Abstract—We propose and experimentally validate a process to dispatch the operation of a distribution feeder with heterogeneous prosumers according to a trajectory with 5 minutes resolution, called dispatch plan, established the day before the operation. The controllable element is a utility-scale grid-connected battery energy storage system (BESS) integrated with a minimally pervasive monitoring infrastructure. The process consists of two stages: day-ahead, where the dispatch plan is determined by using forecast of the aggregated consumption and local distributed generation (prosumption), and real-time operation, where the mismatch between the actual prosumption realization and dispatch plan is compensated for thanks to adjusting the real power injections of the BESS with model predictive control (MPC). MPC accounts for BESS operational constraints thanks to reduced order dynamic grey-box models identified from online measurements. The experimental validation is performed by using a grid-connected 720 kVA/500 kWh BESS to dispatch the operation of a 20 kV distribution feeder of the EPFL campus with both conventional consumption and distributed photo-voltaic generation.

Index Terms—Battery storage plants, Optimal control, Modeling.

I. INTRODUCTION

The progressive displacement of conventional generation in favor of renewables requires restoring an adequate capacity of regulating power to assure reliable operation of interconnected power systems. An emerging concept to tackle this challenge is achieving the controllability of portions of distribution networks by exploiting controllable distributed generation (DG), flexible demand and storage. This paradigm can be traced in a number of frameworks, such as virtual power plants (VPPs) and grid-tied microgrids, which, in broad terms, consist in operating aggregates of heterogeneous resources with the objective of providing ancillary services to the upper grid layer, e.g., [1]–[3]. In general, solutions based on aggregating the capability of DERs require extended ICT infrastructures and efficient control policies to harvest flexibility until the LV distribution level, see e.g. [4]–[6]. As a matter of fact, these solutions are of difficult integration in the existing grid at the current stage because: (i) they might not offer the same reliability level as conventional generation, (ii) they require a radical change of the operational practices, and (iii) their technical requirements are not met or expensive to implement.

An essential aspect to enable the transition towards a smarter grid is the availability of plug-and-play technologies that can provide grid ancillary services in the current operational and regulatory framework with a reduced set of technical requirements and minimal levels of complexity. An example of research efforts in this direction is given by the proposition of directly coupling storage with stochastic renewable DG in order to achieve dispatchable operation, as e.g. in [7]–[9].

In this paper, we merge the two aforementioned concepts and propose to achieve the dispatchability of distribution feeders with DG by controlling utility-scale battery energy storage systems (BESSs). Specifically, we describe a process to dispatch the operation of a group of prosumers (characterized by both conventional demand and photo-voltaic, PV, generation) according to a scheduled prosumption trajectory at 5 minutes resolution, called dispatch plan, established the day before operation by implementing forecast of the local prosumption\(^1\). During real-time operation, the mismatch between the dispatch plan and prosumption realization is corrected by adjusting the real power injections of the BESS converter with model predictive control (MPC). MPC is designed to track the dispatch plan while obeying to BESS voltage, current and state-of-charge (SOC) constraints, which are modeled thanks to prediction models. Both battery and consumption forecasting models are data-driven (namely identified from experimental measurements).

The proposed process is experimentally validated by dispatching the operation of a 20 kV medium voltage active distribution system of the EPFL campus, called dispatchable feeder, by using a grid-connected Lithium Titanate 720 kVA/500 kWh BESS placed at the root of the feeder. The dispatchable feeder includes office buildings with conventional demand and 95 kWp rooftop PV DG. It relies on a minimally invasive monitoring infrastructure and requires only the power flow at the grid connection point (GCP) and the information from the battery management system (BMS).

\(^1\)We therefore do not perform any intra-day redispatch.
The envisaged benefits of achieving dispatched operation by design of distribution systems are two-fold. At the system level, it allows to reduce the uncertainty associated with prosumption operation, which is known to increase balancing power requirements and system costs, see e.g. [10], [11]. At the local level, the dispatch plan is generated in order to meet the requirements of local distribution systems operation, for example, to perform consumption peak-shaving.

In comparison with methodologies that have been proposed in the literature to achieve the controllability of heterogeneous clusters of prosumers, the proposed process is characterized by a low complexity in terms of metering, computation and communication requirements. Indeed, the principal coordination mechanism is given by the commitment of the operator to track the dispatch plan, without involving complex iterations or real-time communication. Moreover, the two-stage formulation similar to the conventional way of operating the power system (day-ahead planning and intra-day operation) could facilitate the integration in the current grid operational paradigm.

The contributions of this paper are:

- formulation of a BESS-based control strategy to dispatch the operation of a group of heterogeneous stochastic and uncontrollable prosumers according to a trajectory established the day before the beginning of the operation;
- a short-term energy management MPC strategy for the BESS which implements predictive constraints on the BESS current, voltage and SOC while retaining a convex formulation of the underlying optimization problem;
- experimental validation of the proposed method on a real-scale and real-life grid.

The paper is organized as follows: Section II states the problem and, to fix ideas, introduces the experimental setup used for the validation. Sections III and IV respectively describe day-ahead and real-time operation. Section V presents the experimental results, and, finally, Section VI concludes the paper by summarizing the contributions.

II. PRELIMINARIES AND PROBLEM STATEMENT

We consider a distribution network populated by an unknown mix of electric loads, possibly with distributed generation too, and equipped with a BESS. The considered scenario is well exemplified by the adopted experimental configuration, which is depicted in Fig. 1. It includes a group of buildings of the EPFL campus with 350 kW peak consumption (equipped with 95 kWp root-top PV installations) and a grid-connected 720 kVA/500 kWh Lithium Titanate BESS. The BESS bidirectional real power flow is denoted by $B$, while $P$ is the composite power flow as seen at the GCP. The former is the control variable, and its actual realization is available from measurements. The latter is known by information from a phasor measurement unit (PMU) placed at the root of the feeder [12]. The aggregated feeder demand is denoted by $L$, and, by neglecting grid losses, it is estimated as $L = P - B$.

![Diagram of dispatchable feeder](image)

In general, $L$ consists of both demand and generation and, in the following, it will be simply denoted as prosumption. For both buildings and BESS, we use the passive sign notation, namely positive values denote consumption, while negative denote injections towards the GCP. At the current stage, we assume that the local distribution feeder has ample capacity and characteristics always to operate within its voltage and lines ampacities technical constraints.

The problem consists in dispatching the feeder such that the composite power transit at the GCP follows a power consumption sequence with 5 minutes resolution, called dispatch plan, which is established the day before the beginning of the operation. Similarly to the conventional way of planning power system operation, the control strategies consists in a two-stage process:

- **Day-ahead operation.** The dispatchable feeder operator determines the dispatch plan based on forecast on the prosumption and accounting also for the energy necessary to restore an adequate BESS state-of-energy (SOE) to establish a minimum level of up/down regulation capacity. The dispatch plan is a sequence of power consumption values with 5 minutes resolution that the feeder should follow during the next day of operation. The day-ahead procedure is repeated every day. At the current stage, we arbitrarily choose to perform it one hour before the beginning of the real-time operation. At clock-time 00:00 UTC of the next day, the dispatch plan comes into effect and real-time operation begins. This phase is detailed in Section III.

- **Real-time operation.** This phase starts at clock-time 00:00 UTC of the day of operation, and lasts for the next 24 hours period. The dispatchable feeder operator adjusts the BESS power injection $B$ to compensate for the mismatch between the dispatch plan and the power prosumption realization, which are likely to occur due to forecasting errors. This is accomplished by applying model predictive control (MPC), accounting for the dynamic behavior of the BESS voltage and SOC.
thanks to dynamic grey-box models identified from BESS measurements, and including short-term forecasts of the prosumption. This procedure is detailed in Section IV. The operation sequence is also sketched in the timeline of Fig. 2, which shows the interactions between the load balance responsible (as for example described in [13], the dispatchable feeder operator (which implement the proposed control strategy) and the BESS.

The choice of the 5 minute dispatch interval is according to the envisaged trend for real-time electricity markets. Although not specifically discussed in this work, this formulation potentially allows for day-ahead scheduling considering also dynamic electricity prices, in a similar way as done in [14]–[16].

III. DAY-AHEAD PROBLEM

The objective is to determine the dispatch plan, which is the power consumption profile at 5 minutes resolution that the feeder should follow during the next day operation. The dispatch plan is denoted by the sequence \( \hat{P}_i, i = 0, 1, \ldots, N-1 \) where the index \( i \) represents the 5-minute interval of the day of operation, and \( N = 288 \) is the number of 5-minute intervals in 24 hours. It is defined as:

\[
\hat{P}_i = \hat{L}_i + F^o_i, \quad i = 0, \ldots, N-1, \tag{1}
\]

where \( \hat{L}_0, \ldots, \hat{L}_{N-1} \) are prosumption point predictions, determined as described in III-A, and the sequence \( F^o_0, \ldots, F^o_{N-1} \) is called offset profile (the notation \( ^o \) means “optimal” and denotes the output of an optimization problem, as detailed later in this section). The latter accounts for the amount of BESS energy necessary to restore an adequate level of flexibility. For example, at the end of each day of operation, the BESS residual charge might be close to the upper or lower bounds; the offset plan biases the dispatch plan such that the BESS will discharge or charge, therefore re-establishing a suitable energy level to compensate for the mismatch between the dispatch plan and the prosumption realization. The offset plan is computed by applying a robust optimization problem, as described in III-B. As similarly envisaged in [9], scheduling the BESS energy demand in the dispatch plan to manage its residual charge intrinsically enables continuous-time operation. Especially, it avoids the need of assuming an initial state of energy (SOE) level or imposing a dedicated constraint on the final SOE at the end of the day of operation (see for example [17]–[19]).

A. Day-ahead Prosumption Forecasting

Although being a well established methodology for high levels of aggregation, consumption and generation forecasting at local level is a relatively unexplored topic. It recently come to prominence especially in connection to large scale deployment of distributed generation in distribution networks, microgrids operation, flexible demand management and thanks to the progressive availability of metering infrastructure at the low-voltage level, e.g. [20]–[23]. Local prosumption is characterized by a high volatility due to the reduced spatial smoothing effect, in case for example of PV generation, and prominence of isolated stochastic events, such as induction motors inrushes due to the insertion of pumps or elevators. Because of these reasons, existing forecasting methodologies developed considering high levels of aggregation, e.g. [24], might fail in forecasting low populated aggregates of prosumers.

For the proposed application, we apply a fairly simple non-parametric forecasting strategy that is able to capture the prosumption daily and seasonal components with a decent level of accuracy. As it will be detailed in the following, it consists in selecting and averaging a number of historical power consumption sequences that are relevant for the period to predict according to the value of certain prosumption data features. It is important to note that the described dispatch process is independent of the selected forecasting method. In particular, in this work, we do not claim specific contributions in the context of prosumption forecasting, and a more exhaustive assessment of the performance of the proposed forecasting tool is postponed to future works.

1) Training data set: We consider a series of historical power prosumption measurements collected at the GCP at 5 minutes resolution organized in daily sequences of length \( \mathcal{N} \) (the bold typeface denotes a sequence of scalars obtained by stacking the time evolving value of the referenced variable):

\[
l_y, d = [l_{y,d,0}, l_{y,d,1}, \ldots, l_{y,d,N-1}] \tag{2}
\]

where \( y \) and \( d \) respectively denote the calendar year and calendar day-of-year of the observation. The observations at 5 minutes resolution are obtained by average downsampling the original historical time series, which was with 20 milliseconds resolution. Also, historical measurements of the daily global horizontal radiation (GHI) over a 24 hours period (source MeteoSwiss, metoestation “Lausanne freiland”, GPS coordinates 6°38’56’’ 46°33’33’’).

2) Forecast computation: Assuming being on a given day, the objective is to determine the prosumption forecast at 5 minutes resolution for the time interval from hour 0 UTC (Coordinated Universal Time) to 24 UTC of the incoming day, called target day. The information associated with the
target day are the calendar day-of-year $d^{*}$, calendar year $y^{*}$ and the forecasted global horizontal radiation $R^{*}$. The last information is obtained by applying the model in [25] starting from forecast of the cloud coverage (obtained for Lausanne by using NorwayMet services). The cloud coverage to GHI conversion model requires the clear-sky irradiance, which is computed for Lausanne by using the clear-sky model implementation available in the open source geographical information system GRASS [26], [27], which allows accounting for the topological shading induced by geographical features on the horizon.

The first step to compute the prosumption forecast is to select from the historical data a number of sequences $l_{y,d}$ with similar features as those of the target day. This is done by applying the following heuristic:

- A set $\Omega_A$ is composed by selecting among historical observations the couples $(y,d)$ which refer to calendar non-working days (weekends and bank holidays) if $(y^{*},d^{*})$ correspond to a non-working day, and vice-versa.
- A set $\Omega_B$ is obtained by selecting the first 10 couples $(y,d)$ in $\Omega_A$ chosen in increasing order of distance in time with respect to the target day. The distance in time is evaluated as $\sqrt{(y - y^{*})^2 + (d - d^{*})^2}$.
- A set $\Omega_C$ is obtained by selecting the first 5 couples $(y,d)$ in $\Omega_B$ chosen in increasing order of distance between the respective observed cumulated irradiance $R_{y,d}$ and the target radiance $R^{*}$, evaluated as $|R^{*} - R_{y,d}|$.

Summarizing, the set $\Omega_C$ is composed of 5 couples of indexes $(y,d)$ which are i) closest in time to the target-day, ii) same kind as the target day, and iii) closest in amount of radiation to the GHI forecast for the target-day.

Finally, the couples in $\Omega_C$ are used to select as many sequences of historical prosumption measurements (2), which are regarded to as potential realizations of the next day prosumption. In particular, the set with the estimates of the prosumption realization is given as:

$$\mathcal{L}_i = \{l_{y,d}, \forall (d,y) \in \Omega_C\},$$

for each time interval $i = 0,1,2,\ldots,N-1$. At the current stage, realizations from the uncertainty sets are assumed independent in time, in other words, time correlation is not modeled. The prosumption point predictions $\hat{L}_0,\ldots,\hat{L}_{N-1}$ are obtained by averaging the elements in the respective estimates set:

$$\hat{L}_i = \frac{1}{|\mathcal{L}_i|} \sum_{l \in \mathcal{L}_i} l, \quad i = 0,1,\ldots,N-1,$$

where $|\mathcal{L}_i|$ denotes the number of elements in $\mathcal{L}_i$ (set cardinality). It is to mention that the outdoor temperature is not included as an influential variable in the selection process of the historical profiles: this is because the considered group of buildings are not equipped with electric space heating. Moreover, the dependency between PV generation and temperature is neglected at this stage.

**B. Determination of the Offset Profile**

The BESS injection required during real-time operation to compensate the mismatch between the dispatch plan and the prosumption realization $L_i$ is given as:

$$B_i = \hat{P}_i - L_i = \hat{L}_i + F^i - L_i, \quad i = 0,\ldots,N-1 \quad (5)$$

where (1) has been used in the last equality to render $\hat{P}_i$. Since $L_i$ is unknown at the current time, we model it as a random realization $l_i$ from the uncertainty set $\mathcal{L}_i$. Therefore from (5), an estimate $\hat{B}_i$ of the BESS compensation action at time interval $i$ is:

$$\hat{B}_i = F^i + \hat{L}_i - l_i, \quad l_i \in \mathcal{L}_i. \quad (6)$$

In this phase, we use the notion of BESS SOE, the evolution of which can be conveniently formulated as a function of the BESS power. The evolution of the BESS SOE as a function of the charge (discharge) positive (negative) power flow $\hat{B}_i$ is modeled by:

$$\text{SOE}_{i+1} = \text{SOE}_i + \beta(\hat{B}_i) \cdot \hat{B}_i$$

where $\beta(\hat{B}_i)$ is the charge/discharge efficiency

$$\beta(\hat{B}_i) = \begin{cases} 
\beta^+ = T_s/3600 \cdot \eta, & \hat{B}_i \geq 0, \\
\beta^- = T_s/3600 \cdot 1/\eta, & \hat{B}_i < 0,
\end{cases} \quad (8)$$

$T_s = 300$ is the length of the discretization step, and $\eta = 0.96$ is the conversion efficiency assessed from experimental measurements considering a BESS charge/discharge rate less than C/5. This discharge rate has been selected since it corresponds to the typical values observed during the experiments.

The lowest possible BESS SOE is given by propagating the dynamic equation (7) from an initial state-of-charge $\text{SOE}_i$ accounting for the greatest lower bound of the BESS power:

$$\text{SOE}_{i+1} = \text{SOE}_i + \beta(\hat{B}_i) \cdot \inf \{\hat{B}_i\}, \quad (9)$$

The BESS power lower bound $\inf \{\hat{B}_i\}$ can be expressed by using (6) and considering the largest possible prosumption stochastic realization $\hat{F}_i^\inf$. It is:

$$\inf \{\hat{B}_i\} = F^i + \hat{L}_i - \hat{l}_i^\inf \quad (10)$$

where

$$\hat{l}_i^\inf = \sup \{l_i\}. \quad (11)$$

Replacing (10) into (9) yields:

$$\text{SOE}_{i+1} = \text{SOE}_i + \beta(\hat{B}_i) \cdot \left( F^i + \hat{L}_i - \hat{l}_i^\inf \right). \quad (12)$$

Conversely, the lowest upper bound of the BESS power and highest possible BESS SOE are:

$$\sup \{\hat{B}_i\} = F^i + \hat{L}_i - \hat{l}_i^\sup \quad (13)$$

$$\text{SOE}_{i+1} = \text{SOE}_i + \beta(\hat{B}_i) \cdot \left( F^i + \hat{L}_i - \hat{l}_i^\sup \right), \quad (14)$$

where

$$\hat{l}_i^\sup = \inf \{l_i\}. \quad (15)$$

Since the dispatch plan is computed one hour before the beginning of the day of operation, the initial BESS state-of-charge $\text{SOE}_0$ is unknown and should be therefore predicted. At the current stage we use a persistent predictor, namely $\text{SOE}_0 = \text{SOC}_{-5 \times 12}$. 
The offset profile \( \mathbf{F}^o = [F^o_0, \ldots, F^o_{N-1}] \) is determined by a robust optimization problem such that the BESS compensation action respects the BESS operational constraints. Specifically, the lowest and highest SOE values in (9) and (14), BESS power flow in (10) and (13) must observe their respective allowed limits.

We compute \( \mathbf{F}^o \) as (where \( s^+_0, \ldots, s^+_N, s^-_0, \ldots, s^-_{N-1}, b^+_0, \ldots, b^+_{N-1}, b^-_0, \ldots, b^-_{N-1} \) four sequences of positive slack variables to relax the optimization hard constraints into soft):

\[
\begin{align*}
\arg\min_{\mathbf{F}^o \in \mathbb{R}^N} & \left\{ \sum_{i=1}^N F^o_i + \alpha \sum_{i=1}^N (s^+_i + s^-_i + b^+_i + b^-_i) \right\} \\
\text{s.t.} & \quad \text{SOE}^+_i + \beta(\cdot) \cdot (F_i + \tilde{L}_i - l^+_i) \geq \text{SOE}_{\text{min}} - s^+_i, \\
& \quad \text{SOE}^-_i + \beta(\cdot) \cdot (F_i + \tilde{L}_i - l^-_i) \leq \text{SOE}_{\text{max}} + s^-_i, \\
& \quad F_i + \tilde{L}_i - l^+_i \geq P_{\text{min}} - b^+_i, \\
& \quad F_i + \tilde{L}_i - l^-_i \leq P_{\text{max}} + b^-_i, \\
& \quad \bar{P}_i \leq P_{\text{max}} \\
\end{align*}
\]

subject to

\[
\begin{align*}
\text{SOE}^+_i + \beta(\cdot) \cdot (F_i + \tilde{L}_i - l^+_i) & \geq \text{SOE}_{\text{min}} - s^+_i, \\
\text{SOE}^-_i + \beta(\cdot) \cdot (F_i + \tilde{L}_i - l^-_i) & \leq \text{SOE}_{\text{max}} + s^-_i, \\
F_i + \tilde{L}_i - l^+_i & \geq P_{\text{min}} - b^+_i, \\
F_i + \tilde{L}_i - l^-_i & \leq P_{\text{max}} + b^-_i, \\
\bar{P}_i & \leq P_{\text{max}}
\end{align*}
\]

for \( i = 0, \ldots, N-1 \). The slack variables are weighted in (16) with a large positive coefficient \( \alpha \) (e.g. \( 1 \times 10^6 \)) such to have a predominant cost when they are violated. They allow the optimization problem to converge also when the corresponding hard constraints are not met, even if at a very large cost. Finally, the inequality constraint (21) is to make sure that the dispatch plan is smaller than a tunable threshold, the value of which can be designed by the modeller to peak shave the real power power transit at the GCP (as it will be shown in the Results Section).

The formulation in (16)-(21) is nonconvex due to the relationship (8). In the real implementation, we solve an equivalent convex formulation of it, which is shown in Appendix A.

It is important to note that the analysis proposed in this paper is limited to the formulation and validation of the dispatch plan and of the associated control problem. In particular, aspects related to the BESS sizing problem are beyond the scope of this paper and will be covered in future works. Nevertheless, the two couples of slack variables \( (s^+, s^-) \) and \( (b^+, b^-) \) give already practical indications on the fact if the BESS power and energy capacities are enough to compensate for the estimated prosumption uncertainty.

IV. REAL-TIME OPERATION

The objective of the real-time operation is to adjust the BESS real power injection in order that the average prosumption value in the current 5-minute interval matches the respective set-point from the dispatch plan. The control objective is formalized in Paragraph IV-A. Beforehand, we introduce the following notation:

- the control strategy is actuated with a sample time of 10 s; the period has been chosen in order to capture early time battery dynamics and assure good control performance.
- The index \( k = 0, 1, 2, \ldots, K-1 \) denotes the rolling 10 seconds time interval of the current day of operation, where \( K = 8640 \) is the number of 10 seconds period in 24 hours;
- At the beginning of each interval \( k \), the real power flow at the GCP, the BESS flow, and the real power of the prosumption realization for the previous interval \( k - 1 \) become known thanks to measurements. They are respectively denoted by \( P_{k-1}, B_{k-1} \), and \( L_{k-1} \). We note that, although the BESS power flow is our control variable, its actual realization might differ due to imprecise actuations of the converter and BESS transformer losses.
- The value of the prosumption set-point to match, denoted by \( P^*_k \), is retrieved from the dispatch plan \( \bar{P}_0, \bar{P}_1, \ldots, \bar{P}_{N-1} \) as:

\[
P^*_k = \hat{P}\lfloor \frac{k}{30} \rfloor,
\]

where \( \lfloor \cdot \rfloor \) denotes the nearest lower integer of the argument, and 30 is the number of 10-second intervals in a 5-minute slot;
- The \( k \)-index of the first 10-second interval for the current 5-minute slot is denoted as \( k_0 \) and is:

\[
k_0 = \left\lfloor \frac{k}{30} \right\rfloor \cdot 30.
\]

For example, at clock-time 00:16 UTC (sixteen minutes past midnight), \( k = 96 \), \( P^*_k = \hat{P}_3 \), and \( k_0 = 90 \). Similarly, the \( k \)-index of the last 10-second interval for the current 5-minute slot is:

\[
k_0 = \frac{k + 30 - 1}{30}.
\]

- The control action consists in actuating the set-point for the BESS converter real power demand (in kilowatt, kW). It is denoted by \( B^*_k \) and is piecewise constant in the interval \( k \). It is determined by model predictive control as described in Paragraph IV-B.

The nomenclature is also exemplified in Fig. 3, which sketches, for example, the situation at the beginning of the time interval \( k = 2 \) (clock-time 00:00:20 UTC): the BESS real power set-points \( B^*_0 \) and \( B^*_1 \) were actuated already in the previous two intervals, \( B^*_2 \) has been just determined using the most recent information (namely, the prosumption realizations \( L_0 \) and \( L_1 \)), and the average prosumption set-point to achieve in the 5-minute interval is given by the first value of the dispatch plan, \( P_0 \).
A. Formulation of the Control Objective

Say being at the beginning of the time interval \( k \), the average composite power flow at the GCP (prosumption + BESS injection) for the current 5 minute slot is given by averaging the available information until \( k \). If \( k \) corresponds to the beginning of a 5 minute period, no information is available yet, and we say that the average composite consumption is zero. Formally, it is:

\[
P_k = \begin{cases} 
0 & k = k \\
\frac{1}{(k - k)} \sum_{j=k}^{k-1} (L_j + B_j) & k > k.
\end{cases}
\] (25)

For example: at clock-time 00:05 UTC (300 seconds since the beginning of the day, \( k = k = 20 \)), the average power consumption is zero; twenty seconds later (\( k = 22, k = 20 \)), two samples are available and averaged to determine \( P_{22} \).

We augment the definition (25) including short-term point predictions of the prosumption \( \tilde{L}_{k|k}, \tilde{L}_{k+1|k}, \ldots, \tilde{L}_{k|k} \), in order to calculate the expected average composite consumption over the whole duration of the current 5 minute slot:

\[
P_k^+ = \frac{1}{30} \left( (k - k) \cdot P_k + \sum_{j=k}^{k} \tilde{L}_{j|k} \right).
\] (26)

At the current stage, we use a persistant predictor\(^5\) to model future realizations, namely \( \tilde{L}_{j|k} = L_{k-1}, j = k, \ldots, k \).

The energy error, expressed in kWh, between the realizations and the dispatch plan in the current 5-minute slot (dispatch plan error):

\[
e_k = \frac{300}{3600} \cdot \left( P_k^* - P_k^+ \right),
\] (27)

where 300 and 3600 are respectively the number of seconds in 5 minutes and 1 hour, \( P_k^* \) is as defined in (22), and \( P_k^+ \) is as in (26).

B. Formulation of Model Predictive Control

In general, MPC consists in determining the control action for a system by solving at each time step an optimization problem with updated information, where the system constraints are enforced by implementing prediction models in the optimization problem. MPC has been widely adopted in several engineering fields, and proposed also in application to power system operation, for example for control of demand response and storage [28], [29]. In this work, MPC is to determine the BESS real power injection \( B_k^o \) in order to achieve zero dispatch plan error (27) by the end of each 5-minute slot while obeying to BESS SOC, DC voltage and current operational limits.

The choice of designing the control problem with MPC rather than flavors of PID-based regulators is because MPC allows to:

- explicitly formulate BESS predictive constraints, thus increasing the awareness of the control action;
- distribute the control action in the available amount of time, therefore achieving smoother operation;
- integrate short-term consumption forecast in the formulation.

The proposed MPC is based on a convex formulation of the underlying optimization problem, which is efficient to solve and implement.

There are two potential candidates for decision variables in the optimization problem underlying the proposed MPC strategy: \( i \), the real power injection on the AC side, and \( ii \), the current on the DC side. We adopt the second solution because it admits a convex equivalent formulation of the optimization problem. We explicitly seek for convexity because convex optimization problems, besides having a single minimum point which is global (provided that the solution exists), can be solved efficiently and reliably.

a) BESS Energy Throughput: We model the BESS energy throughput (in kWh) on the AC bus in the discretized time period from \( k \) to \( k \):

\[
E_{\tau|k}(v_k, \ldots, v_{\tau}, i_k, \ldots, i_{\tau}) = \alpha \sum_{j=k}^{\tau} v_j i_j,
\] (28)

where \( v_k \) and \( i_k \) are the battery DC voltage and current (positive when charging and vice-versa), respectively, and the scale factor \( \alpha = 10/3600 \cdot 0.98 \) is to convert from power (in kW) in the discretized 10 seconds time interval to energy (in kWh), and the coefficient 0.98 models the converter losses (estimated from measurements). The expression above can be expressed as the following matrix product:

\[
E_{\tau|k}(\cdot) = \alpha v^T_{\tau|k} i_{\tau|k},
\] (29)

where the bold notation denotes sequences obtained by stacking in column vectors the realizations in time of the referenced variables, e.g. \( v_{\tau|k} = [v_k, \ldots, v_{\tau}]^T \). Since the voltage on the DC bus is modeled by using a linear electrical circuit (as detailed in IV-C1), its dynamic evolution can be expressed as a linear function of the battery current. By applying the transition matrices \( \phi^v, \psi^v, \phi^i, \psi^i \) (which are developed starting from the voltage discrete state-space model representation as described in Section IV-C1 and Appendix B), the battery voltage can be expressed by:

\[
v_{\tau|k} = \phi^v x_k + \psi^v i_{\tau|k} + \psi^i 1,
\] (30)

where \( x_k \) is the state vector of the voltage model and is known from measurements.

It is to note that the voltage evolution also depends on the BESS SOC. As will be detailed in Section IV-C1, this

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\(^5\)The use of more advanced short-term prediction models will be considered in future works.
dependency is captured by identifying five linear models for as many different SOC ranges. The linear transition matrices in (30) are obtained by selecting the appropriate voltage model according to the current measured BESS SOC (model scheduling). Moreover, we assume that the BESS SOC does not vary significantly in the actuation period, so we can write the voltage as a time invariant linear function of the BESS current, therefore retaining linearity.

Replacing (30) into (29) yields to:

$$E_{F|k}(i_{T|k}) = \alpha \left( \phi^T \alpha_{x} + \psi_i^T i_{T|k} + \psi_i^T 1 \right)^T i_{T|k} = \alpha \left( x_k^T \phi^T \alpha_{x} i_{T|k} + i_{N}^T \psi_i^T \alpha_{x} i_{T|k} + 1^T \psi_i^T \alpha_{x} i_{T|k} \right),$$

(31)

where $1$ denotes the all-ones vector.

We now explore the requirements for $E_{F|k}(\cdot)$ in (31) being a convex function of $i_{T|k}$. Since the nonnegative sum between functions preserves convexity, the convexity of (31) requires that all its three addends are convex. The first and third addend are linear in $i_{T|k}$, therefore convex. The second term is a quadratic form of $i_{T|k}$: the necessary and sufficient condition for its convexity is given by $\psi_i^T$ being semidefinite positive. This hypothesis has been verified numerically for all the BESS identified voltage models, which are presented in Section IV-C1. We conclude that $E_{F|k}$ in (31) is convex in $i_{T|k}$.

b) Formulation and Implementation: The energy tracking problem (given by achieving zero dispatch plan error at the end of the 5-minute slot) could be formulated by minimizing the squared deviation of (31) from (27), such as:

$$\left( E_{F|k}(i_{T|k}) - e_k \right)^2.$$

(32)

However, as known from the functions composition rules [30], the convexity of $p(x) = q(r(x))$ when $r(x)$ is convex requires $q$ convex non decreasing, which is not this case because the squared function of the difference is convex but not nondecreasing on all its domain. Therefore, we reformulate the objective and achieve a convex equivalent formulation of the original problem: it consists in maximizing the BESS DC current while imposing that the convex BESS energy throughput (31) is smaller than or equal to the energy target (27); this achieves the energy throughput to hit the upper bound of the inequality, thus achieving the same value as the target energy. The combination of a linear cost function with an inequality constraint in the form “$f(x) \leq 0$, with $f$ convex in $x$” is a convex optimization problem.

Overall, the formulation of the MPC optimization problem is given by augmenting the just described formulation with $i$ constraints on the BESS current and its rate of change and $ii$ open-loop predictive constraints on BESS voltage and BESS state-of-charge (SOC). It is worth noting that, whereas in the day-ahead problem the constraints on the BESS flexibility were enforced by considering the BESS power and SOE, in this case we consider BESS current, SOC and voltage. This allows for a higher degree of modeling details and is indeed more suitable for the control problem, where the primary objective is determining a control decision that is respectful of BESS operation constraints.

Formally, the decision problem is:

$$i_{T|k}^* = \arg\max_{i \in \mathbb{R}^{k-\tau+1}} \left\{ 1^T i_{T|k} \right\}$$

(33)

subject to

$$\alpha \left( x_k^T \phi^T i_{T|k} + i_N^T \psi_i^T i_{T|k} + 1^T \psi_i^T i_{T|k} \right) \leq e_k$$

(34)

$$1 \cdot i_{\min} \leq i_{T|k} \leq 1 \cdot i_{\max}$$

(35)

$$v_{T|k} = \phi^T v_k + \psi_i^T i_{T|k} + \psi_i^T 1$$

(36)

$$v_{T|k} = \phi^T v_k + \psi_i^T i_{T|k} + \psi_i^T 1$$

(37)

$$1 \cdot v_{\min} \leq v_{T|k} \leq 1 \cdot v_{\max}$$

(38)

$$\text{SOC}_{T|k} = \phi^T \text{SOC}_{k} + \psi_i^T i_{T|k}$$

(39)

$$1 \cdot \text{SOC}_{\min} \leq \text{SOC}_{T|k} \leq 1 \cdot \text{SOC}_{\max}$$

(40)

where $i_{T|k}^* \in \mathbb{R}^{k-\tau+1}$ is the computed control action trajectory, $1$ denotes the all-ones column vector, the multiplication $1 \cdot \phi$ denotes the all-$\phi$ column vector, and the symbol $\leq$ is the component-wise inequality. The cost function (33) consists in maximizing the sum of the equally weighted current values over the shrinking horizon from $k$ to $\bar{k}$. This, in combination with the inequality (34), achieves the BESS energy throughput to be as close as possible to $e_k$, as introduced earlier. The inequalities (35) and (36) respectively enforce minimum and maximum magnitude and rate of change for the BESS current, where ($i_{\min}, i_{\max}$) and ($\Delta i_{\min}, \Delta i_{\max}$) are the respective limits and the matrix $H \in \mathbb{R}^{(k-\tau+1) \times (k-\tau+1)}$ is:

$$H = \begin{bmatrix} 1 & 1 & 0 & \cdots & 0 \\ 0 & 1 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{bmatrix}.$$  

(41)

The equality (37) is the electrical equivalent circuit model of the the BESS according to the notation previously discussed for (30), while (39) imposes BESS voltage limits, which are denoted by the couple ($v_{\min}, v_{\max}$).

Analogously, the equality constraint in (39) is the evolution of the BESS SOC as a linear function of the variable $i_{T|k}$, where $\phi^T \text{SOC}, \psi_i^T \text{SOC}$ are transition matrices calculated as shown in Appendix B using the SOC model described in IV-C2. Finally, (40) imposes the limits on the BESS SOC, which are given by the couple ($\text{SOC}_{\min}, \text{SOC}_{\max}$).

The optimization problem (33)-(40) is convex since the cost function is linear and all the inequality constraints are convex in $i_{T|k}$. It is noteworthy that if the real power injection had instead being adopted as the optimization variable, the problem would not have been convex because the BESS voltage evolution is nonlinear in the power and thus the constraints in (30) would have been nonconvex.

The optimization problem is solved at each time step $k$ (with updated information) on a shrinking horizon from the index $k$ to $\bar{k}$, namely from current time until the end of the current 5-minute slot. At each $k$, the control trajectory for the whole residual horizon is available, however only the first component of the current control law is considered for actuation, which we denote by $i_{T|k}^*$. Since the BESS power flow is controlled by using a real power reference signal, it is required to transform
from $i_k^o$ to the power set-point $B_k^o$. By using the same model applied in (28), it is:

$$B_k^o = v_k \cdot i_k^o. \quad (42)$$

Since the control decision is re-evaluated every 10 seconds, errors on the voltage predictions and short-term consumption forecast which arise in the current actuation period are absorbed in the next cycle, where updated measurements are used. A summary and a diagram to summarize the real-time procedure and MPC operation is proposed at the end of this section, in IV-E.

It is noteworthy that the current implementation does not prevent the tracking problem to fail, for example if the accumulated mismatch between realizations and dispatch plan is larger than the BESS capacity. The discussion concerning mechanisms to manage real-time contingencies and proper storage capacity sizing is postponed to future investigations.

C. Prediction Models

1) BESS Voltage:

a) Model Formulation: Battery voltage models for control application are normally based on electric equivalent circuits, which trade detailed modelling of the electrochemical reactions for increased tractability, see for example [31]–[33]. A voltage model commonly adopted in the literature is the so-called two time constants model (TTC): it consists in a linear second order dynamic model, where the value of the model parameters depend on the battery SOC, temperature and C-rate. In this work, we augment the classical TTC formulation by implementing grey-box modelling, a set of rigorous and systematic methods for data-driven dynamic model identification (see for example [23], [34]). It consists in increasing the complexity of the model (by adding a order, for example) until reaching a satisfactory behavior of the model prediction errors, which at the final stage should be i.i.d. (independent and identically distributed) random noise with zero mean. This is with the main objective of obtaining a prediction model that is tuned to capture accurately the dynamics of the BESS that is used in the experimental validation of the control framework. The adopted modelling procedure initially consists in recording a series of experimental measurements of the BESS DC voltage and current while requiring to the BESS a random power flow according to a PRBS (pseudo random binary signal). The PRBS is a two levels square wave (in this case ±200 kW) with on-off periods of random durations. It is normally adopted in model identification because it is able to excite a wide range of system dynamics. As mentioned earlier, model parameters normally depend on the BESS SOC: in order to capture this dependence, a number of PRBS experimental sessions are performed when the BESS is in different SOC ranges (0-20%, 20-40%, 40-60%, 60-80%, 80-100%). The dependencies between model parameters and both C-rate and temperature are not modeled at this stage, and it will the objective of future investigations. Nevertheless, it is worth noting that the latter dependency is expected to play a minor role because the batteries are installed in a temperature controlled environment at 20°C.

The BESS voltage model is formulated by using the continuous time stochastic state-space model representation:

$$dx = A_c(\theta)x + B_c(\theta)u + K_c(\theta)dw \quad (43)$$

$$v_k = Cx_k + D(\theta)u_k + G(\theta)g_k. \quad (44)$$

where $x \in \mathbb{R}^n$ is the system state vector, $n$ the model order, $A_c$ is the system matrix, $B_c$ input matrix, $K_c$ the input disturbance matrix (used later to implement Kalman filtering for state reconstruction), $C$ output matrix, $D$ feedforward matrix, $G$ is the measurement noise matrix, $g_k$ is i.i.d. standard normal noise, $u$ input vector, $\omega$ a n-dimension standard Wiener process, and $\theta$ is the set of model parameters to estimate. Model parameters are estimated by using the greyest function in MATLAB and minimizing the sum of the model one-step-ahead prediction errors. It was found that the model with best performance (namely, with uncorrelated model residuals) for the considered experimental BESS is a three time constant model ($n = 3$) with structure as shown in Fig. 4.

![Fig. 4. Equivalent circuit model of the BESS. The quantities $v$ and $i$ are respectively the BESS terminal voltage and DC current, while $v_{C1}$, $v_{C2}$, and $v_{C3}$ are the components of the state vector $x$ in (43)-(44).](image)

The model is given by applying Kirchhoff laws to the circuit in Fig. 4 and is:

$$x = \begin{bmatrix} v_{C1} \\ v_{C2} \\ v_{C3} \end{bmatrix}, \quad u_{tk} = \begin{bmatrix} i_{tk} \\ 1 \end{bmatrix}^T \quad (45)$$

$$A_c = \begin{bmatrix} \frac{1}{R_{C1}} & 0 & 0 \\ 0 & \frac{1}{R_{C2}} & 0 \\ 0 & 0 & \frac{1}{R_{C3}} \end{bmatrix}, \quad B_c = \begin{bmatrix} \frac{1}{C_1} \\ \frac{1}{C_2} \\ \frac{1}{C_3} \end{bmatrix} \quad (46)$$

$$K_c = \text{diag}(k_1, k_2, k_3),$$

$$C = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}, \quad D = \begin{bmatrix} R_s & E \end{bmatrix}, \quad G = \sigma_g \quad (47)$$

where $R_1, C_1, R_2, C_2, R_3, C_3, k_1, k_2, k_3, R_s, E, \sigma_g$ is the set of parameters to estimate. The estimated values of the system parameters according to the BESS SOC ranges are summarized in Table I.

Fig. 5 shows the autocorrelation function (ACF) of the time series of model prediction errors (or model residuals), that is the difference between the training set of current/voltage measurements and the respective model predictions. Checking for model residuals correlation is a test widely adopted in systematic methods for data-driven dynamic model identification. As mentioned earlier, model parameters normally depend on the BESS SOC: in order to capture this dependence, a number of PRBS experimental sessions are performed when the BESS is in different SOC ranges (0-20%, 20-40%, 40-60%, 60-80%, 80-100%). The dependencies between model parameters and both C-rate and temperature are not modeled at this stage, and it will the objective of future investigations. Nevertheless, it is worth noting that the latter dependency is expected to play a minor role because the batteries are installed in a temperature controlled environment at 20°C.
Table I
Estimated BESS voltage model parameters for different SOC ranges

<table>
<thead>
<tr>
<th>SOC</th>
<th>0-20%</th>
<th>20-40%</th>
<th>40-50%</th>
<th>60-80%</th>
<th>80-100%</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E$</td>
<td>592.2</td>
<td>625.0</td>
<td>652.9</td>
<td>680.2</td>
<td>733.2</td>
</tr>
<tr>
<td>$R_e$</td>
<td>0.029</td>
<td>0.021</td>
<td>0.015</td>
<td>0.014</td>
<td>0.013</td>
</tr>
<tr>
<td>$R_1$</td>
<td>0.095</td>
<td>0.075</td>
<td>0.090</td>
<td>0.079</td>
<td>0.199</td>
</tr>
<tr>
<td>$C_1$</td>
<td>8930</td>
<td>8909</td>
<td>13996</td>
<td>9499</td>
<td>11234</td>
</tr>
<tr>
<td>$R_2$</td>
<td>0.04</td>
<td>0.009</td>
<td>0.009</td>
<td>0.009</td>
<td>0.010</td>
</tr>
<tr>
<td>$C_2$</td>
<td>909</td>
<td>2139</td>
<td>2482</td>
<td>2190</td>
<td>2505</td>
</tr>
<tr>
<td>$R_3$</td>
<td>2.5e-3</td>
<td>4.9e-5</td>
<td>2.4e-4</td>
<td>6.8e-4</td>
<td>6.0e-4</td>
</tr>
<tr>
<td>$C_3$</td>
<td>544.2</td>
<td>789.0</td>
<td>2959.7</td>
<td>100.2</td>
<td>6177.3</td>
</tr>
<tr>
<td>$k_1$</td>
<td>0.639</td>
<td>0.677</td>
<td>0.617</td>
<td>0.547</td>
<td>0.795</td>
</tr>
<tr>
<td>$k_2$</td>
<td>-5.31</td>
<td>-0.22</td>
<td>-0.36</td>
<td>-0.28</td>
<td>0.077</td>
</tr>
<tr>
<td>$k_3$</td>
<td>5.41</td>
<td>40</td>
<td>40</td>
<td>2.83</td>
<td>-0.24</td>
</tr>
<tr>
<td>$\sigma^2$</td>
<td>-1.31</td>
<td>-0.42</td>
<td>0.3426</td>
<td>3.5784</td>
<td>2.7694</td>
</tr>
</tbody>
</table>

The matrices $A, B, K$ in (49)-(51) and $C, D, G$ in (48) constitute the skeleton of the discrete time state-space of the BESS equivalent circuit model. By using the procedure shown in Appendix B, they are used to generate the transition matrices $\phi^k, \psi^k, \psi^k_1$, which are finally used to implement the MPC voltage constraints in (37).

c) State estimation: The components of the system state in (43) are a modelling abstraction and cannot be measured. However, their value must be known in order to compute the BESS voltage predictions. In turns, the state is estimated from measurements of the battery DC voltage by applying Kalman filtering (KF, [37]). As known, it consists in a two-stage procedure, repeated at each discrete time interval: a prediction step to determine the system evolution (state expected value and covariance matrix $P$) solely on the basis of the knowledge on the system

$$x_{k|k-1} = Ax_{k-1|k-1} + Bu_{k-1}$$

and an update stage, where the predicted state is corrected accounting for the last measurement $v_k$

$$x_{k|k} = x_{k|k-1} + G(y_k - Cx_{k|k-1})$$

$$P_{k|k} = \left(P^{-1}_{k|k-1} + CP_{k|k-1}CT + \sigma_g^2C^T \right)^{-1}$$

where $G$ is the Kalman gain:

$$G = P_{k|k-1}CT \left(CP_{k|k-1}CT + \sigma_g^2C^T \right)^{-1},$$

and $\sigma_g$ is the measurement noise (known from the parameters estimation). KF requires full system observability, that in our case is enforced by construction since the model is estimated from measurements.

2) BESS SOC: The BESS SOC model is:

$$SOC_{k+1} = SOC_k + \frac{10}{3600} \frac{i_k}{C_{nom}}$$

where $C_{nom} = 810 \text{ A h}$ (ampere-hour) is BESS capacity from datasheet information. It is worth noting that we neglect here the dependency of the BESS capacity with respect to the C-rate and cells temperature, see for example [38]. The discretized state-space matrix are easily found from (59) and are $A = I, B = 10/3600/C_{nom}, C = 1, D = 0$. They are used as shown in Appendix B to generate the transition matrices $\phi^{SOC}, \psi^{SOC}$ for the SOC predictive constraints in (39).

D. On the use of BESS models in the day-ahead and real-time stages

Open-loop constraints on BESS operation are enforced in both day-ahead and real-time stages. The main differences between the two implementations are the decision variables, the time resolution at which they are computed (5 minutes and 10 s, respectively), and the fact that the latter problem is re-evaluated at each time interval by incorporating new information from real-time measurements. From the point of view of their practical purpose, BESS models are integrated with the objective of modeling the storage capacity and constraints. However, while in the day-ahead formulation they are to
represent the long-term flexibility, in real-time operation they model short-term operational constraints, therefore enabling the computation of reliable control actions which are respectful of the BESS operational limits. This explains the reason why more accurate BESS models are implemented in the latter stage than in the former, where a high level of details is not needed, especially considering that a more conservative plan for the BESS usage can be achieved by adding back-off terms to SOC and power flow constraints in (17)-(20).

E. Implementation of the real-time strategy

The flow of operation during the real-time strategy is sketched in Figure 6. Real-time operation is implemented as a Matlab script and executed daily on the same computer as the day-ahead strategy. Also, information which become available from real-time measurements (power flow at the GCP, DC current, SOC, AC power flow of the BESS) are acquired and stored in a time series database with 1 second resolution.

![Flow chart showing real-time operation during 24 hours.](image)

As introduced in Section II, experiments are performed on a real-scale real-life MV distribution grid using a grid-connected 720 kVA/500 kWh BESS. The BESS is based on the Lithium titanate technology and is rated for 15 thousands cycles. It consists in 9 parallel racks (each is composed by 15 modules in series, and each module is composed by a 20s3p cell pack), a four quadrant power converter, everything installed in a temperature controlled container. A view of the experimental BESS is shown in Fig. 7.

![An outside (top picture) and inside view (bottom picture) of the 720 kVA/500 kWh Lithium Titanate-based BESS used in this experimental work.](image)

V. EXPERIMENTAL RESULTS AND DISCUSSION

In this section, we show the experimental results obtained by dispatching the operation of the MV feeder described in Section II. We start by describing three contiguous days of operation (denoted by day 0, day 1 and day 2) in order to illustrate the ability of the proposed method to manage the SOC and achieve continuous time operation. Results are shown in figures 8-10. Each figure consists in four main plots a, b, c, d, which respectively shows:

(a) the extreme values of the prosumption uncertainty sets \( \left( t_{i}^{1}, t_{i}^{F} \right) \), the expected prosumption \( \hat{L}_{i}, i = 0, \ldots, N - 1 \), and the offset profile \( F^{o} \). The first two quantities are the outcome of the forecasting tool discussed in III-A, while the last is computed as detailed III-B. The second and third quantities are used to generate the dispatch plan according to (1) \( \hat{P}_{i} \);

(b) the GHI day-ahead forecast, the respective realizations, respective average components (denoted with the dashed lines). As described in III-A, the latter quantity is used to determine the total radiation in one day and select the most suitable prosumption profiles among the historical data;

(c) the dispatch plan \( \hat{P}_{i} \), the power transit at the GCP realizations \( P_{i} \) and the prosumption realizations \( L_{i} \);

(d) the measurements of the BESS SOC, DC current \( i \) and voltage \( v \) and their respective limits.

As visible from Fig. 8d, the BESS starts the operation in the day 0 experiment with a very low SOC. In general (and assuming for one moment unbiased prosumption forecast), this is not a desirable situation because the BESS capacity of absorbing...
forecasting errors is not symmetric. The specific objective of the offset profile is to restore a sufficient BESS SOC in order that enough BESS energy capacity is available along the day to compensate for forecasting errors, which are modeled in the optimization problem in (16)-(21) by the prosumption uncertainty sets (shown in Fig. 8a with the shaded band). In this case, the offset profile is such that the dispatch plan overestimates the prosumption, therefore implicitly achieving the BESS to slowly charge thanks to absorbing the increased level of demand. Figure 8b shows that day ahead GHI forecast are fairly accurate in terms of average components, which is what is considered in the adopted forecasting algorithm. As visible from Fig. 8c, during operation the power flow at the GCP $P$ follows precisely the dispatch plan $\hat{P}$, thus denoting the good tracking performance of the MPC. A more accurate numerical comparison is shown later in this section. Fig. 8d shows the details of BESS operation. As described in the paper, the BESS is controlled in order to compensate for the mismatch between prosumption realization and dispatch plan, and, in this case, it slowly charges along the day, as imposed by the offset plan. The BESS constraints, which are enforced in the MPC are generally respected during operation.

The BESS terminates the day 0 experiment with a SOC above the 50% level. This is reflected on the operation of the next day (experiment day 1, in Fig. 9), which is characterized by a slightly negative offset plan ($F^o$ in Fig. 9a) to restore a lower a BESS SOC, as visible in Fig. 9d. As shown in Fig. 9c, the algorithm achieves to track the dispatch plan successfully. As visible by comparing figures 8d and 9d, the BESS SOC variation in the latter case is smaller than in the former, thus denoting a smaller accumulated forecasting error.

Fig. 10 shows the operation of the experiment on day 3. In this case, the BESS SOC is below the 50% level since the previous day of operation, and a positive offset profile $F^o$ (Fig. 10a) is necessary to raise the SOC. During operation, the algorithm is able to control the BESS in order to track the dispatch plan successfully (Fig. 10c). However, the BESS SOC slowly drifts away from the 50% level during the day, as visible in Fig. 10d, to compensate for a moderate overestimation of the average PV production (as visible in Fig. 10b).

Table II summarizes the tracking performance of the proposed control strategy in the experimental results. For each day experiment, two cases are considered:

- **no dispatch**: this corresponds to nowadays conventional operation, namely when prosumption is not dispatched but simply forecasted (although on a higher aggregation level than as done here). In this context, the tracking error is defined as the difference between the prosumption forecast and the prosumption realization;
- **dispatch**: the operation of the feeder is dispatched as described previously. The error here is given by the difference between the dispatch plan and the power flow realizations at the GCP.

Three metrics are considered: the root mean square error (RMSE) along with its mean and maximum absolute value. As visible from Table II, dispatched operation achieve better figures than the base case. The tracking performance of the MPC are fairly accurate, with an RMS value less than 0.5 kW, which is approximately the 0.2% of the feeder average power consumption (200 kW).

Finally, Fig. 11 shows a fourth day of operation where the parameter $P_{\max}$ in (21) has been set to 210 kW to explicitly shows the capability of performing peak shaving. As visible from Fig. 11c, the magnitude of the dispatch plan is limited to the aforementioned value. Although the prosumption realization has peaks of 250 kW, the power flow at the GCP sticks to the dispatch plan, therefore achieving consumption peak shaving as well as dispatched operation.
Table II

<table>
<thead>
<tr>
<th>Experiment</th>
<th>RMSE</th>
<th>Mean</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Day 0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>no dispatch</td>
<td>19.20</td>
<td>4.68</td>
<td>50.82</td>
</tr>
<tr>
<td>dispatch</td>
<td>0.43</td>
<td>&lt; 0.01</td>
<td>1.54</td>
</tr>
<tr>
<td>Day 1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>no dispatch</td>
<td>18.71</td>
<td>-0.39</td>
<td>72.150</td>
</tr>
<tr>
<td>dispatch</td>
<td>0.25</td>
<td>&lt; 0.01</td>
<td>0.740</td>
</tr>
<tr>
<td>Day 2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>no dispatch</td>
<td>18.06</td>
<td>-4.92</td>
<td>54.45</td>
</tr>
<tr>
<td>dispatch</td>
<td>0.42</td>
<td>&lt; 0.01</td>
<td>1.41</td>
</tr>
</tbody>
</table>

Table III summarizes in numerical form the operation during the experiment on day 3.

Table III

<table>
<thead>
<tr>
<th>Experiment</th>
<th>RMS</th>
<th>Mean</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Day 3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>no dispatch</td>
<td>20.579</td>
<td>-5.451</td>
<td>59.270</td>
</tr>
<tr>
<td>dispatch</td>
<td>0.196</td>
<td>&lt; 0.01</td>
<td>0.570</td>
</tr>
</tbody>
</table>

Finally, Table IV shows statistics on the computation time required to complete the day-ahead and real-time procedures. The day-ahead procedure consists in performing the day-ahead prosumption forecasting (which is the most time consuming operation in this phase because it requires to load a large amount of historical data), computing the offset profile by
solving the convex optimization problem (16)-(21), and determining the dispatch plan with (1). The real-time procedure consists in updating the Kalman filtering for reconstructing the state of the BESS voltage model, computing the short-term forecast of the prosumption and solving the convex optimization problem in (33)-(40) to determine the BESS current trajectory. The computation is performed in Matlab on a Debian Linux machine equipped with an Intel i5 processor and 4Gb of RAM. The computation time of both procedures is quite short. In particular, real-time computation (which is the most time critical among the two because it needs to be recomputed every 10 s to determine the control decision) is solved in less than half a second, considerably less than the 10 s period at which it is actuated.

The statistics in Table IV do not include the communication time, which however is as the typical low latency of local area network (in the range of tens of milliseconds, see for example [12]).

<table>
<thead>
<tr>
<th>Procedure</th>
<th>Mean (second)</th>
<th>Standard Deviation (second)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Day-ahead problem</td>
<td>6.5</td>
<td>0.2</td>
</tr>
<tr>
<td>Real-time problem</td>
<td>0.55</td>
<td>&lt; 0.1</td>
</tr>
</tbody>
</table>

VI. CONCLUSIONS AND PERSPECTIVES

We have discussed a control framework to dispatch the operation of a cluster of stochastic prosumers according to a profile that is established the day before the operation by using a BESS as a controllable element. It consists of a two-stage procedure: day-ahead scheduling and real-time operation. In the former phase, the dispatch plan (namely the prosumption profile at 5 minutes resolution that the feeder should follow during operation) is determined as the sum between the prosumption forecasted profile and the so-called offset profile, which allows restoring an adequate level of BESS flexibility for the next day of operation. In the latter phase, MPC is used to compensate for the mismatch between prosumption realization and dispatch plan.

Prediction models that are applied in the problem (prosumption forecasting models and BESS voltage models) are identified from measurements applying data-driven approaches. The optimization problems which were developed as a part of the control framework are all formulated as convex ones. Therefore, they are characterized by an enhanced level of tractability. In particular, the BESS MPC allows formulating the BESS energy throughput in the objective function while retaining the linearity of the expressions of the BESS SOC, DC voltage and current constraints.

The control framework is validated on a real-life real-scale MV system of the EPFL campus, the so-called dispatchable feeder, which consists in a distribution feeder interfacing a number of office buildings (300 kW peak demand) equipped with roof-top PV generation (90 kWp). The controllable element is a grid-connected 720 kVA-500 kWh Lithium titanate battery. The experimental results show that the proposed control framework is able to dispatch the operation of the group of prosumers along multiple days of operation with a good tracking performance (out of 200 kW average prosumption, the tracking error RMS value is 0.5 kW and the mean is below 0.01 kW).

The proposed strategy can be regarded as a solution for the integration of utility-scale BESSs. It allows to achieve dispatched operation of specific MV nodes by design, while relying on a minimally invasive monitoring and control infrastructure (only the BESS and the power flow at the GCP are required) and with minimal coordination requirements with the upper grid layer because all the complexity is masked behind
the commitment of the operator to track the dispatch plan. Future works concern the improvement of forecasting tools performance for low levels of aggregation, the definition of intra-day mechanisms to manage contingencies (for example, when the storage capacity is saturated) and the establishment of planning criteria to size the storage.

APPENDIX A

CONVEX FORMULATION OF DAY-AHEAD PROBLEM

The formulation in (16)-(21) is nonconvex due to the relationship (8), which involves a nonlinear function of the decision variable. We apply the method used in [39] to obtain a convex equivalent formulation of the original problem. It initially consists in expressing the decision variable as the difference of two positive sequences:

$$F_i = F_i^++F_i^-,$$

and explicitly decouple the positive and negative BESS flow, which are therefore weighted accounting for the respective efficiency factor according to (8). The convex formulation is\(^6\):

$$\min_{F^+, F^- \in \mathbb{R}^N, s^+, s^-, b^+, b^- \in \mathbb{R}^N} \left\{ \sum_{i=1}^{N} \left( F_i^{+2} + F_i^{-2} \right) + \alpha \sum_{i=1}^{N} \left( s_i^+ + s_i^- + b_i^+ + b_i^- \right) \right\},$$

subject to:

$$\text{SOC}_i^+ + \beta^+ F_i^+ + \beta^- \left( F_i^- + \hat{L}_i - l_i^+ \right) \geq \text{SOC}_{\text{min}} - s_i^+,$$

$$\text{SOC}_i^- + \beta^- F_i^- + \beta^+ \left( F_i^+ + \hat{L}_i - l_i^- \right) \leq \text{SOC}_{\text{max}} + s_i^-,$$

$$F_i^+ + F_i^- + \hat{L}_i - l_i^+ \geq B_{\text{min}} - b_i^+,$$

$$F_i^+ + F_i^- + \hat{L}_i - l_i^- \leq B_{\text{max}} + b_i^-,$$

$$F_i^+ \geq 0,$$

$$F_i^- \leq 0,$$

$$\hat{P}_i \leq P_{\text{max}}$$

for \(i = 0, \ldots, N - 1\). The term \(F_i^- + \hat{L}_i - l_i^+\) in (62) is weighted with the coefficient \(B^-\) because all the term are negative: indeed \(F_i^-\) is negative by definition, and \(\hat{L}_i - l_i^+\) is negative by construction since \(\hat{L}_i\) is the sample average of the set \(\mathcal{L}_i\), while \(l_i^+\) corresponds to its largest term. The opposite consideration applies to (63).

APPENDIX B

DERIVATION OF THE TRANSITION MATRICES FOR MPC

We consider a linear dynamic model with the following discrete state-space representation (as those developed in Section IV-C for the BESS SOC and voltage):

$$x_{k+1} = Ax_k + Bu_k$$

$$y_k = Cx_k + Du_k$$

\(^6\)We square the single terms rather than the binomial because the objective is to keep the individual components of (60) as small as possible where \(x_k \in \mathbb{R}^n\) is the state vector at discrete time interval \(k\), \(u_k \in \mathbb{R}\) is the input, \(y \in \mathbb{R}\) is the system output, \(A\) is the \(n \times n\) system matrix, \(B\) is the \(n \times 1\) input matrix, \(C\) is the \(1 \times n\) output matrix, and the scalar \(D\) is the feed-forward gain. For the moment, we consider the case with only one input signal: the extension to multiple inputs is shown at the end of this section. The evolution of the state vector \(x\) from a known initial state \(x_0\) as a function of a given input sequence \(u_0, u_1, \ldots, u_N\) using (69) is

$$x_1 = Ax_0 + Bu_0$$

$$x_2 = Ax_1 + Bu_1 = A(Ax_0 + Bu_0) + Bu_1 = A^2x_0 + ABu_0 + Bu_1$$

$$x_3 = Ax_2 + Bu_2 = A^3x_0 + A^2Bu_0 + ABu_1 + Bu_2.$$

Iterating until the time interval \(N\):

$$x_N = A^N x_0 + A^{N-1} Bu_0 + \cdots + A^0 Bu_{N-1}.$$  

Applying (70) to (71)-(74) yields:

$$\begin{bmatrix} y_0 \\ y_1 \\ y_2 \\ \vdots \\ y_N \end{bmatrix} = \begin{bmatrix} C \\ CA \\ CA^2 \\ \vdots \\ CA^N \end{bmatrix} \begin{bmatrix} x_0 \\ \vdots \\ \vdots \end{bmatrix}$$

$$+ \begin{bmatrix} D \\ CA_0B \\ CA_1B \\ \vdots \end{bmatrix} \begin{bmatrix} 0 \\ \vdots \\ \vdots \end{bmatrix}$$

$$+ \begin{bmatrix} CA_{N-1}B \\ \vdots \end{bmatrix} \begin{bmatrix} u_0 \\ \vdots \\ u_{N-1} \end{bmatrix}$$

which we write in compact form as:

$$y = \phi x_0 + \psi u.$$  

where \(y = y_0, \ldots, y_N\), \(x = x_0, \ldots, x_N\) and \(u = u_0, \ldots, u_N\). For the case of multiple inputs, we add an input \(r = r_0, \ldots, r_N\) to the state-space model (71)-(74):

$$x_{k+1} = Ax_k + Bu_k + Br_k$$

$$y_k = Cx_k + Du_k + Dr_k.$$  

The system output is written by applying the transformation \(\psi\) to \(r\):

$$y = \phi x_0 + \psi u + \psi r.$$  

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