# A MODEL TO CALCULATE DYNAMICAL AND STEADY-STATE BEHAVIOUR OF ICE PARTICLES IN ICE SLURRY STORAGE TANKS 

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Storage of ice slurries is advantageous to save energy and to produce a continuous controlled long-time operation of an ice-slurry system. To guarantee homogeneous ice particle fields and to prevent a system of blocking by ice clusters, up-to-present a good mixing of the slurries was proposed. But mixing can lead to twenty percent energy demand of the total energy consumption of an ice slurry system, including the production of cold. Therefore, new methods are invented to operate systems without a mixing in the storage tank, but still with a control of the fluid and flow conditions in the piping circuits. The theoretical basis to calculate distributions of ice particle fields, respectively ice concentration distributions in unmixed ice slurry storage tanks with stratifications is given and first numerical simulation results are presented.

## 1. INTRODUCTION

Thermal energy storage systems have always been recognized as essential for energy savings in cold and hot systems [1], e.g. for refrigeration or air conditioning applications. Energy storage in Phase Change Materials (PCM's) has several advantages, for example:

- a high storage capacity caused by a high enthalpy density of the PCM's
- a temperature stabilization caused by the phase change [2].

In cooling technology storage with phase change materials, e.g. see Ref. [3], has frequently been applied in practical systems. On the other hand in thermal heating applications latent heat energy storage has only seldomly overcome the stage of research and development (for a realization in practice see Refs. [4] and [5]). Phase change slurries - e.g. micro-encapsulated PCM's in carrier fluids [6] or ice slurries [7] - have a further very important advantage:

- The transport fluid and the storage medium are the same substance. Therefore, no additional heat exchangers are necessary.

Storage is especially interesting, if the power consumption prize is a widely fluctuating function of time. Then with storage possibilities the size of the production unit for hot or cold can be reduced. At present in several countries the costs for electricity at day and night time do not differ substantially; in others, e.g. in France or Japan, the prize policy for electricity in some cases favours the use of energy storage technologies [8].

In a storage application the tank is the main component. The storage tank must be well insulated, so that the cold or hot losses are minimized. If the storage medium is an ice slurry, a mixing device mounted into the storage vessel is recommended. If no mixing device exists, due to buoyancy the ice crystals accumulate at the top of the tank. Especially when the system is not tight - and moist air gets into contact with the ice slurry surface - the ice crystals freeze together, leading to large plate-like structures. At a certain stage of this process it becomes practically impossible to dissolve the frozen mush (the coherent ice particle sheets) and to transport the resulting inhomogeneous suspension out into the system.

## 2. ICE SLURRY STORAGE TANKS WITHOUT MIXING

Because mixing is very energy consuming, ideas to store an ice slurry without mixing are welcome. Meili et al. showed that in a closed system a stratified ice slurry after fifteen hours can be brought back to a homogeneous state [9]. Therefore, if a system does not require cold for long-time periods, e.g during the night, to save energy the mixer (see FIG. 1 a) may be switched off.

If the dissolvability of the agglomerated ice particles remains for even longer periods of time, new methods to run ice slurry systems may be developed. FIG. 1b shows an idea, which has not yet been experimentally tested. A system with a storage tank contains no mixer. Therefore, the buoyant motion of ice particles lead to a highly stratified ice slurry field. With a displacement pump high-concentration ice slurry [10] is transported from the top region of the tank to a nearby located mixing device (eventually a mixing valve). Instead of forced convection maybe free convection is sufficient to transport the ice particles with less energy consumption to this valve. The high ice concentration slurry is mixed with an ice slurry with low ice fraction from the bottom region. The mixing guarantees a full control of the ice fraction in the system and completely safe running conditions.


Figure 1: Schematic drawing of a usual method of storage and mixing (a), and a proposal for a new system without a mixer in the tank (b). This method allows to save electrical energy. The inlets and outlets shown are parts of to the secondary circuits.

## 3. THEORY FOR ISOTHERMAL CONDITIONS

In this chapter the differential equations and boundary conditions for the ice particle density and the ice fractions to solve stratification problems in storage tanks are derived. The number of particles $d N$ in a differential mass element $d m$ is
$d N=n d m$.
The quantity $n$ describes the particle density. With the fundamental relation
$d m=\rho d V$,
it follows that
$d N=\rho n d V$.
Now the particle conservation equation is derived term by term. The alteration per unit time is
$(1)=\frac{\partial}{\partial t} d N d t=\frac{\partial}{\partial t}(\rho n) d V d t$.
The diffusion is described by Fick's law
$\vec{j}=-D \nabla \tilde{n}$,
with the particle density per unit volume
$\tilde{n}=\rho n$.
With this equation and (3/5) the diffusion term is
$-\operatorname{div} \vec{j}=\operatorname{div}[D \nabla(\rho n)] d V d t$
Now the Laplace operator is introduced, which is the divergence of the gradient
$\Delta:=\operatorname{div} \nabla$,
to derive from (3/7b)
(2) $=-d i v \vec{j}=[\nabla D \nabla(\rho n)+D \Delta(\rho n)] d V d t$.

It is assumed that basically the overall fluid is at rest, and that only a relative motion of ice particles and an in opposite direction streaming additive/water flow occurs. If a small density difference of the flow components "ice" and "additive/water" is present, a small bulk flow results. This problem of internal motion is well known in the domain of multi-component fluids [11] and also in the phenomenological theory of quantum fluids (see Ref. [12]). Densities of the components are introduced by the following definition

$$
\begin{equation*}
\hat{\rho}_{\chi}=\frac{\delta m_{\chi}}{\delta V}, \quad \chi \in\{I, A W\} . \tag{3/10a,b}
\end{equation*}
$$

Adding (3/10a) and (3/10b) it follows that

$$
\begin{equation*}
\rho=\hat{\rho}_{I}+\hat{\rho}_{A W}, \quad \vec{v}=\frac{\hat{\rho}_{I} \vec{v}_{I}+\hat{\rho}_{A W} \vec{v}_{A W}}{\hat{\rho}_{I}+\hat{\rho}_{A W}}, \tag{3/11a,b}
\end{equation*}
$$

where the velocity of the component "ice" is identical to the ice particle velocity $\vec{v}_{I}=\vec{v}_{P}$. The continuity equations are independently valid for both components

$$
\begin{equation*}
\frac{\partial \hat{\rho}_{\chi}}{\partial t}+\operatorname{div}\left(\hat{\rho}_{\chi} \vec{v}_{\chi}\right)=0, \quad \chi \in\{I, A W\} . \tag{3/12a,b}
\end{equation*}
$$

If the two equations (3/12a) and (3/12b) are added, with ((3/11a,b) the continuity equation of the entire fluid is obtained

$$
\frac{\partial \rho}{\partial t}+\operatorname{div}(\rho \vec{v})=0 .
$$

The relative motion of the ice particles in the fluid also contributes to the particle conservation equation by the following term

$$
\begin{equation*}
\text { (3) }=-\operatorname{div}\left(\rho n \vec{v}_{I}\right) d V d t \text {. } \tag{3/14}
\end{equation*}
$$

A fourth contribution is given by a term describing a source, respectively a sink. This term is important to describe processes of ice particle creation, respectively destruction. In ice slurries the process of particle generation is related to freezing, whereas destruction is caused by melting
(4) $=q_{p} d m d t=q_{p} \rho d V d t$.

Now the conservation law of ice particles is given by

$$
\begin{equation*}
(1)=(2)+(3)+(4) \text {. } \tag{3/16}
\end{equation*}
$$

By substituting the previous introduced equations for (1) to (4), and by additionally dividing all the terms by $d V d t$, it follows
$\frac{\partial}{\partial t}(\rho n)+\operatorname{div}\left(\rho n \vec{v}_{I}\right)-[\nabla D \nabla(\rho n)]-D \Delta(\rho n)-\rho q_{p}=0$.
By substituting
$\Delta(\rho n)=\rho \Delta n+2(\nabla \rho \nabla n)+n \Delta \rho$,
into (3/17) a basic equation for particle conservation occurs

$$
\begin{equation*}
\frac{\partial}{\partial t}(\rho n)+\operatorname{div}\left(\rho n \vec{v}_{I}\right)-\rho[\nabla D \nabla n]-n[\nabla D \nabla \rho]-\rho D \Delta n-2 D(\nabla \rho \nabla n)-D n \Delta \rho-\rho q_{p}=0 . \tag{3/19}
\end{equation*}
$$

For a small control volume the mass of the ice particles is

$$
\begin{equation*}
\delta m_{I}=\rho n \delta V V_{p} \rho_{p}=\rho n \delta V V_{p} \rho_{I}, \tag{3/20a,b}
\end{equation*}
$$

where $V_{p}$ denotes the (mean) volume of a single particle. The mass ice fraction is defined by

$$
\begin{equation*}
c_{I}=\frac{\delta m_{I}}{\delta m}=\frac{\rho n \delta V V_{p} \rho_{I}}{\rho \delta V}=\rho_{I} n V_{p} . \tag{3/21a-c}
\end{equation*}
$$

From this it is straightforward to conclude that for spherical particles the ice particle density is

$$
\begin{equation*}
n=\frac{c_{I}}{\rho_{I} V_{p}}=\frac{6}{\pi} \frac{c_{I}}{\rho_{I} d_{p}^{3}}, \quad A=n A_{p}=\pi n d_{p}{ }^{2} . \tag{3/22a-d}
\end{equation*}
$$

The ice particle density, calculated with equation (3/22b), is shown in FIG. 2a. In FIG. 2b for numerous ice fractions the total surface areas of the large amount of particles in one kilogram ice slurry are plotted, which were calculated by applying equation (3/22d). Bel estimated for $c_{l}=50 \%$ and a particle diameter of $d_{p}=400 \mu \mathrm{~m}$ a total surface area of $8.2 \mathrm{~m}^{2}$ [13], which is in good agreement with FIG. 2b.


Figure 2: Particle density as a function of particle diameter for numerous ice fractions (a), and the total surface area of all the particles in one kilogram ice slurry (b). These graphics are valid for ice slurries with any additives, but only for spherical particles.

Now two approximations are introduced:

1) $\quad V_{p}=$ const ,
2) $\rho_{I}=$ const .

The first approximation states a constant mean volume of the particles. This is identical with the requirement of no time behaviour or the approach of the time asymptotic limit. The second statement is approximately fulfilled when the temperature alterations are not very high. With these approximations, by substituting (3/22a) into (3/19), it follows that

$$
\begin{equation*}
\frac{\partial}{\partial t}\left(\rho c_{I}\right)+\operatorname{div}\left(\rho c_{I} \vec{v}_{I}\right)-\rho\left[\nabla D \nabla c_{I}\right]-c_{I}[\nabla D \nabla \rho]-D \rho \Delta c_{I}-2 D\left(\nabla \rho \nabla c_{I}\right)-D c_{I} \Delta \rho-\rho_{I} \rho V_{p} q_{p}=0 \tag{3/25}
\end{equation*}
$$

Because only isothermal stratifications are discussed, in equation (3/25) the last term can be neglected. Furthermore - in a first approach to the problem - we assume that the diffusion constant is no function of temperature and ice fraction, and that the density changes are much smaller than those of the ice fraction. Combined stratification, diffusion and melting will be solved later and presented elsewhere. With these assumptions (3/25) may be simplified
$\frac{\partial}{\partial t}\left(\rho c_{I}\right)+\operatorname{div}\left(\rho c_{I} \vec{v}_{I}\right)-D \rho \Delta c_{I}=0$.
A simple relation is derived

$$
\begin{equation*}
\rho c_{I}=\frac{\delta m}{\delta V} \frac{\delta m_{I}}{\delta m}=\frac{\delta m_{I}}{\delta V}=\hat{\rho}_{I} . \tag{3/27a-c}
\end{equation*}
$$

By substituting (3/27c) into (3/26) one obtains

$$
\begin{equation*}
\frac{\partial}{\partial t}\left(\hat{\rho}_{I}\right)+\operatorname{div}\left(\hat{\rho}_{I} \vec{v}_{I}\right)-D \rho \Delta c_{I}=0 \tag{3/28}
\end{equation*}
$$

which - if D is set equal to zero - is identical to the continuity equation (3/12a). The initial condition is defined by a homogeneous ice fraction throughout the fluid domain $\Gamma$ in the storage tank (see FIG. 1). Then the density is constant and with (3/27c) it is found that

$$
\begin{equation*}
c_{I}(t=0, \vec{x})=c_{I 0}=\text { const }, \quad \forall \vec{x} \in \Gamma, \quad \Rightarrow \quad \hat{\rho}_{I}(t=0, \vec{x})=\hat{\rho}_{I 0}=\text { const }, \quad \forall \vec{x} \in \Gamma . \tag{3/29a-d}
\end{equation*}
$$

The boundary condition states that no ice particles cross the bounding walls of the storage tank, respectively the free surface of the ice slurry, which are denoted by $\partial \Gamma$. With (3/5), $(3 / 14),(3 / 22 a),(3 / 23)$ and (3/24) one derives the boundary condition

$$
\begin{equation*}
\left[\hat{\rho}_{I} \vec{v}_{I}-D \nabla \hat{\rho}_{I}\right] \vec{\gamma}(t, \vec{x})=0, \quad \forall \vec{x}, \vec{\gamma} \in \partial \Gamma . \tag{3/30}
\end{equation*}
$$

Equations (3/28), (3/29c,d) and (3/30) define mathematically the storage problem with buoyancy-induced motion of the ice particles in the ice slurry with diffusion, but without the occurrence of melting.

## 4. THE HINDRED SETTLING VELOCITY

The vertical velocity is caused by two forces. The driving force is the buoyancy force, which is proportional to the density difference between the solid ice particle and the sourrounding ice slurry suspension in the gravitational field of the earth
$F_{B}=\frac{4}{3} \pi r_{\vec{p}}^{3}\left(\rho-\rho_{I}\right)$.

In equation (4/1) $\rho$ denotes the density of the ice slurry and $\rho_{I}$ the density of the ice. The damping force is caused by friction. In the viscous ice slurry the flow around the rising particle is assumed to be laminar. Therefore, Stokes law
$F_{D}=6 \pi \eta r_{p} v$
applies [11]. In a rising particle with constant velocity these forces are in equilibrium. Therefore, the velocity can be calculated by applying equation (4/1) and (4/2) (see also in Ref. [14])

$$
\begin{equation*}
v=\frac{1}{18} \frac{g d_{p}^{2}}{\eta}\left(\rho-\rho_{I}\right) . \tag{4/3}
\end{equation*}
$$

This formula is valid approximately up to ice fractions of $15-20 \%$. For higher ice fractions the formula must be generalized for the Bingham behaviour of ice slurries. This will be performed in a future work. The packing factor of the ice particles in the storage tank is
$p=\frac{V_{I}}{V}, \quad p_{\max }=\frac{V_{I \max }}{V}$,
where $p_{\max }$ denotes the maximal packing factor at the top of the tank. It can be estimated by pure geometrical means, e.g. in a idealized consideration of crystal-like structures. If particle size distributions are taken into consideration higher maximal packing factors can be obtained. For the ice fraction it follows
$c_{I}=\frac{m_{I}}{m}=\frac{\rho_{I}}{\rho} \frac{V_{I}}{V}=\frac{\rho_{I}}{\rho} p \quad \Rightarrow \quad c_{I \max }=\frac{\rho_{I}}{\rho} p_{\max }$.
The density is described as a function of the temperature and the ice fraction (see chapter 5). Then for the hindred settling velocity the formula is written in a twofold manner
$v_{I}\left(\vartheta, c_{I}\right)= \begin{cases}\frac{1}{18} \frac{g d_{P}{ }^{2}}{\eta\left(\vartheta, c_{I}\right)}\left[\rho\left(\vartheta, c_{I}\right)-\rho(\vartheta, 1)\right], & c_{I}<c_{I \max } \\ 0, & c_{I} \geq c_{I \max },\end{cases}$
with $\rho(\vartheta, 1)=\rho_{I}(\vartheta)$. Because the density of the ice slurry decreases towards the top of the tank, it approaches the density of pure ice. Therefore, the density difference - which is the
driving force of an ice particle - and the ice particle velocity decrease (see equation (4/6a)). When an ice particle reaches the region of maximal packing factor, it is mechanically hindred to further rise. This phenomenon is described by equation (4/6b). It guarantees that the particle comes to rest as soon as it reaches this domain.

## 5. A GENERALIZED PHYSICAL-PROPERTIES MODEL

### 5.1 The density

In Ref. [15] a model to calculate physical properties was published. It relates only to thermal equilibrium states, which are described in more detail in this reference. Vuarnoz et al. have observed for more than two hours a stratification of ice particles in an insulated storage tank at constant temperature [10]. Therefore, it is possible to create conditions with deviating ice contents, which can be lower or higher than the most probable occurring equilibrium ice fraction. And these deviating states are also in thermal equilibrium. To calculate physicalproperties for these additional states, the physical-properties model must be generalized. This is performed in detail in Ref. [16]. Here only one main result is summarized. The density is

$$
\begin{equation*}
\rho\left(\vartheta, c_{I}\right)=\frac{1}{\frac{c_{I}}{\rho_{I}(\vartheta)}+\mu(\vartheta)\left[\frac{\left(1-c_{I}\right) c A^{*}(\vartheta)}{\rho_{A}(\vartheta)}+\frac{1-c A^{*}(\vartheta)-\left[1-c A^{*}(\vartheta)\right] c_{I}}{\rho_{W}(\vartheta)}\right]} . \tag{5.1/1}
\end{equation*}
$$

Setting $c_{l}=1$ one immediately obtains

$$
\begin{equation*}
\rho(\vartheta, 1)=\rho_{I}(\vartheta) . \tag{5.1/2}
\end{equation*}
$$

To calculate the hindred settling velocity equations (5.1/1) and (5.1/2) are inserted into (4/6a).

### 5.2 The dynamic viscosity

To calculate the dynamic viscosity, measured data obtained at the French institute Cemagref and at the University of Applied Sciences of Central Switzerland (UASCS) are considered [17]. It is assumed that the viscosity mainly depends on the ice fraction and that the influence of the temperature is of minor importance. The data and an empirical formula - determined by curve fitting - are shown together with results calculated with the Thomas model (see in FIG. 3). The Thomas model is presented, e.g. in Ref. [13]. The original reference is [18]. It leads to

$$
\begin{equation*}
\eta=\eta_{L}\left[1+2.5 p+10.05 p^{2}+0.00273 \exp (16.6 p)\right], \quad p=\frac{\rho}{\rho_{I}} c_{I} \tag{5.2/1a,b}
\end{equation*}
$$

In numerical calculations the following equation is used [13]

$$
\begin{equation*}
\eta_{L}=3.8914+0.0673 \vartheta+0.0469 \vartheta^{2} . \tag{5.2/2}
\end{equation*}
$$

In an approximation to obtain the curve in FIG. 3a, the density ratio was set equal to one. In FIG. 3a and FIG. 3 b the dynamic viscosity of the liquid $\eta_{L}$ was taken to be 3.8 mPa s, which corresponds to $\vartheta=-5.0^{\circ} \mathrm{C}$.

The experimental results of Frei and Egolf [17] compare well with the Thomas model. They are described by the following empirical quadratic equation
$\eta=0.227 c_{I}{ }^{2}-0.0167 c_{I}+0.00353$.
By curve fitting to the experimental data, which were obtained at Cemagref, it follows that

$$
\begin{equation*}
\eta=1.790 c_{I}^{2}-0.000731 c_{I}+0.00885 \tag{5.2/4}
\end{equation*}
$$

The curve fitting procedure to the Cemagref data leads to too high results at $c_{l}=0$.


Figure 3: Measurements of the dynamic viscosity [17] and the curve fitting results (a). To numerically simulate the stratification process four typs of approximations are taken into consideration (b), characterizing the mean age of the ice particles.

The difference between the curves of UASCS and Cemagref is explained by an occurring time behaviour, which is described in Ref. [17]). To refer to this phenomenon four empirical equations are introduced, which describe the viscosity for four different mean life times of the ice particles after their creation occured. Unfortunately, it is not possible to exactly define the corresponding mean life times to each of these curves. Because of this reason they are only distinguished by Roman numbers. Curve I represents the "oldest" ice slurry, II and III inter-
mediate times and IV the "youngest" ice slurry. Considering the experimental devices and procedures an estimation of the mean life time of the particles could be performed.

The related equations are:
$\eta=\alpha c_{I}{ }^{2}+\eta_{L} \quad\left\{\begin{array}{c}I: \alpha=0.09 \\ \text { II: } \alpha=0.66 \\ \text { III: } \alpha=1.23 \\ I V: \alpha=1.80 .\end{array}\right.$
The quantity $\eta_{L}$ is calculated with equation (5.2/2).

## 6. NUMERICALLY CALCULATED STRATIFICATIONS

In this paper an academic example is presented to explain the basic phenomena of ice particle stratification. Calculations of stratification with the same method of more practical storage problems are presented in Ref. [16]. This example was also important for testing of the numerical simulation program, which was developed in Borland Delphi 4. Dealing with front propagation is not very simple, because discontinuities are sources of numerical instabilities. In a first approach, choosing a constant propagation velocity, the solutions showed extreme instabilities. They could be avoided by programming two phenomena occurring also in nature. The first was by defining a particle velocity, which decreases in field regions of higher ice concentration. Therefore, the velocity was calculated by the following linear distribution

$$
\begin{equation*}
v_{I}\left(c_{I}\right)=v_{I}(0)\left(1-c_{I}\right) . \tag{6/1}
\end{equation*}
$$

Similar behaviour is also described by the Stokes law (4/6a). It also leads to a reduction of velocity in field domains of higher ice particle concentrations. The second method was to introduce diffusivity. - In this example the maximal packing factor was chosen to be $p_{\max }=100 \%$. Some main parameters of the numerical simulations are given in Table 1.

| Height of ice slurry in the tank | 1600 mm |
| :--- | :--- |
| Density of ice (model assumption!) | $1000 \mathrm{~kg} / \mathrm{m}^{3}$ |
| Initial ice concentration | $40 \%$ |
| Maximal packing factor / maximal ice concentration | $100 \% / 100 \%$ |
| Diffusion constant (no function of $\vartheta$ and $c_{I}$ ) | $2.010^{-5} \mathrm{~m}^{2} \mathrm{~s}^{-1}$ |
| Spatial resolution, number of steps | 200 |
| Time of particle propagation | $0 \mathrm{~s}, 533 \mathrm{~s}, 1067 \mathrm{~s}, 1600 \mathrm{~s}$ |
| Time steps | 10000 |
| Maximal velocity of particles | $1 \mathrm{~mm} / \mathrm{s}$ |

Table 1: Parameters for numerical simulation of a first academic example describing isothermal ice particle stratification.

In FIG. 4 ice fraction distributions are shown for three different times of a stratification process in an initially homogeneous ice particle field (a). The particles start to rise and the lowest located particles constitute the front. Here the front velocity is identical to the particle velocity. It is calculated by equation ( $6 / 1$ )

$$
v_{F 1}=v_{I}\left(c_{I}\right)=v_{I}(0)\left(1-c_{I 0}\right)=(1-0.4) \frac{1 \mathrm{~mm}}{s}=0.6 \frac{\mathrm{~mm}}{\mathrm{~s}}, \quad f_{1}=0.6 \frac{\mathrm{~mm}}{s} 533 \mathrm{~s}=320 \mathrm{~mm} . \quad(6 / 2 \mathrm{a}-\mathrm{f})
$$

After 533 s the height of the front is $f_{l}=320 \mathrm{~mm}$ (FIG. 4b).


Figure 4: The dynamical behaviour of stratification shown at four different times from the beginning (a) to a first intermediate state (b) to a second (c), and toward the final state with maximal packing in the upper region of the tank (d). In this simple example the integral (area under the concentration profiles) is conserved. Therefore, areas $\boldsymbol{A}$ and $B$ are identical (see FIG. 4b). This conservation is directly related to the total number of ice particles $N$ in the tank, which in systems - which are in isothermal conditions - also is a conservation quantity.

The upward directed motion leads to an accumulation of ice particles in the top region, where the ice concentration increases up to its maximal value, which is directly related to the maximal packing factor. In our case the maximal ice concentration was chosen to be $100 \%$. It is assumed that in reality the maximal packing factor is an increasing function of time. But this has not been taken into consideration. The front propagation velocity of the second (upper) front $v_{F 2}$ is not identical to the particle velocity. In this case it follows that
$v_{F 2}=\frac{0.4}{0.6} v_{F 1}=\frac{2}{3} 0.6 \frac{\mathrm{~mm}}{\mathrm{~s}}=0.4 \frac{\mathrm{~mm}}{\mathrm{~s}}, \quad \quad f_{2}=1600 \mathrm{~mm}-0.4 \frac{\mathrm{~mm}}{\mathrm{~s}} 533 \mathrm{~s}=1387 \mathrm{~mm} . \quad$ (6/3a-e)

The numerical simulation results agree with this value, which is calculated by simple kinematical considerations. Analogous determinations of front positions may be performed for $t=$ 1133 s (see FIG. 4c).

At a time equal or larger than the stratification time $t_{S}=1600 \mathrm{~s}$, the field shows two regions, one with $0 \%$ and the other with $100 \%$ ice fraction (FIG. 4d). This is the steady state describing maximal stratification. Because of diffusion the front is slightly smeared out and not perfectly discontinuous.

In FIG. 5a the particle velocity is shown. In the region on the right the velocity is $1 \mathrm{~mm} / \mathrm{s}$, but this is meaningless, because in this domain the ice concentration is zero; no particles occur. In the core region the velocity is $0.6 \mathrm{~mm} / \mathrm{s}$ (equation (6/2d)). The domain on the right shows no particle velocity, because here the maximal packing has been obtained and the particles have settled down.
a)

b)


Figure 5: The distribution of the ice particle velocity (a). It is directly proportional to the difference of the maximal ice concentration and the actual occurring concentration. Furthermore the ice concentration flux density is shown (b). It is positive, which states that the ice particle flux is directed to the right hand side.

FIG. 5b presents the ice mass flux density, which is proportional to the ice particle flux density. Its numerical value is calculated by the following formula

$$
\begin{equation*}
j=\rho_{I S} c_{I} v_{I}=1000 \frac{\mathrm{~kg}}{\mathrm{~m}^{3}} 0.4 \cdot 0.0006 \frac{\mathrm{~m}}{\mathrm{~s}}=0.24 \frac{\mathrm{~kg}}{\mathrm{~m}^{2} \mathrm{~s}}, \tag{6/4}
\end{equation*}
$$

in agreement with numerical simulation results.

## 7. CONCLUSIONS AND OUTLOOK

Conservation equations for the particle density and ice fractions are derived. A new formula for the density of an ice slurry with two independent variables - the temperature and the ice fraction - is presented. A method to calculate stratification of ice particles in storage tanks by numerical simulations is outlined. Its usefulness and correctness is demonstrated by a simple example. Calculations of more realistic stratification processes will be presented in [16]. In these calculations the equations of chapter 4 and 5 will be applied. Their implementation leads to complicated couplings between the ice fraction, the density, the viscosity of the ice slurry, and the settling velocity of the ice particles.

## NOMENCLATURE

## Standard

| $A$ | surface area | $\mathrm{m}^{2}$ |
| :--- | :--- | :--- |
| $c$ | concentration | $(-)$ |
| $c_{p}$ | specific heat at constant pressure | $\mathrm{J} \mathrm{kg}^{-1} \mathrm{~K}^{-1}$ |
| $d$ | diameter | m |
| $D$ | diffusion "constant", resp. function | $\mathrm{m}^{2} \mathrm{~s}^{-1}$ |
| $d i v$ | divergence operator | $\mathrm{m}^{-1}$ |
| $\vec{j}$ | particle flux density / | $\mathrm{m}^{-2} \mathrm{~s}^{-1 /}$ |
|  | ice mass flux density | $\mathrm{kg} \mathrm{m}^{2} \mathrm{~s}^{-1}$ |
| $m$ | mass | kg |
| $n$ | particle density per unit of mass | $\mathrm{kg}^{-1}$ |
| $\tilde{n}$ | particle density per unit of volume | $\mathrm{m}^{-3}$ |
| $N$ | number of particles | - |
| $p$ | packing factor | - |
| $q$ | source and sink | $\mathrm{kg}^{-1} \mathrm{~s}^{-1}$ |
| $r$ | radius | $\mathrm{m}^{2}$ |
| $t$ | time | s |
| v | velocity component in axial direction | $\mathrm{m} \mathrm{s}^{-1}$ |
| $\vec{v}$ | vectorial velocity | $\mathrm{m} \mathrm{s}^{-1}$ |
| $V$ | volume | $\mathrm{m}^{3}$ |

## Greek

| $\alpha$ | constant | $\mathrm{m}^{2} \mathrm{~s}^{-1}$ |
| :--- | :--- | :--- |
| $\partial \Gamma$ | boundary of ice slurry in storage vessel | - |
| $\vec{\gamma}$ | normal boundary vector | - |
| $\Gamma$ | domain of ice slurry in storage vessel | - |
| $\eta$ | dynamic viscosity | $\mathrm{m}^{2} \mathrm{~s}^{-1}$ |
| $\vartheta$ | temperature | ${ }^{\circ} \mathrm{C}$ |
| $\rho$ | density | $\mathrm{kg} \mathrm{m}^{-3}$ |
| $\hat{\rho}$ | density of component of |  |
|  | two-component ice slurry | $\mathrm{kg} \mathrm{m}^{-3}$ |
| $\nabla$ | gradient operator | - |
| $\Delta$ | Laplace operator | - |

## Indices

| $A W$ | additive/water |
| :--- | :--- |
| $B$ | buoyancy |
| D | drag |
| $I$ | ice |
| $L$ | liquid |
| $P$ | particle (ice particle) |
| $S$ | Stratification |

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