The behavioral dimension of optimization

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Outline

Transportation systems

Two dimensions

- Supply = infrastructure
- Demand = behavior, choices
- Congestion = mismatch

Transportation systems

Objectives

Minimize costs

Maximize satisfaction

Transportation systems

Maximize revenues

Revenues = Benefits - Costs

Costs: examples

- Building infrastructure
- Operating the system
- Environmental externalities

Benefits: examples

- Income from ticket sales
- Social welfare

Demand-supply interactions

Operations Research

- Given the demand...
- configure the system

Johnson City Enterprise.

Published Every Saturday,

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SATURDAY, APRIL 7, 1883.

TIME TABLE

E. T., V. & G. R. R.

PASSENGER,	ARRIVES,
No. 1, West,	6:37, a. m.
No. 2, East,	9:45, p. m.
No. 3, West,	11:51, p.m.
No. 4, East,	3:56, a. m.
LOCAL FREIGHT,	ARRIVES,
No. 5,	7:20, a. m.
No. 8,	6:20, p. m.

JNO. W. EAKIN, Agent.

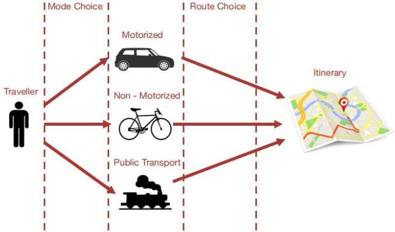
E. T. & W. N. C. R. R.

Passenger, leaves,	7, a. m.
" arrives,	6, p. m.

J. C. HARDIN, Agent.

Behavioral models

- Given the configuration of the system...
- predict the demand



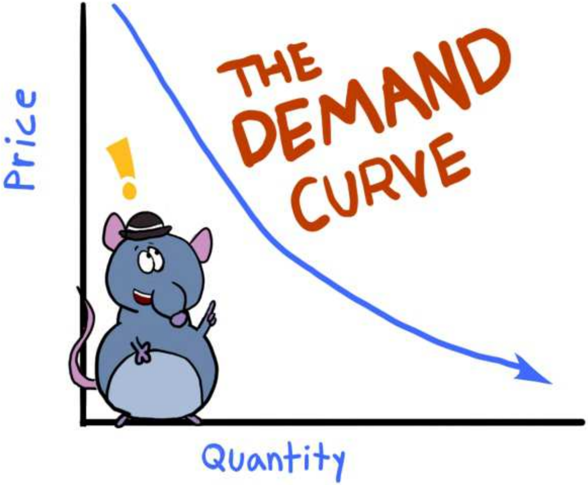
Research objectives

Framework for demand-supply interactions

- General: not designed for a specific application or context.
- Flexible: wide variety of demand and supply models.
- Scalable: the level of complexity can be adjusted.
- Integrated: not sequential.
- Operational: can be solved efficiently.

Outline

Aggregate demand



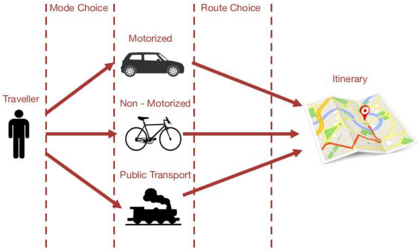
Aggregate demand

- Homogeneous population
- Identical behavior
- Price (P) and quantity (Q)
- Demand function: $Q = f(P)$
- Demand curve: $P = f^{-1}(Q)$

Disaggregate demand

- Heterogeneous population
- Different behaviors
- Many variables:
  - Attributes: price, travel time, reliability, frequency, etc.
  - Characteristics: age, income, education, etc.
- Complex demand/inverse demand functions.

Disaggregate demand



Behavioral models

- Demand = combination of individual choices.
- Modeling demand = modeling choice.
- Behavioral models: choice models.

Choice models

Daniel McFadden

- UC Berkeley 1963, MIT 1977, UC Berkeley 1991
- Laureate of *The Bank of Sweden Prize in Economic Sciences in Memory of Alfred Nobel 2000*
- Owns a farm and vineyard in Napa Valley
- “Farm work clears the mind, and the vineyard is a great place to prove theorems”

2000

Decision rules

Neoclassical economic theory

Preference-indifference operator $\succsim$

1. reflexivity

$$a \succsim a \quad \forall a \in \mathcal{C}_n$$

2. transitivity

$$a \succsim b \text{ and } b \succsim c \Rightarrow a \succsim c \quad \forall a, b, c \in \mathcal{C}_n$$

3. comparability

$$a \succsim b \text{ or } b \succsim a \quad \forall a, b \in \mathcal{C}_n$$

Decision rules

Utility

$$\exists\ U_n : \mathcal{C}_n \longrightarrow \mathbb{R} : a \rightsquigarrow U_n(a) \text{ such that}$$

$$a \gtrsim b \Leftrightarrow U_n(a) \geq U_n(b) \quad \forall a, b \in \mathcal{C}_n$$

Remarks

- Utility is a latent concept
- It cannot be directly observed

Decision rules

Choice

- Individual n
- Choice set $\mathcal{C}_n = \{1, \ldots, J_n\}$
- Utilities U_{in}, $\forall i \in \mathcal{C}_n$
- i is chosen iff $U_{in} = \max_{j \in \mathcal{C}_n} U_{jn}$
- Underlying assumption: no tie.

Example

Two transportation modes

$$\begin{aligned} U_1 &= -\beta t_1 - \gamma c_1 \\ U_2 &= -\beta t_2 - \gamma c_2 \end{aligned}$$

with β, $\gamma > 0$

Mode 1 is chosen if

$$U_1 \geq U_2 \text{ iff } -\beta t_1 - \gamma c_1 \geq -\beta t_2 - \gamma c_2$$

that is

$$-\frac{\beta}{\gamma} t_1 - c_1 \geq -\frac{\beta}{\gamma} t_2 - c_2$$

or

$$c_1 - c_2 \leq -\frac{\beta}{\gamma}(t_1 - t_2)$$

Example

Trade-off

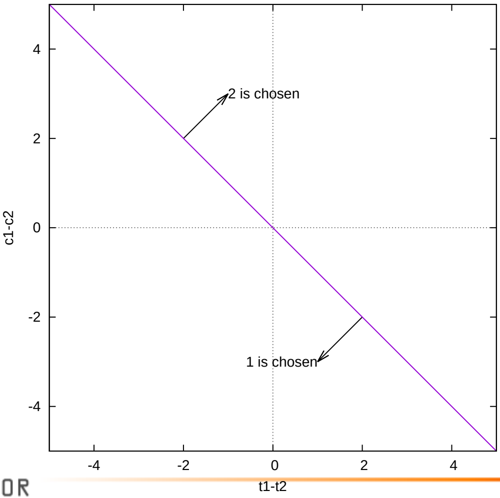
$$c_1 - c_2 \leq -\frac{\beta}{\gamma}(t_1 - t_2)$$

- $c_1 - c_2$ in currency unity (CHF)
- $t_1 - t_2$ in time units (hours)
- β/γ: CHF/hours

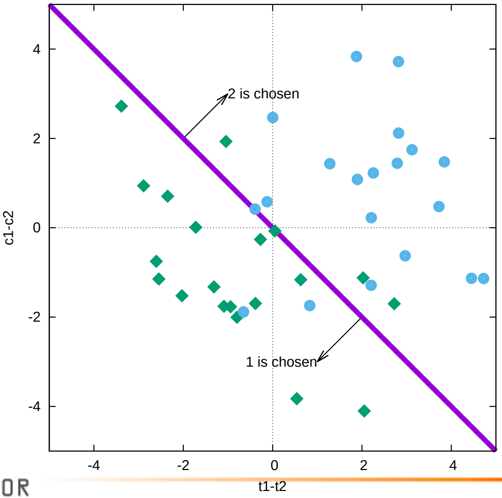
Value of time

Willingness to pay to save travel time.

Example



Example



Assumptions

Decision-maker

- perfect discriminating capability
- full rationality
- permanent consistency

Analyst

- knowledge of all attributes
- perfect knowledge of $\gtrsim$ (or $U_n(\cdot)$)
- no measurement error

Must deal with uncertainty

- Random utility models
- For each individual n and alternative i

$$U_{in} = V_{in} + \varepsilon_{in}$$

and

$$P(i|\mathcal{C}_n) = P[U_{in} = \max_{j \in \mathcal{C}_n} U_{jn}] = P(U_{in} \geq U_{jn} \; \forall j \in \mathcal{C}_n)$$

Logit model

Utility

$$U_{in} = V_{in} + \varepsilon_{in}$$

Availability

$$y_{in} \in \{0, 1\}$$

- Decision-maker n
- Alternative $i \in \mathcal{C}_n$

Choice probability: logit model

$$P_n(i|\mathcal{C}_n) = \frac{y_{in} e^{V_{in}}}{\sum_{j \in \mathcal{C}} y_{jn} e^{V_{jn}}}.$$

Variables: $x_{in} = (z_{in}, s_n)$

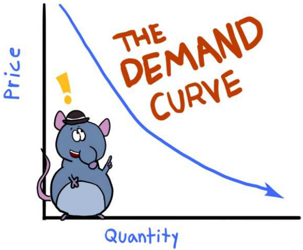
Attributes of alternative i: z_{in}

- Cost / price
- Travel time
- Waiting time
- Level of comfort
- Number of transfers
- Late/early arrival
- etc.

Characteristics of decision-maker n: s_n

- Income
- Age
- Sex
- Trip purpose
- Car ownership
- Education
- Profession
- etc.

Demand curve



Disaggregate model

$$P_n(i|p_{in}, z_{in}, s_n)$$

Total demand

$$D(i) = \sum_n P_n(i|p_{in}, z_{in}, s_n)$$

Difficulty

Non linear and non convex in p_{in} (and z_{in})

Outline

Optimization problem

Given...

the demand

Find...

the best configuration of the transportation system.

Example: airline

Context

- An airline considers to propose various destinations $i = \{1, \ldots, J\}$ to its customers.
- Each potential destination i is served by an aircraft, with capacity c_i.
- The price of the ticket for destination i is p_i.
- The demand is known: W_i passengers want to travel to i.
- The fixed cost of operating a flight to destination i is F_i.
- The airline cannot invest more than a budget B.

Question

What destinations should the airline serve to maximize its revenues?

Example: airline

Decisions variables

$y_i \in \{0, 1\}$: 1 if destination i is served, 0 otherwise.

Maximize revenues

$$\max \sum_{i=1}^{J} \min(W_i, c_i) p_i y_i$$

Constraints

$$\sum_{i=1}^{J} F_i y_i \leq B$$

Example: airline

Integer linear optimization problem

- Decision variables are integers.
- Objective function and constraints are linear.
- Here: knapsack problem.

Solving the problem

- Branch and bound
- Cutting planes

Example: airline

Pricing

- What price p_i should the airline propose?

$$\max \sum_{i=1}^{J} \min(W_i, c_i) p_i y_i$$

Issues

- Non linear objective
- Unbounded problem

Example: airline

Unbounded problem

- As demand is constant, the airline can make money with very high prices.
- We need to take into account the impact of price on demand.

Logit model

$$W_i = \sum_n P_n(i|p_i, z_{in}, s_n)$$

$$P_n(i|p_i, z_{in}, s_n) = \frac{y_i e^{V_{in}(p_i, z_{in}, s_n)}}{\sum_{j \in \mathcal{C}} y_j e^{V_{jn}(p_j, z_{jn}, s_n)}}.$$

The problem becomes highly non linear.

Outline

The main idea

The main idea

Linearization

Hopeless to linearize the logit formula (we tried...)

First principles

Each customer solves an optimization problem

Solution

Use the utility and not the probability

A linear formulation

Utility function

$$U_{in} = V_{in} + \varepsilon_{in} = \sum_k \beta_k x_{ink} + f(z_{in}) + \varepsilon_{in}.$$

Simulation

- Assume a distribution for ε_{in}
- E.g. logit: i.i.d. extreme value
- Draw R realizations ξ_{inr}, $r = 1, \ldots, R$
- The choice problem becomes deterministic

Scenarios

Draws

- Draw R realizations ξ_{inr}, $r = 1, \ldots, R$
- We obtain R scenarios

$$U_{inr} = \sum_k \beta_k x_{ink} + f(z_{in}) + \xi_{inr}.$$

- For each scenario r, we can identify the largest utility.
- It corresponds to the chosen alternative.

Comparing utilities

Variables

$$\mu_{ijnr} = \begin{cases} 1 & \text{if } U_{inr} \geq U_{jnr}, \\ 0 & \text{if } U_{inr} < U_{jnr}. \end{cases}$$

Constraints

$$(\mu_{ijnr} - 1)M_{nr} \leq U_{inr} - U_{jnr} \leq \mu_{ijnr} M_{nr}, \forall i, j, n, r.$$

where

$$|U_{inr} - U_{jnr}| \leq M_{nr}, \forall i, j,$$

Comparing utilities

$$(\mu_{ijnr} - 1) M_{nr} \leq U_{inr} - U_{jnr} \leq \mu_{ijnr} M_{nr}, \forall i, j, n, r.$$

Constraints: $\mu_{ijnr} = 1$

$$0 \leq U_{inr} - U_{jnr} \leq M_{nr}, \forall i, j, n, r.$$

$$U_{jnr} \leq U_{inr}, \forall i, j, n, r.$$

Constraints: $\mu_{ijnr} = 0$

$$-M_{nr} \leq U_{inr} - U_{jnr} \leq 0, \forall i, j, n, r.$$

$$U_{inr} \leq U_{jnr}, \forall i, j, n, r.$$

Comparing utilities

$$(\mu_{ijnr} - 1)M_{nr} \leq U_{inr} - U_{jnr} \leq \mu_{ijnr} M_{nr}, \forall i, j, n, r.$$

Equivalence if no tie

$$\begin{aligned}
\mu_{ijnr} = 1 &\implies U_{inr} \geq U_{jnr} \\
\mu_{ijnr} = 0 &\implies U_{inr} \leq U_{jnr} \\
U_{inr} > U_{jnr} &\implies \mu_{ijnr} = 1 \\
U_{inr} < U_{jnr} &\implies \mu_{ijnr} = 0
\end{aligned}$$

Accounting for availabilities

Motivation

- If $y_i = 0$, alternative i is not available.
- Its utility should not be involved in any constraint.

New variables: two alternatives are both available

$$\eta_{ij} = y_i y_j$$

Linearization:

$$\begin{aligned} y_i + y_j &\leq 1 + \eta_{ij}, \\ \eta_{ij} &\leq y_i, \\ \eta_{ij} &\leq y_j. \end{aligned}$$

Comparing utilities of available alternatives

Constraints

$$M_{nr}\eta_{ij} - 2M_{nr} \leq U_{inr} - U_{jnr} - M_{nr}\mu_{ijnr} \leq (1-\eta_{ij})M_{nr}, \forall i,j,n,r.$$

$\eta_{ij} = 1$ and $\mu_{ijnr} = 1$

$$0 \leq U_{inr} - U_{jnr} \leq M_{nr}, \forall i,j,n,r.$$

$\eta_{ij} = 1$ and $\mu_{ijnr} = 0$

$$-M_{nr} \leq U_{inr} - U_{jnr} \leq 0, \forall i,j,n,r.$$

Comparing utilities of available alternatives

Constraints

$$M_{nr}\eta_{ij} - 2M_{nr} \leq U_{inr} - U_{jnr} - M_{nr}\mu_{ijnr} \leq (1-\eta_{ij})M_{nr}, \forall i, j, n, r.$$

$\eta_{ij} = 0$ and $\mu_{ijnr} = 1$

$$-M_{nr} \leq U_{inr} - U_{jnr} \leq 2M_{nr}, \forall i, j, n, r,$$

$\eta_{ij} = 0$ and $\mu_{ijnr} = 0$

$$-2M_{nr} \leq U_{inr} - U_{jnr} \leq M_{nr}, \forall i, j, n, r,$$

Comparing utilities of available alternatives

Valid inequalities

$$\mu_{ijnr} \leq y_i, \qquad \forall i,j,n,r,$$
$$\mu_{ijnr} + \mu_{jinr} \leq 1, \qquad \forall i,j,n,r.$$

The choice

Variables

$$w_{inr} = \begin{cases} 1 & \text{if } n \text{ chooses } i \text{ in scenario } r, \\ 0 & \text{otherwise} \end{cases}$$

Maximum utility

$$w_{inr} \leq \mu_{ijnr}, \forall i, j, n, r.$$

Availability

$$w_{inr} \leq y_i, \forall i, n, r.$$

The choice

One choice

$$\sum_{i \in \mathcal{C}} w_{inr} = 1, \forall n, r.$$

Demand and revenues

Demand

$$W_i = \frac{1}{R} \sum_{n=1}^{n} \sum_{r=1}^{R} w_{inr}.$$

Revenues

$$R_i = \frac{1}{R} \sum_{n=1}^{N} p_i \sum_{r=1}^{R} w_{inr}.$$

Outline

A simple example

Data

- $\mathcal{C}$: set of movies
- Population of N individuals
- Utility function:
  $U_{in} = \beta_{in} p_{in} + f(z_{in}) + \varepsilon_{in}$

Decision variables

- What movies to propose? y_i
- What price? p_{in}

Demand model

Logit model

Probability that n chooses movie i:

$$P(i|y, p_n, z_n) = \frac{y_i e^{\beta_{in} p_{in} + f(z_{in})}}{\sum_j y_j e^{\beta_{jn} p_{jn} + f(z_{jn})}}$$

Total revenue:

$$\sum_{i \in C} y_i \sum_{n=1}^{N} p_{in} P(i|y, p_n, z_n)$$

Non linear and non convex in the decision variables

Example: programming movie theaters

Data

- Two alternatives: my theater (m) and the competition (c)
- We assume an homogeneous population of N individuals

$$U_c = 0 + \varepsilon_c$$
$$U_m = \beta_c p_m + \varepsilon_m$$

- $\beta_c < 0$
- Logit model: ε_m i.i.d. EV

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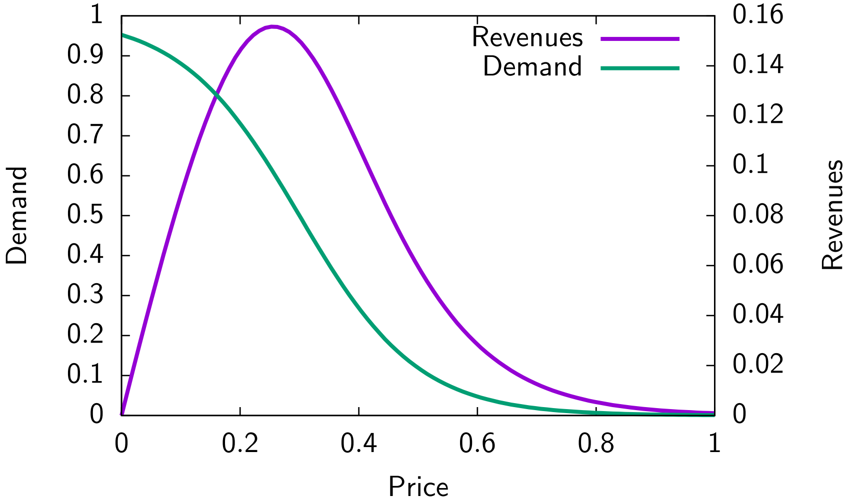
Demand and revenues



Optimization (with GLPK)

Data

- $N = 1$
- $R = 100$
- $U_m = -10p_m + 3$
- Prices: 0.10, 0.20, 0.30, 0.40, 0.50

Results

- Optimum price: 0.3
- Demand: 56%
- Revenues: 0.168

Heterogeneous population

Two groups in the population

$$U_{in} = \beta_n p_i + c_n$$

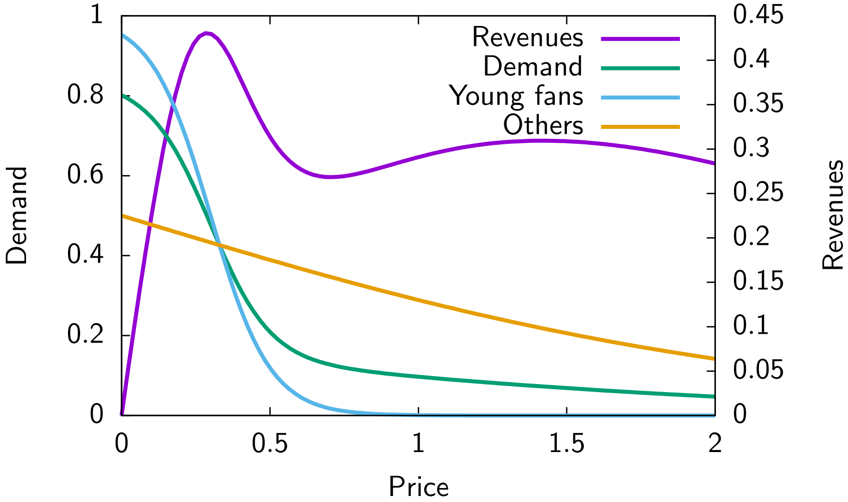
Young fans: 2/3

$\beta_1 = -10, \; c_1 = 3$

Others: 1/3

$\beta_1 = -0.9, \; c_1 = 0$

Demand and revenues



Optimization

Data

- $N = 3$
- $R = 100$
- $U_{m1} = -10p_m + 3$
- $U_{m2} = -0.9p_m$
- Prices: 0.3, 0.7, 1.1, 1.5, 1.9

Results

- Optimum price: 0.3
- Customer 1 (fan): 60% [theory: 50 %]
- Customer 2 (fan) : 49% [theory: 50 %]
- Customer 3 (other) : 45% [theory: 43 %]
- Demand: 1.54 (51%)
- Revenues: 0.48

Two theaters, different types of films

Two theaters, different types of films

Theater m

- Expensive
- Star Wars Episode VII

Theater k

- Cheap
- Tinker Tailor Soldier Spy

Heterogeneous demand

- Two third of the population is young (price sensitive)
- One third of the population is old (less price sensitive)

Two theaters, different types of films

Data

- Theaters m and k
- $N = 6$
- $R = 10$
- $U_{mn} = -10p_m + \textcircled{4}$, $n = 1, 2, 4, 5$
- $U_{mn} = -0.9p_m$, $n = 3, 6$
- $U_{kn} = -10p_k + \textcircled{0}$, $n = 1, 2, 4, 5$
- $U_{kn} = -0.9p_k$, $n = 3, 6$
- Prices m: 1.0, 1.2, 1.4, 1.6, 1.8
- Prices k: half price

Theater m

- Optimum price m: 1.6
- 4 young customers: 0
- 2 old customers: 0.5
- Demand: 0.5 (8.3%)
- Revenues: 0.8

Theater k

- Optimum price m: 0.5
- Young customers: 0.8
- Old customers: 1.5
- Demand: 2.3 (38%)
- Revenues: 1.15

Two theaters, same type of films

Theater m

- Expensive
- Star Wars Episode VII

Theater k

- Cheap
- Star Wars Episode VIII

Heterogeneous demand

- Two third of the population is young (price sensitive)
- One third of the population is old (less price sensitive)

Two theaters, same type of films

Data

- Theaters m and k
- $N = 6$
- $R = 10$
- $U_{mn} = -10p_m + \textcircled{4}$, $n = 1, 2, 4, 5$
- $U_{mn} = -0.9p_m$, $n = 3, 6$
- $U_{kn} = -10p_k + \textcircled{4}$, $n = 1, 2, 4, 5$
- $U_{kn} = -0.9p_k$, $n = 3, 6$
- Prices m: 1.0, 1.2, 1.4, 1.6, 1.8
- Prices k: half price

Theater m

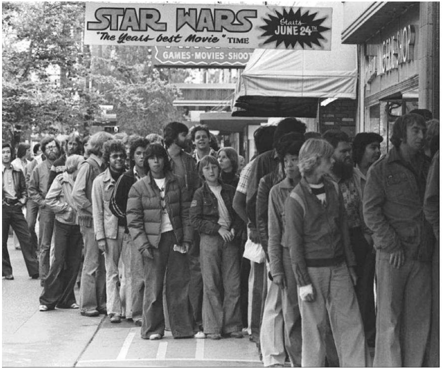
- Optimum price m: 1.8
- Young customers: 0
- Old customers: 1.9
- Demand: 1.9 (31.7%)
- Revenues: 3.42

Theater k

Closed

Extension: dealing with capacities

- Demand may exceed supply
- Not every choice can be accommodated
- Difficulty: who has access?
- Assumption: priority list is exogenous



Outline

Summary

Demand and supply

- Supply: prices and capacity
- Demand: choice of customers
- Interaction between the two

Discrete choice models

- Rich family of behavioral models
- Strong theoretical foundations
- Great deal of concrete applications
- Capture the heterogeneity of behavior
- Probabilistic models

Optimization

Discrete choice models

- Non linear and non convex
- Idea: use utility instead of probability
- Rely on simulation to capture stochasticity

Proposed formulation

- General: not designed for a specific application or context.
- Flexible: wide variety of demand and supply models.
- Scalable: the level of complexity can be adjusted.
- Integrated: not sequential.
- Operational: can be solved efficiently.

Ongoing research

Revenue management

Airlines, train operators, etc.

Decomposition methods

- Scenarios are (almost) independent from each other (except objective function)
- Individuals are also loosely coupled (except for capacity constraints)

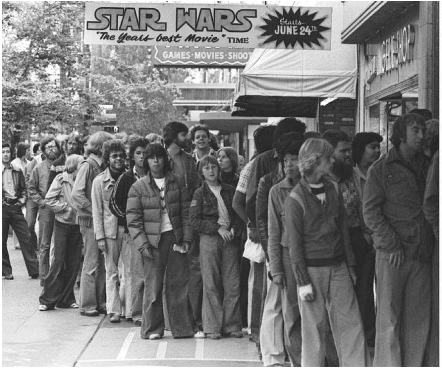
Thank you!

Questions?

Outline

Dealing with capacities

- Demand may exceed supply
- Not every choice can be accommodated
- Difficulty: who has access?
- Assumption: priority list is exogenous



Priority list

Application dependent

- First in, first out
- Frequent travelers
- Subscribers
- ...

In this framework

The list of customers must be sorted

Dealing with capacities

Variables

- y_{in}: decision of the operator
- y_{inr}: availability

Constraints

$$\sum_{n=1}^{N} w_{inr} \leq c_i$$

$$y_{inr} \leq y_{in}$$

$$y_{i(n+1)r} \leq y_{inr}$$

Constraints

$$c_i(1 - y_{inr}) \leq \sum_{m=1}^{n-1} w_{imr} + (1 - y_{in})c_{\max}$$

$y_{in} = 1,\ y_{inr} = 1$

$$0 \leq \sum_{m=1}^{n-1} w_{imr}$$

$y_{in} = 1,\ y_{inr} = 0$

$$c_i \leq \sum_{m=1}^{n-1} w_{imr}$$

$y_{in} = 0,\ y_{inr} = 0$

$$c_i \leq \sum_{m=1}^{n-1} w_{imr} + c_{\max}$$

Constraints

$$\sum_{m=1}^{n-1} w_{imr} + (1 - y_{in}) c_{\max} \leq (c_i - 1) y_{inr} + \max(n, c_{\max})(1 - y_{inr})$$

$y_{in} = 1,\ y_{inr} = 1$

$$1 + \sum_{m=1}^{n-1} w_{imr} \leq c_i$$

$y_{in} = 1,\ y_{inr} = 0$

$$\sum_{m=1}^{n-1} w_{imr} \leq \max(n, c_{\max})$$

$y_{in} = 0,\ y_{inr} = 0$

$$\sum_{m=1}^{n-1} w_{imr} + c_{\max} \leq \max(n, c_{\max})$$

Two theaters, different types of films

Data

- Theaters m and k
- Capacity: 2
- $N = 6$
- $R = 5$
- $U_{mn} = -10p_m + 4, \; n = 1, 2, 4, 5$
- $U_{mn} = -0.9p_m, \; n = 3, 6$
- $U_{kn} = -10p_k + 0, \; n = 1, 2, 4, 5$
- $U_{kn} = -0.9p_k, \; n = 3, 6$
- Prices m: 1.0, 1.2, 1.4, 1.6, 1.8
- Prices k: half price

Theater m

- Optimum price m: 1.8
- Demand: 0.2 (3.3%)
- Revenues: 0.36

Theater k

- Optimum price m: 0.5
- Demand: 2 (33.3%)
- Revenues: 1.15

Example of two scenarios

Customer	Choice	Capacity m	Capacity k
1	0	2	2
2	0	2	2
3	k	2	1
4	0	2	1
5	0	2	1
6	k	2	0

Customer	Choice	Capacity m	Capacity k
1	0	2	2
2	k	2	1
3	0	2	1
4	k	2	0
5	0	2	0
6	0	2	0

Two theaters: all prices divided by 2

Data

- Theaters m and k
- Capacity: 2
- $N = 6$
- $R = 5$
- $U_{mn} = -10p_m + 4, \; n = 1, 2, 4, 5$
- $U_{mn} = -0.9p_m, \; n = 3, 6$
- $U_{kn} = -10p_k + 0, \; n = 1, 2, 4, 5$
- $U_{kn} = -0.9p_k, \; n = 3, 6$
- Prices m: 0.5, 0.6, 0.7, 0.8, 0.9
- Prices k: half price

Theater m

- Optimum price m: 0.5
- Demand: 1.4
- Revenues: 0.7

Theater k

- Optimum price m: 0.45
- Demand: 1.6
- Revenues: 0.72

Example of two scenarios

Customer	Choice	Capacity m	Capacity k
1	0	2	2
2	0	2	2
3	0	2	2
4	k	2	1
5	k	2	0
6	0	2	0

Customer	Choice	Capacity m	Capacity k
1	k	2	1
2	k	2	0
3	0	2	0
4	m	1	0
5	0	1	0
6	m	0	0