Simple Thermal Noise Estimation of OTA-based Switched-capacitor Filters

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***Abstract*—This paper presents a new analytical method to simply estimate the integrated thermal noise in switched-capacitor filters (SCF). It is shown how the Bode theorem, which is theoretically valid only for continuous-time passive filters, can be extended to also estimate the noise in active SCF built with operational transconductance amplifiers (OTAs).**

I. Introduction

THE flicker noise due to the amplifiers used in switched-capacitor filters (SCF) can be significantly reduced thanks to the autozero technique [1]. The remaining noise is therefore ultimately dominated by the aliased noise due to sampling of the broadband thermal noise coming from the amplifiers and switches [1], [2]. Although today it is possible to simulate the noise of SCF [3], it remains difficult to have a 1^{st}-order analytical expression to estimate the noise sufficiently accurately, in order to optimize the circuit [4]. This paper presents a new analytical method to simply estimate the integrated thermal noise in SCF. It is shown how the Bode theorem [5], which is theoretically valid only for continuous-time passive filters, can be extended to also estimate the thermal noise in active SCF built with operational transconductance amplifiers (OTAs).

II. Noise Mechanisms in SC filters

Fig. 1 shows a typical integrator, part of a larger SCF and implemented by an OTA having a transconductance G_{mi} and an integrating capacitor C_i. The integrator virtual ground is connected to a switched-capacitor (SC) C_{1i} and a directly coupled capacitor C_{2i}. The noise in the SCF is generated from the noise sources in the OTAs and switches which is then sampled on the SCs and then transferred to the filter output [6]. The continuous-time noise seen at the filter output node during each phase (also called the direct noise) adds to this sampled and transferred noise [6]. The noise sampled on all the SCs connected to the virtual ground of the OTA_i is modelled in Fig. 1 by a noise charge Q_{ni} injected into the virtual ground at the end of each phase Φ. The variance of this noise charge Q_{ni}^2 is then the sum of the variances of the noise charge due to the different noise sources active in each phase Φ (switches and OTAs) and sampled at the end of each phase Φ on each SC connected to the virtual ground of OAT_i

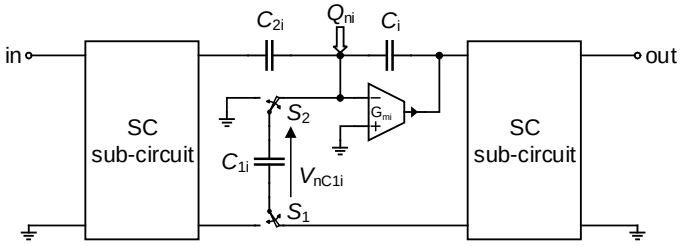


Fig. 1. General case of an SCF circuit that encompass at least one integrator.

$$Q_{ni}^2 = \sum_{\Phi} Q_{ni}^2|_{\Phi} \quad \text{with} \quad Q_{ni}^2|_{\Phi} = \sum_{k} C_{ki}^2 \cdot V_{nC_{ki}}^2|_{\Phi}. \quad (1)$$

The variances of the injected noise charges $Q_{ni}^2|_{\Phi}$ are calculated in each phase from the variances of the voltages $V_{nC_{ki}}^2$ across each switched-capacitor C_{ki} connected to the virtual ground i. The z-transfer function $H_{ni}(z)$ from the injected noise charge Q_{ni} to the output is then evaluated. The variance of the output noise voltage is then given by [7]

$$V_{nout}^2 = \sum_{i} \frac{Q_{ni}^2}{C_i^2} \cdot \sum_{k=0}^{+\infty} h_{nik}^2 + V_{nout,direct}^2, \quad (2)$$

where h_{nik} is the k^{th} term of the impulse response corresponding to the transfer function from the virtual ground i to the output. In addition to the sampled noise, the variance of the continuous-time noise at the filter output $V_{nout,direct}^2$ has to be added. The gain from the noise charge injector to the output can also be calculated directly from the transfer function $H_{ni}(z)$ as [7]

$$\sum_{k=0}^{+\infty} h_{nik}^2 = \frac{1}{j2\pi} \oint_{unit\ circle} H_{ni}(z) \cdot H_{ni}(z^{-1}) \cdot z^{-1} dz. \quad (3)$$

Note that the later integral can be evaluated using the residues theorem according to [7]

$$\sum_{k=0}^{+\infty} h_{nik}^2 = \sum_{k} Res\left\{H_{ni}(z) \cdot H_{ni}(z^{-1}) \cdot z^{-1}\right\}. \quad (4)$$

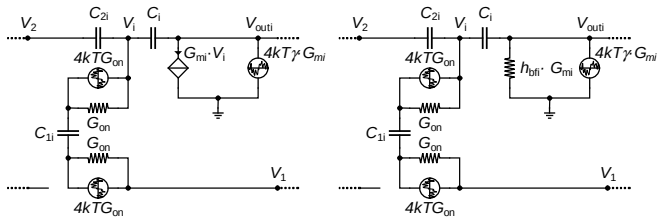


Fig. 2. a) Equivalent linear schematic of the integrator of Fig. 1 during charge transfer phase Φ_2. b) Simplified equivalent linear schematic.

III. Simplified Estimation of the Thermal Noise Charge Variance

Calculating the thermal noise voltage variances $V^2_{nC_{ki}}$ across each switched-capacitor C_{ki} connected to each virtual ground i can be complicated in most SCF circuits. In this Section, a new simple method is presented which extends the Bode theorem to the case of active SCF with OTAs.

Replacing all the MOS switches and OTAs in Fig. 1 by their linear equivalent circuit including their thermal noise sources leads to the schematic shown in Fig. 2a, where G_{on} is the on-conductance of the various switches which are assumed to be equal. The voltage V_i at the virtual ground node i is a linear combination of the output voltage of OTA_i V_{outi} and the output voltages of the other OTAs coupled to node i

$$V_i = h_{fbi}(\omega) \cdot V_{outi} + \beta_1(\omega) \cdot V_1 + \beta_2(\omega) \cdot V_2. \qquad (5)$$

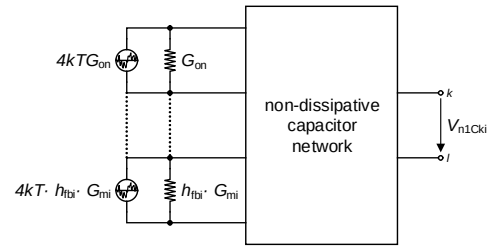
The transfer functions $\beta_1(\omega)$ and $\beta_2(\omega)$ are proportional to $\alpha_1 = \frac{C_{1i}}{C_i}$ and $\alpha_2 = \frac{C_{2i}}{C_i}$, respectively. Assuming that the filter cut-off frequency is much lower than the sampling frequency implies that $\alpha_1, \alpha_2 \ll 1$ and hence $\beta_1(\omega), \beta_2(\omega) \ll 1$. Consequently, the voltage V_i representing the OTA_i input is essentially a function of the OTA output voltage V_{outi}

$$V_i \cong h_{fbi}(0) \cdot V_{outi}. \qquad (6)$$

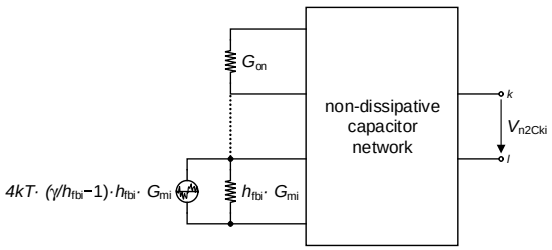
The voltage-controlled current source (VCCS) in Fig. 2a can then be replaced by a conductance of value $h_{fbi} \cdot G_{mi}$ as depicted in Fig. 2b. Since the OTA thermal noise source has a power spectral density (PSD) proportional to G_{mi}, the equivalent circuit can actually be considered as passive. The noisy resistors of the switches have a noise temperature T whereas the noisy resistor of the equivalent conductance of the VCCS can be considered to have a temperature $\frac{\gamma T}{h_{fbi}}$, where γ is the noise excess factor of the OTA_i. Therefore, the noise voltage $V^2_{nC_{ki}}$, across capacitor C_{ki}, can be decomposed into the sum of two terms

$$V^2_{nC_{ki}} = V^2_{n1C_{ki}} + V^2_{n2C_{ki}}, \qquad (7)$$

where $V^2_{n1C_{ki}}$ corresponds to the noise observed when all the conductances have the same noise temperature as shown in Fig. 3a, whereas $V^2_{n2C_{ki}}$ accounts for the excess noise present in the conductances $\gamma_i \cdot h_{fbi} \cdot G_{mi}$ representing the OTAs as shown in Fig. 3b. Note that the schematic of Fig. 3b can



(a) All the conductances have the same noise temperature



(b) Excess noise temperature present in the conductances representing the OTAs

Fig. 3. Simplified schematic of an OTA based SCF.

be further simplified considering that $G_{on} \gg h_{fbi} \cdot G_{mi}$ so that the on-conductances can be replaced by short-circuits without significant loss in accuracy. Therefore, $V^2_{n1C_{ki}}$ can be calculated by applying the Bode theorem on the simplified schematics of Fig. 3a according to [5]

$$V^2_{n1C_{ki}} = kT \cdot \left(\frac{1}{C_{\infty ki}} - \frac{1}{C_{0ki}} \right), \qquad (8)$$

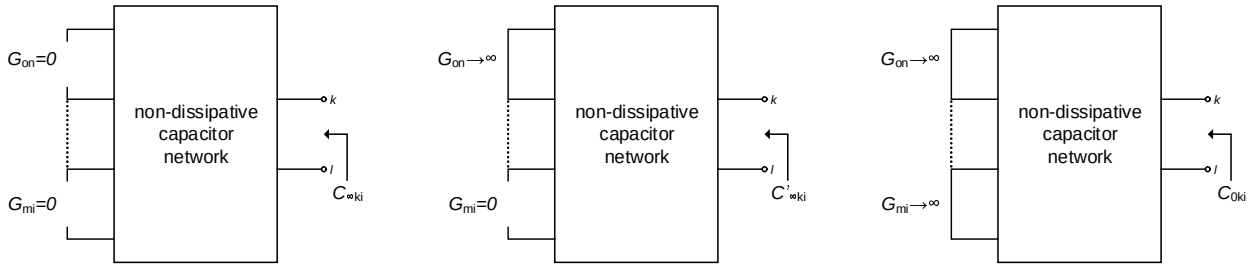
where $C_{\infty ki}$ is the capacitance as seen between the nodes k and l when all switches and OTAs of the SC circuit are removed and C_{0ki} is the capacitance as seen between the nodes k and l when switches that are closed during the clock phase in consideration are replaced by short-circuits and all OTAs of the SC circuit have their output shorted to ground. In the same way, $V^2_{n2C_{ki}}$ can be calculated as

$$V^2_{n2C_{ki}} = kT \cdot \left(\frac{\gamma}{h_{fbi}} - 1 \right) \cdot \left(\frac{1}{C'_{\infty ki}} - \frac{1}{C_{0ki}} \right), \qquad (9)$$

where $C'_{\infty ki}$ is the capacitance as seen between the nodes k and l when switches that are closed during the clock phase in consideration are replaced by short-circuits and all OTAs of the SC circuit are removed. Note that the thermal noise variances $V^2_{n1C_{ki}}$ and $V^2_{n2C_{ki}}$ are simply obtained by inspection of the different equivalent circuits used to calculate $C_{\infty ki}$, $C'_{\infty ki}$ and C_{0ki} which are only made of connected capacitors. The total noise voltage variance (7) is then given by

$$V^2_{nC_{ki}} = kT \cdot \left[\frac{1}{C_{\infty ki}} - \frac{1}{C'_{\infty ki}} + \frac{\gamma}{h_{fbi}} \cdot \left(\frac{1}{C'_{\infty ki}} - \frac{1}{C_{0ki}} \right) \right] \qquad (10)$$

where $C_{\infty ki}$, $C'_{\infty ki}$ and C_{0ki} are obtained by inspection of the three equivalent circuits depicted in Fig. 4.



(a) All switches and OTAs of the SC circuit are removed

(b) All switches that are closed during the clock phase in consideration are replaced by short-circuits and all OTAs of the SC circuit are removed

(c) All switches that are closed during the clock phase in consideration are replaced by short-circuits and all OTAs of the SC circuit have their output shorted to ground

Fig. 4. Extention of the Bode theorem to OTA based SC filters.

IV. Example: First-order Low Pass SC Filter

In order to better understand the noise calculation method presented above, this Section presents the example of a SC low pass filter as depicted in Fig. 5. The z-transfer function of this low-pass SCF is given by

$$H(z) = \frac{V_{in}(z)}{V_{out}(z)} = \frac{\alpha_1 \cdot z^{-1}}{1 + \alpha_2 - z^{-1}}, \tag{11}$$

where $\alpha_1 = \frac{C_1}{C}$ and $\alpha_2 = \frac{C_2}{C}$. Note that α_1 and α_2 are much smaller than unity in the usual case where the cut-off frequency is much smaller than the sampling frequency. In order to simplify the calculations, it is considered that $\alpha_1 = \alpha_2 = \alpha$. At the end of phase Φ_1 and Φ_2, the noise charges sampled on capacitors C_1 and C_2 are transferred to the integrating capacitor C. This sampled noise charge is modeled by the charge injector Q_n which injects a noise charge at the end of phase Φ_2 that represent the total noise charge injected onto the integrating capacitor C during one cycle (phase Φ_1 + phase Φ_2). The variance of this noise charge Q_n^2 has to be calculated in each phase by evaluating the voltage variance on each switched-capacitor.

A. Noise variance estimation in phase Φ_1

The method presented in the previous section is applied to calculate the noise voltage variances across the SCs connected to the virtual ground C_1 and C_2. Thus $V_{nC_1}^2|_{\Phi_1}$ and $V_{nC_2}^2|_{\Phi_1}$ are calculated using (10). The calculation of capacitors C_∞, C'_∞ and C_0 for the voltages across capacitors C_1 and C_2 at the end of phase Φ_1 is shown in Fig. 6 resulting in

$$V_{nC_1}^2|_{\Phi_1} = kT \cdot \left(\frac{1}{C_1} + 0 - 0\right) = \frac{kT}{C_1}, \tag{12a}$$

$$V_{nC_2}^2|_{\Phi_1} = kT \cdot \left(\frac{1}{C_2} + 0 - 0\right) = \frac{kT}{C_2}. \tag{12b}$$

The noise charge variance injected at the virtual ground at the end of phase Φ_1 can then be expressed as

$$Q_n^2|_{\Phi_1} = C_1^2 \cdot V_{nC_1}^2|_{\Phi_1} + C_2^2 \cdot V_{nC_2}^2|_{\Phi_1} = kT \cdot 2\alpha C. \tag{13}$$

B. Noise variance estimation in phase Φ_2

Following the same steps as in the previous Section, the capacitors C_∞, C'_∞ and C_0 for the voltages across capacitors C_1 and C_2 at the end of phase Φ_2 are first calculated using the schematics shown in Fig. 7. The feedback gain in phase Φ_2 is calculated using the equivalent circuit shown in Fig. 7a resulting in

$$h_{fb} = \frac{V}{V_{out}} = \frac{1+\alpha}{1+2\alpha}. \tag{14}$$

By applying the method and assuming $\alpha \ll 1$, we find

$$\begin{aligned} V_{nC_1}^2|_{\Phi_2} &= \frac{kT}{\alpha C} \cdot \frac{(1+\alpha)\cdot(C_L + \alpha\gamma C)}{(\alpha)^2 C + C_L + \alpha\cdot(C+2C_L)} \\ &\simeq \frac{kT}{\alpha C} \cdot \frac{C_L + \alpha\gamma C}{\alpha C + C_L}, \end{aligned} \tag{15a}$$

$$V_{nC_2}^2|_{\Phi_2} = \frac{kT}{\alpha C} \cdot \frac{1+\alpha+\gamma\alpha^2}{(1+\alpha)^2} \simeq \frac{kT}{\alpha C}. \tag{15b}$$

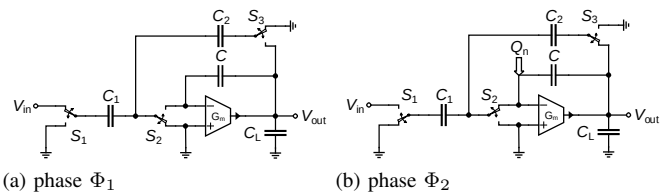
Thus, the variance of the total noise charge injected in the virtual ground at the end of phase Φ_2 is given by

$$Q_n^2|_{\Phi_2} = kT \cdot \alpha C \cdot \left(1 + \frac{\alpha\gamma C + C_L}{\alpha C + C_L(1+2\alpha)}\right). \tag{16}$$

C. Total noise variance estimation

The total noise charge variance injected at the end of every cycle into the virtual ground is then given by

$$Q_n^2 = Q_n^2|_{\Phi_1} + Q_n^2|_{\Phi_2}. \tag{17}$$



(a) phase Φ_1 (b) phase Φ_2

Fig. 5. SC low pass filter.

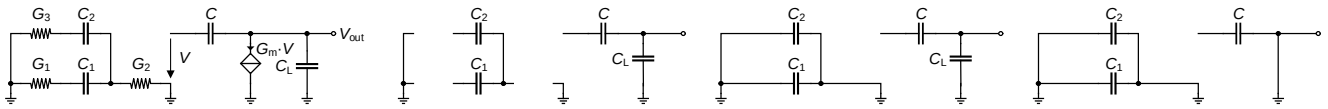


Fig. 6. Equivalent circuit schematics for noise variance calculation in Phase Φ_1.

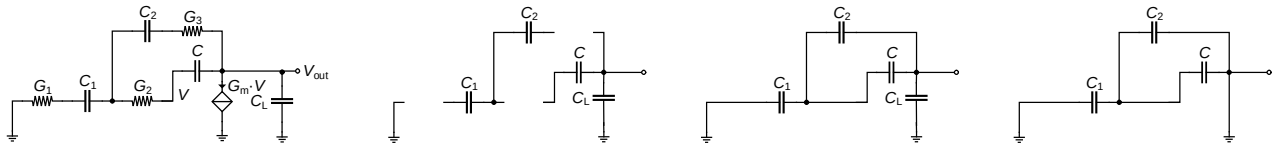


Fig. 7. Equivalent circuit schematics for noise variance calculation in Phase Φ_2.

The noise charge Q_n is then transferred to the integrating capacitor C, generating an output voltage V_{out} calculated using (2) and given by

$$V_{out}(z) = H_n(z) \cdot \frac{Q_n}{C} \text{ with } H_n(z) = \frac{1}{1 + \alpha - z^{-1}}. \quad (18)$$

Thus,

$$Res\left\{H_{ni}(z) \cdot H_{ni}(z^{-1}).z^{-1}\right\} = \frac{1}{\alpha(2+\alpha)} \simeq \frac{1}{2\alpha}. \quad (19)$$

The output noise voltage variance is then calculated based on (3) and (4)

$$V^2_{nout} \simeq \frac{3kT}{2C} + \frac{kT}{2C} \cdot \frac{\alpha\gamma C + C_L}{\alpha C + (1+2\alpha) \cdot C_L} + V^2_{nout,direct}. \quad (20)$$

For the 1^{st}-order low pass filter of this example, the direct noise can be calculated as

$$V^2_{nout,direct} = kT \cdot \frac{\gamma}{C_L + \alpha C}. \quad (21)$$

In order to validate this result, noise simulations are performed on the circuit of Fig. 5 for $\alpha = 0.1$, $C = 5pF$ and $\frac{G_m}{G_{on}} = 0.1$ where G_m is the OTA transconductance and G_{on} the on-conductance of switches ($G_{on} = G_1 = G_2 = G_3$ in the schematics of Fig. 6a and 7a). Both ELDO Transient noise and SpectreRF Pnoise simulations were performed. Fig. 8. shows the simulated RMS noise voltage at the output of the SCLPF, for $C_L = C$, together with the calculated noise using (20). It shows an excellent match between both the simulation and the analytical calculations, validating the presented noise estimation methodology.

V. Conclusion

A simple method for the analytical estimation of thermal noise in OTA-based SCF is presented. The proposed method extends the Bode theorem to the case of OTA-based SCF. It allows the sampled noise to be estimated by simple inspection of the SCF circuits. The new method is illustrated with the example of an OTA-based first-order SC low-pass filter. The estimated noise show an excellent matching compared to noise simulations using SpectreRF Pnoise and Eldo Transient-Noise.

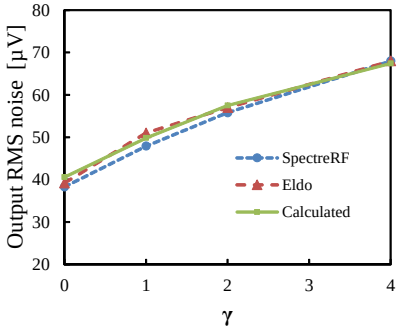


Fig. 8. Simulated output noise with SpectreRF Pnoise and Eldo Trensient noise versus calculated noise in (24).

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