PARTICLE FIELDS OF ICE SLURRIES WITH STRATIFICATIONS AND MELTING

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ABSTRACT

Results on isothermal stratification of ice particles in ice slurry storage tanks are presented. Furthermore - in an extension - the melting of ice particles by heat fluxes across the boundaries of storage vessels is also taken into consideration. A conservation equation for the ice particles, respectively the ice fraction, is then accompanied by a second basic differential equation, the energy conservation equation. It is identical to the continuous-properties model and describes the melting phenomena. To obtain numerical results, a generally accepted physical-properties model of ice slurries had to be generalized. Further main results are the ice fraction distributions of stratification processes.

NOMENCLATURE

Standard

A	surface area
c_p	specific heat at constant pressure
c	concentration, fraction
d	diameter
div	divergence operator
D	diffusion “constant”, resp. function
div	divergence operator
$\vec{j}$	particle flux density
k	thermal conductivity
m	mass
n	particle density per mass unit
q_p	source and sink
$\dot{q}$	heat flux density
r	radius
t	time
T	absolute temperature
U	overall heat transfer coefficient
v	velocity component in axial direction
$\vec{v}$	vectorial velocity
V	volume
W	work (heat) to melt a single ice particle

Greek

α	thermal diffusivity
χ	general variable
Δ	Laplace operator
$\vec{\gamma}$	normal boundary vector
Γ	domain of ice slurry in storage vessel
ϑ	temperature
ρ	density
$\hat{\rho}$	density of ice (particle) component
∇	gradient operator

Indices

A	ambient
i	quantity of generalized phys.-prop. model
I	ice
m	melting
max	maximal
h	quantity of ordinary phys.-prop. model
p	particle (ice particle)

INTRODUCTION

Storage of ice slurries for air-conditioning applications in Japan have been performed by Kozawa and Tanino, e.g. see Ref. [1]. In their work the behaviour of ice rich layers, details of the melting process, etc. were experimentally and numerically determined. The authors, who peformed their work with a strong connection to practical applications, also present results on the ice building-up process in top layers of multiple connected tanks, permeability of water through ice layers, etc. Further work relates to dynamic-type ice storage systems by using supercooled water [2]. Hong, Kawaji, and Goldstein have numerically calculated flow fields in storage tanks with a mixing device [3].

Their work aims toward visualization of flows in storage vessels to obtain knowledge for an optimization of mixing devices. Meili et al. proved that in closed systems over medium time periods no cluster creation of ice slurries in storage tanks with intermittent mixing occurs [4]. Vuarnoz et al. studied experimentally the stratification process in unmixed storage vessels with ice slurry under isothermal conditions [5]. The same scientific group also performed numerical simulations of ice slurries in unmixed storage tanks without an occurrence of melting [6]. The work of the two last references is continued in this article.

ISOTHERMAL STRATIFICATION

In this paper the differential equation describing non-isothermal stratification is only summarized, because a detailed derivation, starting with the ice particle density *n*, is presented in [6]

$$\frac{\partial \hat{\rho}_I}{\partial t} + div\left(\hat{\rho}_I \vec{v}_I\right) - D\rho\Delta c_I - \rho\rho_I V_p q_p = 0 \qquad (1)$$

The weighted density of the ice (particle) component is

$$\hat{\rho}_I = \rho c_I , \qquad (2)$$

in agreement with the theory of multi-component fluids. In equation (1) the diffusion function *D* is assumed to be constant. Because the mean particle size of the ice slurries, which we investigate, is approximately 200 μm, the diffusion is expected to be rather small. Experimentally observed fluctuations of the propagating fronts are expected to be a result of disturbances by displacement movements of ice particles and the additive/water suspension. Furthermore, in a rough approximation, it is assumed that gradients in ice concentration are larger than those in density. The last term in (1) presents the source and sink term, which describes the creation or destruction of ice particles. In [6] frequently applied initial and boundary conditions are defined. In this reference also solutions and graphics of numerical calculations are presented. Now we proceed a step further by adding a numerically favorable model of melting and freezing.

NON-ISOTHERMAL SETTLING OF ICE PARTICLES

The source and sink term, represented by the last term in equation (1), leads to a direct coupling with the thermodynamic description of the system. To construct this coupling the work to destroy one ice particle is derived

$$W = \rho_I V_p h_I , \qquad (3)$$

with the enthalpy density of the ice h_I.

The number of particles destroyed by melting multiplied by the work to destroy one particle is identical to the heating power per mass unit

$$q_p \; W = \frac{1}{\rho} div\dot{q} . \qquad (4)$$

From this equation, and by also substituting the right-hand side of equation (3), it follows that

$$\rho_I V_p q_p = \frac{1}{\rho h_I} div\dot{q} . \qquad (5)$$

This is introduced into (1) to obtain

$$\frac{\partial \hat{\rho}_I}{\partial t} + div\left(\hat{\rho}_I \vec{v}_I\right) - D\rho\Delta c_I - \frac{1}{h_I} div\dot{q} = 0. \qquad (6)$$

Fourier's law connects the heat flux density $\dot{q}$ with the gradient of the temperature field

$$\dot{q} = -k\nabla T . \qquad (7)$$

New is that in cases with non-homogeneous ice particle fields the physical properties are not only defined by the temperature, but also by the ice fraction, e.g. $\rho(\vartheta, c_I)$ (see in the remainder). By substituting (7) into (6) and applying a product differentiation, it follows

$$\frac{\partial \hat{\rho}_I}{\partial t} + div\left(\rho_I \vec{v}_I\right) - D\rho\Delta c_I + \frac{1}{h_I}\left[\left(\nabla k \nabla T\right) + k\Delta T\right] = 0.$$

This can be rewritten to become (8)

$$\frac{\partial \hat{\rho}_I}{\partial t} + div\left(\rho_I \vec{v}_I\right) - D\rho\Delta c_I + \frac{1}{h_I}\left[\frac{dk}{dT}\left(\nabla T\right)^2 + \frac{dk}{dc_I}\left(\nabla c_I \nabla T\right) + k\Delta T\right] = 0. \qquad (9)$$

In the case of melting this equation must be combined with a melting model. We apply the <u>C</u>ontinuous-<u>P</u>roperties <u>M</u>odel (*CPM*), which is generally introduced in Ref. [7] and applied to ice slurries in Ref. [8]. In this new consideration the *CPM* must also refer to the second variable *"ice concentration"* in the physical properties

$$\frac{\partial T}{\partial t} - \alpha\left\{\frac{1}{k}\left[\frac{dk}{dT}\left(\nabla T\right)^2 + \frac{dk}{dc_I}\left(\nabla c_I \nabla T\right)\right] + \Delta T\right\} = 0$$

$$\alpha = \frac{k}{\rho \; c_p} . \qquad (10)$$

The two initial conditions are defined by a homogeneous temperature and ice particle distribution in the fluid domain Γ (for example see in Ref. [6], FIG. 1 and 4a)

$$\chi(t=0,\vec{x}) = \chi_0 = c, \quad \forall \vec{x} \in \Gamma, \quad \chi \in \{T, c_I\} \qquad (11)$$

The boundary condition for the particle flux is

$$\left[\hat{\rho}_I \vec{v}_I (t,\vec{x}) - D\nabla\hat{\rho}_I (t,\vec{x}\right] \vec{\gamma} = 0. \qquad (12)$$

The second boundary condition describes the heat flux density, which is caused by a temperature difference between the ice slurry temperature T at the inside of the tank wall and the ambient temperature T_A of the storage tank

$$\left[T_A - T(t,\vec{x})\right] = -\frac{k}{U}\left[\nabla T(t,\vec{x})\vec{\gamma}\right] \ ,\forall \vec{x}, \vec{\gamma} \in \partial\Gamma \qquad (13)$$

Equations (9) to (13) define mathematically the storage problem with buoyancy-induced motion of the ice particles and a simultaneously occurring melting of the ice slurry.

THE HINDERED SETTLING VELOCITY

Details on the derivation of the hindered settling velocity are presented in Ref. [6]. The main result is of twofold structure

$$v = \begin{cases} \dfrac{g d_p^2}{18\eta(\vartheta, c_I)}\left[\rho(\vartheta, c_I) - \rho(\vartheta, 1)\right] \ , & c_I < c_{I_{\max}} \\ 0, & c_I = c_{I_{\max}}. \end{cases} \qquad (14)$$

The occurring density difference enters into the model by introducing the Archimedes force. To calculate the velocity (14), the density and the dynamic viscosity must be determined as a function of the temperature and ice concentration.

A GENERALIZED PHYSICAL-PROPERTIES MODEL

The density

In Ref. [8] a model to calculate physical properties was published and is used as basis for the generalization in this article. The previous model relates to ice slurries, which are in thermal equilibrium - as described below - and in an overall homogeneous state. If ice crystals of lower temperature than the fluid and a low thermal relaxation time are added to a fluid an unique temperature/ice fraction relation occurs. All the quantities which describe such states are described by a superscript (h), e.g. $\rho^{(h)}$, $c_I^{(h)}$, etc.

Vuarnoz et al. have observed in an insulated tank at a constant temperature for more than two hours a separation of ice particles [5]. From this it follows that it is possible to create conditions with deviating ice contents, which can be lower or higher than $c_I^{(h)}$. Such states can also be in thermodynamic equilibrium and (locally) homogeneous. Physical quantities of these states are denoted with the superscript (i), e.g. $\rho^{(i)}$, $c_I^{(i)}$, etc. This observation motivated a generalization of the physical-properties model. Notify that states denoted with superscript (h) are special cases of those denoted with superscript (i).

To develop the new model an idea presented in Fig. 1 is very helpful. By mixing the ice slurry at a constant temperature a homogeneous distribution is produced (FIG. 1a). Then the mixer is switched off, and without a change of temperature the particles rise (FIG. 1b) and lead to a steady-state stratification (FIG. 1c). An exchange process, characterizing stratification, is idealized in this last figure by replacing one ice particle from the bottom with an element of the additive/water mixture of identical volume from the top region in the tank. This process does not disturb the thermal equilibrium.

As long as the equations are simultaneously valid for quantities with superscript (h) and (i), they are not distinguished by their superscripts. The density is described by the following formula

$$\rho = \frac{m}{V} = \frac{m}{V_I + V_{AW}}, \quad V_{AW} = \mu(V_A + V_W), \qquad (15)$$

with the volume contraction function μ (see Ref. [8]). Inserting the third Eq. of (15) into the second one and dividing the nominator and denominator by m it follows that

$$\rho = \frac{1}{\dfrac{c_I}{\rho_I} + \mu\left(\dfrac{c_A}{\rho_A} + \dfrac{c_W}{\rho_W}\right)}. \qquad (16)$$

In the previous model $\rho=\rho^{(h)}$ is only a function of the temperature ϑ. We assume that the density is a function of the temperature and ice fraction (17)

$$\rho^{(i)} = \frac{1}{\dfrac{c_I^{(i)}}{\rho_I(\vartheta)} + \mu(\vartheta)\left[\dfrac{c_A(\vartheta, c_I^{(i)})}{\rho_A(\vartheta)} + \dfrac{c_W(\vartheta, c_I^{(i)})}{\rho_W(\vartheta)}\right]}.$$

The idea is to describe c_A and c_W by the temperature ϑ and the ice concentration $c_I^{(i)}$. For this purpose two further equations must be derived. The first equation is derived

$$\delta m_A + \delta m_W + \delta m_I = \delta m \quad \Rightarrow \quad c_A + c_W + c_I = 1 \quad \Rightarrow$$

$$c_W = 1 - c_A - c_I. \qquad (18)$$

by dividing the first Eq. (18) by δm. A second relation for c_A is now demanded.

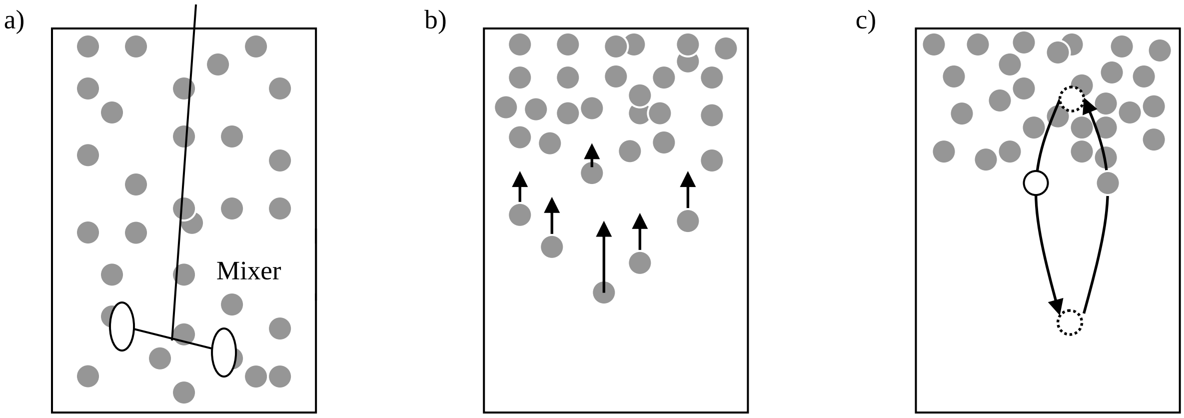

Fig. 1 The idea of the generalized physical-properties model is explained by the three graphical presentations. Explanations are given in the text of this article: homogeneous field, e.g. generated by mixing (a), stratification process (b), and steady-state condition and the explanation of the model by a displacement of a volume element from the top to a lower region and vice versa.

The following relation is valid

$$c_A^{(h)} = c_{A0}, \qquad \forall \vartheta, \tag{19}$$

where c_{A0} is the initial additive content above the freezing point (without any ice). In the exchange process shown in FIG. 1c in the remaining fluid the additive to water ratio does not alter. In FIG. 1a the quantities are denoted by superscript (h), in FIG 1b and FIG. 1c by superscript (i). Then it follows that

$$\frac{c_A^{(i)}}{c_W^{(i)}} = \frac{c_A^{(h)}}{c_W^{(h)}} = \frac{c_{A0}}{c_W^{(h)}} \quad \Rightarrow \quad c_A^{(i)} = \frac{c_{A0}}{c_W^{(h)}} c_W^{(i)}. \tag{20}$$

Now for $c_W^{(h)}$ and $c_W^{(i)}$ the right-hand side of the third Eq. (18) is substituted to obtain

$$c_A^{(i)} = \frac{1 - c_A^{(i)} - c_I^{(i)}}{1 - c_{A0} - c_I^{(h)}} c_{A0}, \tag{21}$$

where also equation (19) was used to simplify the relation. Solving for $c_A^{(i)}$ one obtains

$$c_A^{(i)} = \frac{1 - c_I^{(i)}}{1 - c_I^{(h)}} c_{A0}. \tag{22}$$

In Ref. [8] an equation for $c_I^{(h)}$ is published

$$c_I^{(h)} = 1 - \frac{c_{A0}}{c_A{}^*}, \qquad c_A{}^* = \frac{m_A}{m_A + m_W}. \tag{23}$$

The quantity c_A* denotes the additive concentration in the remaining fluid with ice creation (see second Eq. (23)). In the appendix of Ref. [8] an equation for this concentration c_A* for the additive talin, an industrial ethanol, is presented. Now the first Eq. (23) is substituted into (22), which leads to

$$c_A^{(i)}(\vartheta, c_I^{(i)}) = \left(1 - c_I^{(i)}\right) c_A^*(\vartheta). \tag{24}$$

This is substituted into the third Eq. ((18) with superscript (i) to obtain

$$c_W^{(i)}(\vartheta, c_I^{(i)}) = 1 - c_A^*(\vartheta) - \left[1 - c_A^*(\vartheta)\right] c_I^{(i)}. \tag{25}$$

Now (24) and (25) are inserted into (17) to obtain the final equation (26) for the density

$$\rho^{(i)} = \frac{1}{\dfrac{c_I^{(i)}}{\rho_I} + \mu\left[\dfrac{(1 - c_I^{(i)})c_A^*}{\rho_A} + \dfrac{1 - c_A^* - \left[1 - c_A^*\right] c_I^{(i)}}{\rho_W}\right]}$$

An important special case is

$$\rho^{(i)}(\vartheta, 1) = \rho_I(\vartheta). \tag{27}$$

Equations (26) and (27) describe the densities, which must be inserted into the formula for the hindred settling velocity (14).

The enthalpy density

The enthalpy density is described by the following equation

$$h^{(i)} = c_I^{(i)} h_I + \left(c_A^{(i)} + c_W^{(i)}\right) h_{AW}. \tag{28}$$

When (24) and (25) are inserted numerous terms cancel out. The result is not very surprising

$$h^{(i)} = c_I^{(i)} h_I + \left(1 - c_I^{(i)}\right) h_{AW}. \tag{29}$$

The enthalpy of the ice is

$$h_I = c_{pI} \vartheta + h_m, \tag{30}$$

with the melting enthalpy of ice h_m. The enthalpy of additive/water simply is

$$h_{AW}(\vartheta) = c_{pAW}\,\vartheta . \tag{31}$$

Inserting Eq. (30) and (31) into (29) it follows Eq. (32)

$$h^{(i)}(\vartheta, c_I^{(i)}) = c_I^{(i)}\left(c_{pI}\,\vartheta + h_m\right) + \left(1 - c_I^{(i)}\right)\, c_{pAW}\,\vartheta .$$

This is the final result to calculate the enthalpy densities of ice slurries.

The specific heat

The overall specific heat is defined by the total derivative of the enthalpy density as a function of the absolute temperature

$$c_p^{(i)} = \frac{dh^{(i)}}{dT} = \frac{\partial h^{(i)}}{\partial \vartheta} + \frac{dh^{(i)}}{dc_I^{(i)}}\frac{\partial c_I^{(i)}}{\partial \vartheta} . \tag{33}$$

By differentiation of equation (32), one obtains the final result

$$c_p^{(i)} = c_I^{(i)} c_{pI} + \left(1 - c_I^{(i)}\right)\, c_{pAW} + \left[(c_{pI} - c_{pAW})\ \ \vartheta + h_m\right]\frac{\partial c_I^{(i)}}{\partial \vartheta} . \tag{34}$$

This equation is related to the technical definition

$$h(0°C) = 0. \tag{35}$$

We have stated that this theory has a special case, namely the model with quantities with superscript (h). In this model there is a well-defined relation between ice concentration and temperature. Therefore, it follows with (23)

$$c_p^{(h)} = c_I^{(h)} c_{pI} + \left(1 - c_I^{(h)}\right)\, c_{pAW} + \left[(c_{pI} - c_{pAW})\ \ \vartheta + h_m\right]\frac{1}{c_A^{*\,2}} \cdot \frac{dc_A^*}{d\vartheta} . \tag{36}$$

OTHER PHYSICAL PROPERTIES

Diffusion “constant”

Not much knowledge is yet available on diffusion of ice particles in ice slurries. Special experiments to investigate the diffusive behaviour and the dependence of D on the temperature and ice particle concentration would be useful. On the other hand, because of the large size of the ice particles, it may be expected that Brown’s motion can be neglected. Experimentally observed fluctuations of propagating ice particle fronts may occur due to disturbances by the displacement motions of the stratification of ice particles.

In our numerical studies a small diffusion is introduced to describe these effects only approximately and to damp numerical instabilities [6].

The dynamic viscosity

The dynamic viscosity is calculated with the Thomas model as explained in Ref’s [6] and [9]. The original literature introducing the Thomas model is given in [9]. As in Ref. [6], in this article the numerical calculations are also performed for a ten mass-percent ethanol/water solution.

The thermal conductivity

The thermal conductivity is calculated by applying the model of Jeffrey as presented in Ref. [9].

STRATIFICATION PROCESS

An isothermal stratification process is shown in FIG. 2. The initial ice fraction is twenty mass percent, and the maximal value 74 %. The other parameters are identical to those presented in Ref. [6].

CONCLUSIONS AND OUTLOOK

An extension of the physical-properties model of ice slurries is presented in detail. Furthermore, the theory of ice slurry stratification is generalized to also describe non-isothermal processes. The melting of ice particles is taken into consideration. Further numerical simulations, applying the full models presented in this article, shall be published in a following paper.

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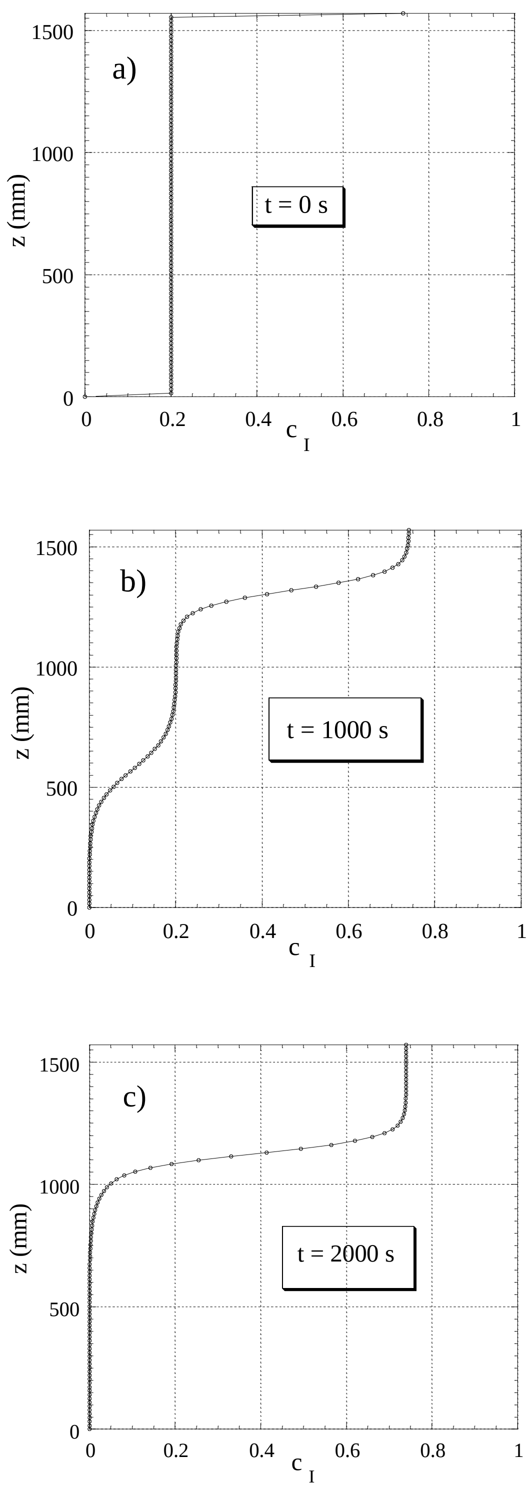

Fig. 2 The height in the tank is shown as a function of the ice concentration for three different times: initial homogeneous state (a), intermediate state of dynamical stratification process (b), and finally the steady state (c).

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