Note on the relation between profile scalelengths and λ's

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Abstract

The calculation of the inverse scalelengths R/L_T, R/L_n may depend on the choice of the radial variable, depending on the definition of the scalelength. The value of λ_T and λ_n, as defined in [O. Sauter *et al*, Phys. Plasmas **21** (2014) 055906], do depend on the radial variable used in the exponential. We discuss the various options and the typical values obtained with C-Mod and TCV equilibria.

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I. DISCUSSION

The gradient scalelength has been defined in various forms which are often not constant on a flux surface. To avoid this, a good definition for the average inverse scalelength of the profile $T(\rho)$ yields:

$$\frac{R_0}{L_T} = -\frac{R_0}{T}\frac{dT}{d\rho} < |\nabla\rho| >, \tag{1}$$

which does not depend on the choice of the radial variable ρ. R_0 is used to normalize the scalelength and $< . >$ stands for the flux surface average. In ref. [1], we have proposed to use λ_T as the inverse scalelength parameter with respect to $\rho_{\rm V} = \sqrt{V/V_a}$, where V_a is the volume at the last-closed flux surface. Thus the core profile, between the sawtooth inversion radius and the pedestal region (in both L- and H-mode), can be expressed with the exponential:

$$T(\rho_{\rm V}) = T_0\ e^{-\lambda_T\ (\rho_{\rm V}-\rho_{inv})} \qquad \text{for } \rho_{inv} < \rho_{\rm V} > \rho_{ped}. \tag{2}$$

This yields the following relation:

$$\lambda_T = -\frac{1}{T}\frac{dT}{d\rho_{\rm V}}. \tag{3}$$

We can introduce this into Eq.(1) and we obtain:

$$\frac{R_0}{L_T} = -\frac{R_0}{T}\frac{dT}{d\rho_{\rm V}} < |\nabla\rho_{\rm V}| >= \frac{R_0}{a}\ \lambda_T\ [a < |\nabla\rho_{\rm V}| >], \tag{4}$$

where a value of 0.85 was mentioned for TCV in Ref. [1] for $a < |\nabla\rho_{\rm V}| >$, with a the plasma minor radius. We want here to compare this term with C-Mod equilibria, compare with using $\rho_{\phi N} = \sqrt{\Phi/\Phi_a}$ (normalized "toroidal" radial coordinate) and comment its general expected value.

Let us start by using ρ_ϕ, from $\sqrt{\Phi/(\pi B_0)}$ where Φ is the toroidal flux. We expect $< |\nabla\rho_\phi| >= 1$ for a large aspect ratio case and $\rho_{\phi,a} = a\sqrt{\kappa}$ the edge value, with κ the plasma elongation. Therefore we should have:

$$a < |\nabla\rho_{\phi N}| >= a < |\nabla\rho_\phi| > \frac{1}{\rho_{\phi,a}} = \frac{1}{\sqrt{\kappa}}, \tag{5}$$

which explains the value of 0.85 in TCV, assuming $\rho_{\phi N} \approx \rho_{\rm V}$. Both represent well the average configuration space average minor radius and thus do not depend too much on the q profile or the value of β (Shafranov shift). They are almost equivalent as you can see in Fig. 1, where we plot $a < |\nabla\rho_{\rm V}| >$ in solid lines and $a < |\nabla\rho_{\phi N}| >$ in dashed lines for C-Mod (blue) and TCV (red) typical cases. First we see that $a < |\nabla\rho_{\rm V}| >$ is almost constant across the radius, which is why $\rho_{\rm V}$ was used in Ref. [1] and which means that $\frac{R_0}{L_T} \approx \frac{R_0}{a\sqrt{\kappa}}\,\lambda_{T,\rm V}$. Therefore $\lambda_{T,\rm V}$ using $\rho_{\rm V}$ is representative of the inverse scalelength over the whole radius. Using the normalized toroidal radius yields very similar values, with about a 20% variation between the plasma center and the edge, as seen in Fig. 1. Note that κ actually also varies over the minor radius, however not so much inside $\rho \approx 0.8$ and $1/\sqrt{\kappa}$ even less, as seen in Fig. 2 which also shows that the relative variations is the same in these C-Mod and TCV examples.

To summarize, we first note that $a < |\nabla\rho_{\rm V}| >$ is almost constant across the minor radius. Nevertheless, one can use $\rho_{\phi N}$ instead of $\rho_{\rm V}$ in the profile representation and the definition of λ, since $a < |\nabla\rho_{\phi N}| >$ does not vary much and its average value is near the value of $a < |\nabla\rho_{\rm V}| >$. To compare the inverse scalelength of different tokamaks and scenarios, if the plasma elongation is similar, one needs to simply compare the values of:

$$\frac{R_0}{L_T} \approx \frac{R_0}{a}\,\lambda_T. \tag{6}$$

In the cases considered here, we have $R_0/a = 3$ for C-Mod and 3.6 for TCV, thus one would expect 20% higher values for λ's in C-Mod than in TCV if it is R_0/L_T which is significant in the core, or not if it is the value of λ_T.

References

[1] O. Sauter *et al*, Phys. Plasmas **21** (2014) 055906.

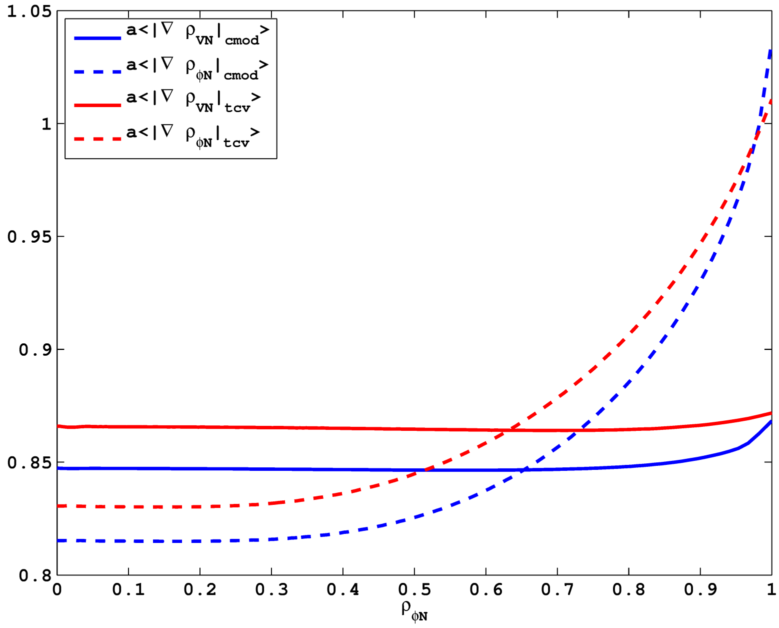

FIG. 1: $a < |\nabla \rho_\mathrm{V}| >$ in solid lines and $a < |\nabla \rho_{\phi N}| >$ in dashed lines for C-Mod, shot 1120620027 (blue), and TCV, shot 48050 (red).

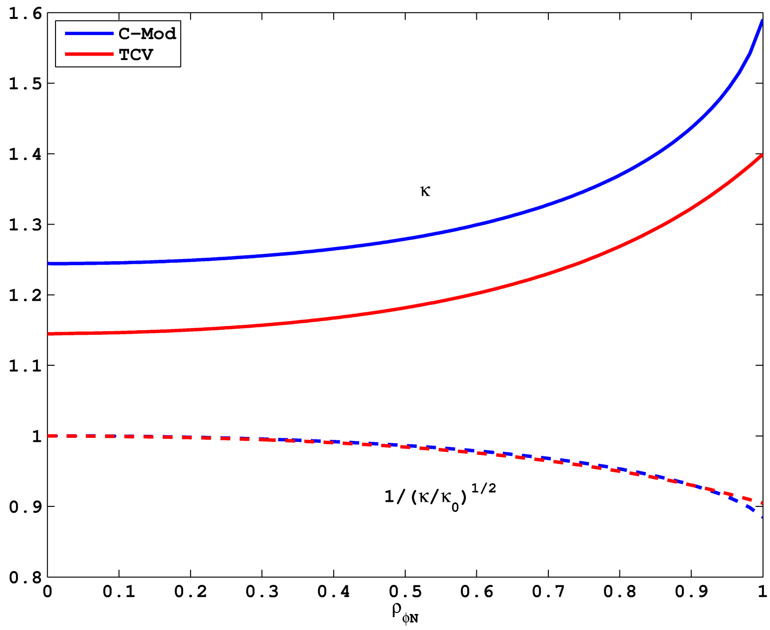

FIG. 2: $\kappa >$ in solid lines and $\frac{1}{\sqrt{\kappa/\kappa_0}}$ in dashed lines for C-Mod (blue) and TCV (red) for the equilibriua considered in Fig. 1.