Reinforcement of gas transmission networks with MIP constraints and uncertain demands [1]

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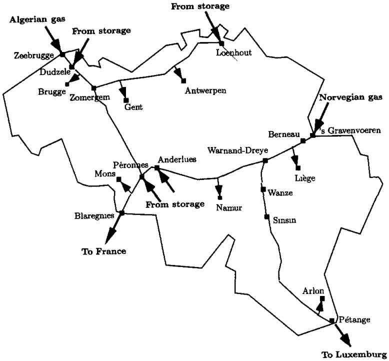
1. Operations and design of gas transmission networks
2. Robust optimization to deal with uncertain demands
3. Numerical experiments (preliminary)

Gas transmission networks



Objective of this research: Extend the continuous and deterministic formualtion of (*F. Babonneau, Y. Nesterov and J.-P. Vial. Design and operations of gas transmission networks. Operations Research, Operations Research, 60(1):34-47, 2012*) to uncertain demands, fixed investment costs, and commercial diameters.

Operations of gas transmission networks

Find a flow x and a system of pressures p such that

$$l_a \beta \frac{x_a|x_a|}{D_a{}^5} = p_i{}^2 - p_j{}^2, \; a = (i,j), a \in E_p \tag{1a}$$

$$\underline{\phi} \leq Ax \leq \bar{\phi} \tag{1b}$$

$$(p_i{}^2 - p_j{}^2)x_a \leq 0, x_a \geq 0, \; a = (i,j), a \in E_c \tag{1c}$$

$$(p_i{}^2 - p_j{}^2)x_a \geq 0, x_a \geq 0, \; a = (i,j), a \in E_r \tag{1d}$$

$$\underline{p}_i \leq p_i \leq \bar{p}_i, \; i \in V. \tag{1e}$$

$\Rightarrow$ Nonlinear and non convex set of inequalities. We rely on a two-step procedure of find a feasible pair of flows and pressures

First step: Finding feasible flows

The first problem computes flows

$$\min_x \quad \mathcal{E}(x) - \langle f, x\rangle - \langle d, Ax\rangle \tag{2a}$$

$$\underline{\phi} \leq Ax \leq \bar{\phi} \tag{2b}$$

$$x_a \geq 0,\ \ a \in E_c \cup E_r. \tag{2c}$$

The function $\mathcal{E}(x) = \sum_{a\in V} \mathcal{E}_a(x_{a)}$ is separable and is defined by

$$\mathcal{E}_a(x_a) = \quad l_a \frac{\beta}{D_a^5}\frac{|x_a|^3}{3}, \quad a \in E_p \tag{3}$$

Second step: Finding compatible pressures

The second one computes compatible pressures. *Given x^*, find a system of pressures p and an action vector f such that*

$$
\begin{aligned}
f + A^T(p)^2 &= \mathcal{E}'(x^*) && \text{(4a)}\\
f_a &= 0,\ a \in E_p && \text{(4b)}\\
p_i^2 - p_j^2 &\leq 0, \text{ if } x_a^* > 0,\ a = (i,j), a \in E_c && \text{(4c)}\\
p_i^2 - p_j^2 &\geq 0, \text{ if } x_a^* > 0,\ a = (i,j), a \in E_r && \text{(4d)}\\
\underline{p}_i \leq p_i &\leq \bar{p}_i,\ i \in V. && \text{(4e)}
\end{aligned}
$$

By applying the simple change of variable $P_i = p_i^2$, system (4) is linear in f and P.

Reinforcement problem

Let E_n to be the set of arcs on which reinforcement takes place and $\bar{C}$ be an upper bound on the total investment cost. The investment problem can be stated as

$$\min_{x} \min_{(D_a,\ a \in E_n)} \sum_{a \in E_n} \mathcal{E}_a(x_a; D_a) + \sum_{a \in E_p} \mathcal{E}_a(x_a) \tag{5a}$$

$$\mathcal{I}(D) \leq \bar{C} \tag{5b}$$

$$\underline{\phi} \leq Ax \leq \bar{\phi} \tag{5c}$$

$$x_a \geq 0,\ a \in E_c \cup E_r. \tag{5d}$$

Assumption

Data analysis shows that the investment cost can be approximated by

$$\mathcal{I}(D) = l \times (k_1 \times D^{2.5} + k_2), \tag{6}$$

where l is the length of the arc and $D \leq \bar{D}$.

Convex and continuous formulation of reinforcement problem

Let us perform the change of variable $y_a = D_a^{2.5}$, $a \in E_n$,

$$\min_{x} \min_{(y_a,\ a \in E_n)} \sum_{a \in E_n} \left(l_a \frac{\beta}{3} \frac{|x_a|^3}{y_a^2} \right) + \sum_{a \in E_p} l_a \frac{\beta}{3} \frac{|x_a|^3}{D_a^5} \tag{7a}$$

$$\sum_{a \in E_n} l_a \times k_1^a \times y_a \leq \bar{C} \tag{7b}$$

$$\underline{\phi} \leq Ax \leq \bar{\phi} \tag{7c}$$

$$x_a \geq 0,\ a \in E_c \cup E_r. \tag{7d}$$

The function $|x|^3/y^2$ is jointly convex in x and y.

Extension to integer considerations

Let k be the set of commercial diameters.

$$
\begin{aligned}
\min_{x,y,z} \quad & \left(\sum_{a \in E_n} l_a \frac{\beta}{3} \frac{|x_a|^3}{y_a^2} \right) + \sum_{a \in E_p} \mathcal{E}(x_a) \\
& \sum_{a \in E^n} \sum_{k} z_{ak} l_a (k_1^a D_k^{5/2} + k_2^a) \leq \bar{C} \\
& y_a = \sum_{k} z_{ak} D_k^{5/2}, \; k = 1, \ldots, K \\
& \sum_{k} z_{ak} \leq 1, \; k = 1, \ldots, K \\
& \underline{\phi} \leq Ax \leq \bar{\phi} \\
& x_a \geq 0, \; a \in E_c \cup E_r \\
& z_{ak} \in [0, 1] \; a \in E_n, \; k = 1, \ldots, K.
\end{aligned}
\tag{8}
$$

Relaxed formulation of reinforcement problem

By partial dualization, the problem is

$$\max_{\alpha \geq 0} \min_{x} \min_{(y_a,\, a \in E_n)} \quad \mathcal{E}(x;y) + \mathcal{E}(x) + \alpha(\mathcal{I}(y) - \bar{C}) \tag{9a}$$

$$\underline{\phi} \leq Ax \leq \bar{\phi} \tag{9b}$$

$$x_a \geq 0, \;\; a \in E_c \cup E_r. \tag{9c}$$

with the inner minimization problem

$$\mathcal{C}_a(x_a) = \min_{y_a} \left\{ l_a \frac{\beta}{3} \frac{|x_a|^3}{y_a^2} + \alpha l_a k_1^a y_a \right\}, a \in E_n. \tag{10}$$

Solving the inner minimization problem

Theorem (Babonneau, Nesterov, Vial)

$\mathcal{C}_a(x_a)$ is convex and is given by

$$\mathcal{C}_a(x_a) = \quad l_a \beta^{1/3} \left(\frac{3\alpha k_1}{2} \right)^{2/3} |x_a|$$

The optimal diameter in problem (11) is

$$D_a^* = \quad \left(\frac{2\beta}{3\alpha k_1^a} \right)^{\frac{2}{15}} |x_a|^{\frac{2}{5}}$$

Simplified formulation of the reinforcement problem

For a given α

$$\min_x \quad \sum_{a\in E_n} l_a \beta^{1/3}\left(\frac{3\alpha k_1^a}{2}\right)^{2/3}|x_a| + \sum_{a\in E_p} l_a \frac{\beta}{3}\frac{|x_a|^3}{D_a^5} \tag{11a}$$

$$A_i x \geq \phi_i, \quad i \in V_d \tag{11b}$$

$$\underline{\phi}_i \leq A_i x \leq \bar{\phi}_i, \quad i \notin V_d \tag{11c}$$

$$x_a \geq 0, \ a \in E_c \cup E_r. \tag{11d}$$

Uncertainty model

We consider the demand parameter at delivery nodes *i* as uncertain such that

$$\phi_i = \phi_i^n + \xi_i \hat{\phi}_i$$

where $\phi_i^n = \underline{\phi}_i$ is the nominal demand, $\hat{\phi}_i^n = \gamma \underline{\phi}_i$ is the demand dispersion and ξ_i is a random factor with support $[-1, 1]$.

The problem of reinforcement gas transmission networks is now a **two-stage problem with recourse**. In the first stage, reinforcement investment is selected and in the second stage the decision concerns the flow (and the activity of compressor and regulator stations) to satisfy observed demands.

Affine decision rules

Given the demand model, we can define a decision rule as a function from the space of demands realizations to the space of recourse flow decisions in order to capture the fact that flows can be adjusted to fit observed demands. We propose affine decision rules (ADR)

$$x_a = \nu_a^0 + \sum_{i \in V_d} \xi_i \nu_a^i, \quad \forall a \in E_p.$$

In that formulation, the new decision variables are the coefficients $\nu_a^0 \in \mathbb{R}$ and $\nu_a^i \in \mathbb{R}$.

Uncertain formulation with robust constraints

We can now replace x and ϕ by their definition.

$$H(\alpha) = \min_{t,\nu} \quad \sum_{a \in E_n} \left(\frac{3\alpha k_1^a}{2} \right)^{2/3} t_a + \sum_{a \in E_p} l_a \frac{\beta}{3} \frac{t_a^3}{D_a^5} \tag{12a}$$

$$|\nu_a^0 + \sum_{i \in V_d} \xi_i \nu_a^i| \leq t_a \quad a \in E_n \quad \forall \xi \in \Xi \tag{12b}$$

$$A_i(\nu_a^0 + \sum_{i \in V_d} \xi_i \nu_a^i) \geq \phi_i^n + \xi_i \hat{\phi}_i, \quad i \in V_d \quad \forall \xi \in \Xi \tag{12c}$$

$$\underline{\phi}_i \leq A_i(\nu_a^0 + \sum_{i \in V_d} \xi_i \nu_a^i) \leq \bar{\phi}_i, \quad i \notin V_d \quad \forall \xi \in \Xi \tag{12d}$$

$$\nu_a^0 + \sum_{i \in V_d} \xi_i \nu_a^i \geq 0, \; a \in E_c \cup E_r \quad \forall \xi \in \Xi. \tag{12e}$$

Applying robust optimization

We can now state the robust equivalent of the robust constraint.

Theorem (Ben-Tal, El Ghaoui, Nemirovski)

Let ξ_i, $i = 1, \ldots, m$ be independent random variables with values in interval $[-1, 1]$ and with average zero: $E(\xi_i) = 0$, the robust equivalent of the constraint

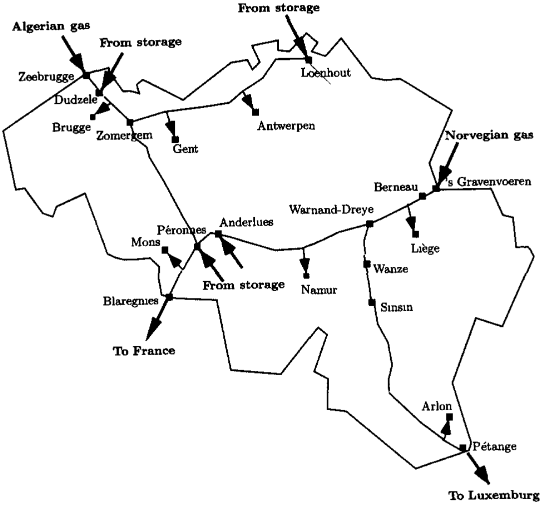
$$\bar{a}^T x + (P^T x)^T \xi \leq b, \text{ for all } \xi \in \Xi = \{\xi \mid ||\xi||_2 \leq k\},$$

is

$$\bar{a}^T x + k||P^T x||_2 \leq b,$$

with an associated satisfaction probability of $(1 - \exp(-\frac{k^2}{2.5}))$

Belgian instance



Belgian instance

#Nodes	Label	$\underline{\phi}$	$\bar{\phi}$	$\underline{p}$	$\bar{p}$
1	Zeebrugge	8.87	11.594	0	77
2	Dudzele	0	8.4	0	77
3	Brugge	$-\infty$	-3.918	30	80
4	Zomergem	0	0	0	80
5	Loenhout	0	4.8	0	77
6	Antwerpen	$-\infty$	-4.034	30	80
7	Gent	$-\infty$	-5.256	30	80
8	Voeren	20.344	22.012	50	66.2
9	Berneau	0	0	0	66.2
10	Liège	$-\infty$	-6.365	30	66.2
11	Warnand	0	0	0	66.2
12	Namur	$-\infty$	-2.12	0	66.2
13	Anderlues	0	1.2	0	66.2
14	Péronnes	0	0.96	0	66.2
15	Mons	$-\infty$	-6.848	0	66.2
16	Blagneries	$-\infty$	-15.616	50	66.2
17	Wanze	0	0	0	66.2
18	Sinsin	0	0	0	63
19	Arlon	$-\infty$	-0.222	0	66.2
20	Pétange	$-\infty$	-1.919	25	66.2

- We increase the bounds of the demands and the supplies with a factor 1.3 to make the existing design under-sized.
- We assume demand variability of 5%.
- Mosek conic MIP optimizer (beta version)

Impact of fixed costs on reinforcement

	Formulation					
#Arc	No fixed costs	Fixed costs	No fixed costs	Fixed costs	No fixed costs	Fixed costs
1	-	-	-	-	-	-
2	-	-	-	-	-	-
3	-	-	-	-	-	-
4	-	-	-	-	-	-
5	-	-	-	-	-	-
6	-	-	-	-	-	-
7	-	-	-	-	-	-
8	-	-	-	-	-	-
9	-	-	-	-	-	-
10	-	798.7	-	664.7	361.3	898.4
11	-	-	-	-	361.3	-
12	-	798.7	-	-	361.3	898.4
13	-	-	-	-	361.3	-
14	-	-	-	-	-	-
15	-	-	-	-	-	-
16	-	-	-	-	-	-
17	-	-	-	-	-	-
18	-	-	-	-	-	-
19	-	922.9	465.5	807.8	666.0	1000.0
20	-	-	-	-	163.5	-
21	-	-	-	-	178.2	-
22	170.5	397.6	264.0	-	316.2	428.5
23	170.5	-	264.0	359.7	316.2	428.5
24	122.0	-	234.1	-	288.8	401.6
Costs	1’508	1’508	1’770	1’770	3’320	3’317
CPU	0.1	140.4	0.1	72.0	0.1	77.4

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Impact of commercial diameters on reinforcement

	Investment budget with fixed costs					
	1’508		1’770		3’320	
#Arc	Continuous	Commercial	Continuous	Commercial	Continuous	Commercial
1	-	-	-	-	-	-
2	-	-	-	-	-	-
3	-	-	-	-	-	-
4	-	-	-	-	-	-
5	-	-	-	-	-	-
6	-	-	-	-	-	-
7	-	-	-	-	-	-
8	-	-	-	-	-	-
9	-	-	-	-	-	-
10	798.7	1000	664.7	1000	898.4	600
11	-	-	-	-	-	-
12	798.7	800	-	800	898.4	1000
13	-	-	-	-	-	-
14	-	-	-	-	-	-
15	-	-	-	-	-	-
16	-	-	-	-	-	-
17	-	-	-	-	-	-
18	-	-	-	-	-	-
19	922.9	800	807.8	1000	1000	1000
20	-	-	-	-	-	-
21	-	-	-	-	-	-
22	397.6	400	-	400	428.5	400
23	-	-	359.7	-	428.5	400
24	-	-	-	600	401.6	400
CPU	140.4	233.0	72.0	318.0	77.4	450.9

Robust investment solutions for $\alpha = 10$

#	Arc (O,D)	Existing	Determinist	Probability satisfaction 10%(k =0.5)	33% (k =1)	80% (k =2)
1	(1,2)	890	0	0	0	0
2	(1,2)	890	0	0	0	0
3	(2,3)	890	0	0	0	0
4	(2,3)	890	0	0	0	0
5	(3,4)	890	316	357	365	324
6	(5,6)	590.1	0	0	0	0
7	(6,7)	590.1	0	0	0	0
8	(7,4)	590.1	0	0	0	0
9	(4,14)	890	0	0	0	0
10	(8,9)	890	536	536	539	575
11	(8,9)	395.5	0	0	0	0
12	(9,10)	890	536	536	539	575
13	(9,10)	395.5	0	0	0	0
14	(10,11)	890	0	0	0	0
15	(10,11)	395.5	0	0	0	0
16	(11,12)	890	0	0	0	0
17	(12,13)	890	0	0	0	0
18	(13,14)	890	0	0	0	0
19	(14,15)	890	701	717	730	730
20	(15,16)	890	320	375	395	395
21	(11,17)	395.5	199	211	221	221
22	(17,18)	315.5	329	334	340	340
23	(18,19)	315.5	329	334	340	340
24	(19,20)	315.5	301	308	313	313

Next steps

- Validation procedure : generate a set of demand scenarios and find compatible pressures. Compute a quality of service.
- Combine all features.
- Improve compressor modelling

Thanks !!!