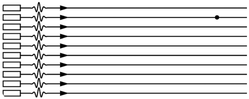

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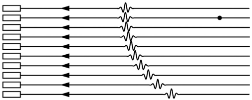

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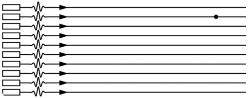

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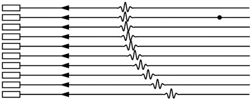

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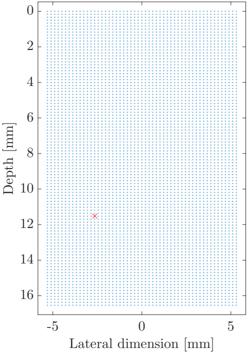
  - Fourier-based approaches [Lu *et al.*, 1997] [Garcia *et al.*, 2013] [Bernard *et al.*, 2014]

### Delay-and-sum beamforming




(a)

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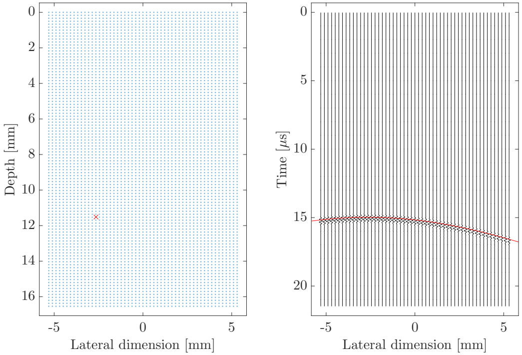


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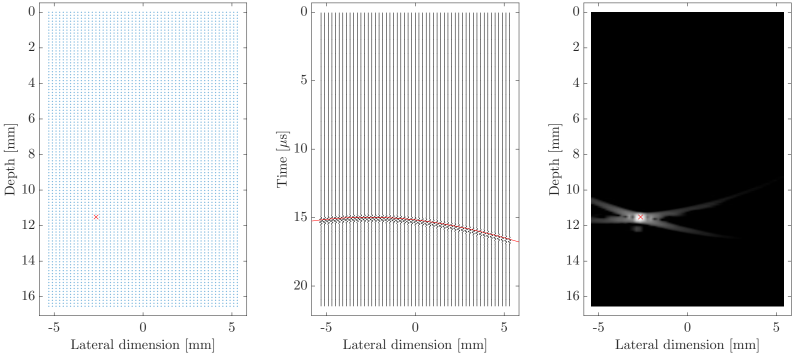


(a) (b) (c)

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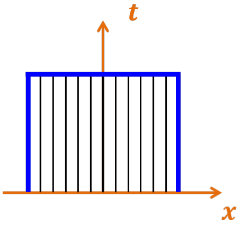


(a) Backscattered echoes

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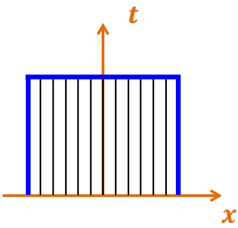


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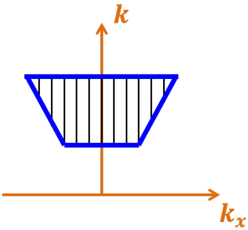


(b) Fourier spectrum

# Ultrafast Ultrasound Imaging
## Fourier-based approaches

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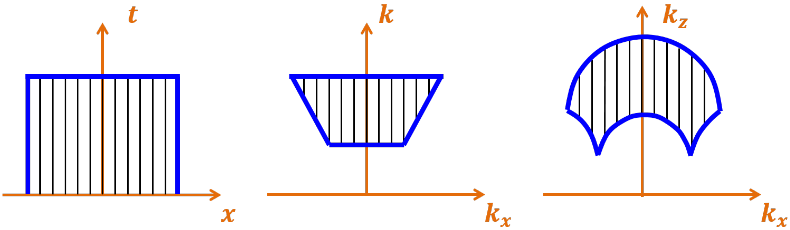


(a) Backscattered echoes

(b) Fourier spectrum

(c) Remapped Fourier spectrum

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# Ultrafast Ultrasound Imaging

## Main drawbacks of the classical approaches

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Classical methods are **filtered-backprojection** solutions of the inverse problem

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**Image reconstruction** An analysis-based problem is solved ($\ell_1$-minimization)

$$\min_{\bar{\boldsymbol{s}} \in \mathbb{C}^N} \|\Psi^H \bar{\boldsymbol{s}}\|_1 \text{ subject to } \|\boldsymbol{r} - \mathrm{H}\bar{\boldsymbol{s}}\|_2 \leq \epsilon$$

Spatial-based approach

## Spatial-based approach

- In the continuous domain:

$$r(x_i, t) = \iint_{(x,z) \in \Omega(x_i,t)} s(x, z)\, dxdz$$

$\Omega(x_i, t)$ propagation delay curve

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### Fourier-based approach [Besson *et al.*, 2016]

# Sparse regularization for ultrasound imaging
## Measurement model

### Spatial-based approach

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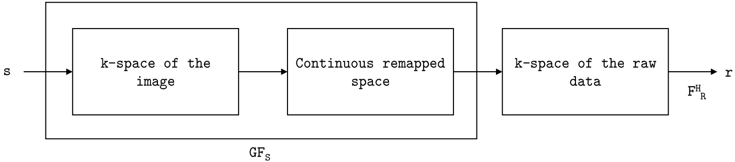
### Fourier-based approach [Besson *et al.*, 2016]

$$\boldsymbol{r} = \mathrm{F_R^H G F_S}\boldsymbol{s} + \boldsymbol{n}$$



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# Sparse regularization for ultrasound imaging

## Sparsifying model and reconstruction algorithm

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### Image reconstruction problem

- Analysis-based problem:

$$\min_{\bar{\boldsymbol{s}} \in \mathbb{C}^N} \|\Psi^H \bar{\boldsymbol{s}}\|_1 \text{ subject to } \|\boldsymbol{r} - \mathrm{H}\bar{\boldsymbol{s}}\|_2 \leq \epsilon$$

  with $\Psi$ the SA model and H Spatial-based or Fourier-based reconstruction model
- Alternating Direction of Minimizers (ADMM) used to solve the problem

# Outline

Problem & Objective

# Applications of the proposed approach
## Sparse regularization for image quality enhancement

### Problem & Objective

- Image reconstruction poses an ill-posed problem
- Discretization of the measurement models leads to inaccuracies [Carrillo *et al.*, 2015]
- Use sparse regularization to enhance image quality

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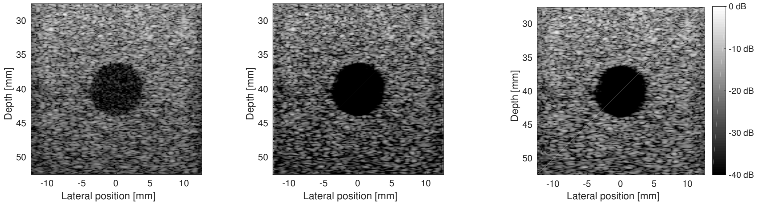
### Numerical study of the contrast

- 8-mm anechoic inclusion positioned at 4 cm embedded in a medium with high density of scatterers
- Insonification with 1 plane wave
- Reconstruction with classical approaches and sparse regularization

# Applications of the proposed approach
## Sparse regularization for image quality enhancement

### B-mode images of the anechoic inclusion




(a) Classical
CR = 5.59 dB

(b) Sparse spatial
CR = 9.10 dB

(c) Sparse Fourier
CR = 9.62 dB

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## Compressed beamforming

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# Applications of the proposed approach
## Compressed beamforming

### Problem & Objective

- Acquisition with few transducer elements
- Creation of a new measurement vector: $\boldsymbol{r_u} = \mathrm{P}\boldsymbol{r}$ with $\mathrm{P} \in \mathbb{R}^{R \times M}$ and $R \ll M$
- Retrieve high quality images from this undersampled measurement vector [Besson *et al.*, 2016]

$$\min_{\bar{\boldsymbol{s}} \in \mathbb{C}^N} \|\Psi^H \bar{\boldsymbol{s}}\|_1 \text{ subject to } \|\boldsymbol{r_u} - \mathrm{H_u}\bar{\boldsymbol{s}}\|_2 \leq \epsilon$$

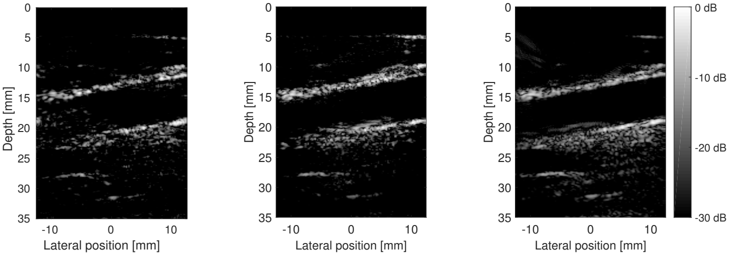
with $\mathrm{H_u} = \mathrm{PH} \in \mathbb{R}^{P \times N}$

### Experiments

- Acquisition of an in-vivo carotid with a Verasonics US system
- 1 PW insonification with a 128 transducer-elements probe
- Random selection of 30 transducer elements in receive
- Reconstruction with the sparse regularization approach

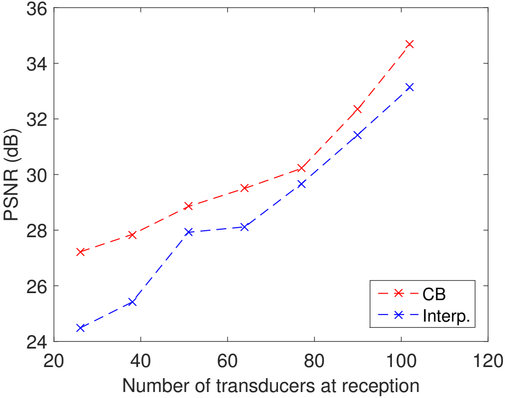
# Applications of the proposed approach
## Compressed beamforming

B-mode images obtained with 30 transducer elements




(a) Fourier (b) Spatial (c) Reference

Peak-signal-to-noise-ratio against number of transducers



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Framework for sparse regularization in US imaging

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Perspectives

- Extension to 3D imaging
- More robust reconstruction algorithms which take into account spatially variable point spread functions

# Thank you!