Integrated demand and supply optimization

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Outline

Transportation systems

Two dimensions

- Supply = infrastructure
- Demand = behavior, choices
- Congestion = mismatch

Transportation systems

Objectives

Minimize costs

Maximize satisfaction

Transportation systems

Maximize revenues

Revenues = Benefits - Costs

Costs: examples

- Building infrastructure
- Operating the system
- Environmental externalities

Benefits: examples

- Income from ticket sales
- Social welfare

Demand-supply interactions

Operations Research

- Given the demand...
- configure the system

Behavioral models

- Given the configuration of the system...
- predict the demand

Johnson City Enterprise.

Published Every Saturday,

$1. per year—Advance Payment.

SATURDAY, APRIL 7, 1883.

TIME TABLE

E. T., V. & G. R. R.

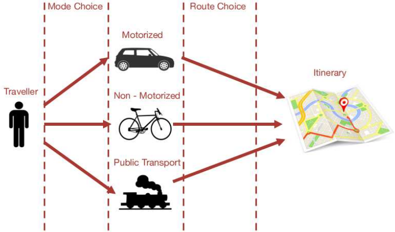
PASSENGER,	ARRIVES,
No. 1, West,	6:37, a. m.
No. 2, East,	9:45, p. m.
No. 3, West,	11:51, p.m.
No. 4, East,	3:56, a. m.
LOCAL FREIGHT,	ARRIVES,
No. 5,	7:20, a. m.
No. 8,	6:20, p. m.

JNO. W. EAKIN, Agent.

E. T. & W. N. C. R. R.

Passenger, leaves,	7, a. m.
" arrives,	6, p. m.

J. C. HARDIN, Agent.



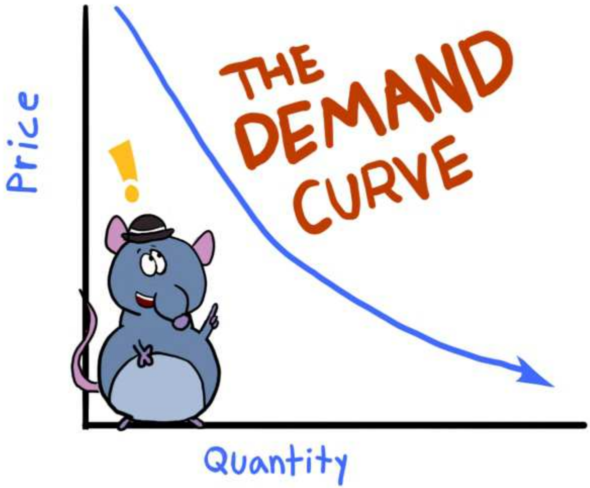
Research objectives

Framework for demand-supply interactions

- General: not designed for a specific application or context.
- Flexible: wide variety of demand and supply models.
- Scalable: the level of complexity can be adjusted.
- Integrated: not sequential.
- Operational: can be solved efficiently.

Outline

Aggregate demand



Aggregate demand

- Homogeneous population
- Identical behavior
- Price (P) and quantity (Q)
- Demand function: $Q = f(P)$
- Demand curve: $P = f^{-1}(Q)$

Disaggregate demand

- Heterogeneous population
- Different behaviors
- Many variables:
  - Attributes: price, travel time, reliability, frequency, etc.
  - Characteristics: age, income, education, etc.
- Complex demand/inverse demand functions.

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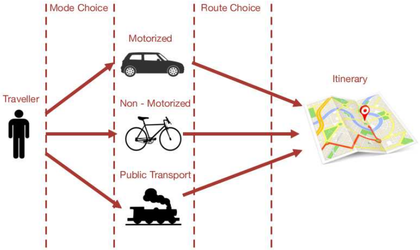
Disaggregate demand



Behavioral models

- Demand = combination of individual choices.
- Modeling demand = modeling choice.
- Behavioral models: choice models.

Choice models

Daniel McFadden

- UC Berkeley 1963, MIT 1977, UC Berkeley 1991
- Laureate of *The Bank of Sweden Prize in Economic Sciences in Memory of Alfred Nobel 2000*
- Owns a farm and vineyard in Napa Valley
- "Farm work clears the mind, and the vineyard is a great place to prove theorems"

2000

Decision rules

Neoclassical economic theory

Preference-indifference operator $\succsim$

1. reflexivity

$$a \succsim a \quad \forall a \in \mathcal{C}_n$$

2. transitivity

$$a \succsim b \text{ and } b \succsim c \Rightarrow a \succsim c \quad \forall a, b, c \in \mathcal{C}_n$$

3. comparability

$$a \succsim b \text{ or } b \succsim a \quad \forall a, b \in \mathcal{C}_n$$

Decision rules

Utility

$$\exists\ U_n : \mathcal{C}_n \longrightarrow \mathbb{R} : a \rightsquigarrow U_n(a) \text{ such that}$$

$$a \gtrsim b \Leftrightarrow U_n(a) \geq U_n(b) \quad \forall a, b \in \mathcal{C}_n$$

Remarks

- Utility is a latent concept
- It cannot be directly observed

Decision rules

Choice

- Individual n
- Choice set $\mathcal{C}_n = \{1, \ldots, J_n\}$
- Utilities U_{in}, $\forall i \in \mathcal{C}_n$
- i is chosen iff $U_{in} = \max_{j \in \mathcal{C}_n} U_{jn}$
- Underlying assumption: no tie.

Example

Two transportation modes

$$\begin{aligned} U_1 &= -\beta t_1 - \gamma c_1 \\ U_2 &= -\beta t_2 - \gamma c_2 \end{aligned}$$

with β, $\gamma > 0$

Mode 1 is chosen if

$$U_1 \geq U_2 \text{ iff } -\beta t_1 - \gamma c_1 \geq -\beta t_2 - \gamma c_2$$

that is

$$-\frac{\beta}{\gamma} t_1 - c_1 \geq -\frac{\beta}{\gamma} t_2 - c_2$$

or

$$c_1 - c_2 \leq -\frac{\beta}{\gamma}(t_1 - t_2)$$

Example

Trade-off

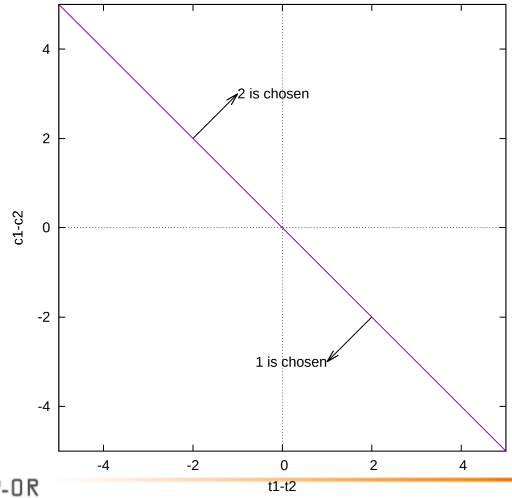
$$c_1 - c_2 \leq -\frac{\beta}{\gamma}(t_1 - t_2)$$

- $c_1 - c_2$ in currency unity (CHF)
- $t_1 - t_2$ in time units (hours)
- β/γ: CHF/hours

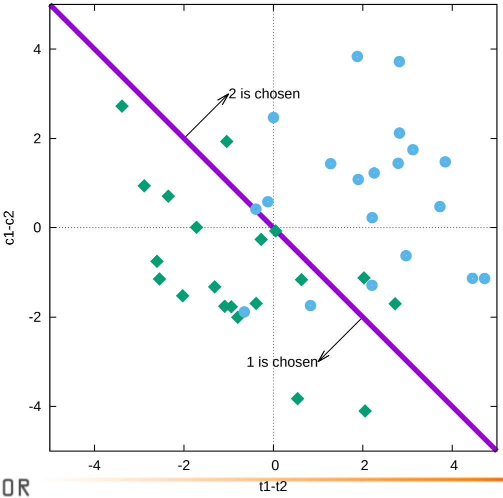
Value of time

Willingness to pay to save travel time.

Example



Example



Assumptions

Decision-maker

- perfect discriminating capability
- full rationality
- permanent consistency

Analyst

- knowledge of all attributes
- perfect knowledge of $\gtrsim$ (or $U_n(\cdot)$)
- no measurement error

Must deal with uncertainty

- Random utility models
- For each individual n and alternative i

$$U_{in} = V_{in} + \varepsilon_{in}$$

and

$$P(i|\mathcal{C}_n) = P[U_{in} = \max_{j \in \mathcal{C}_n} U_{jn}] = P(U_{in} \geq U_{jn}\ \forall j \in \mathcal{C}_n)$$

Logit model

Utility

$$U_{in} = V_{in} + \varepsilon_{in}$$

- Decision-maker n
- Alternative $i \in \mathcal{C}_n$

Choice probability: logit model

$$P_n(i|\mathcal{C}_n) = \frac{y_{in} e^{V_{in}}}{\sum_{j \in \mathcal{C}} y_{jn} e^{V_{jn}}}.$$

Variables: $x_{in} = (z_{in}, s_n)$

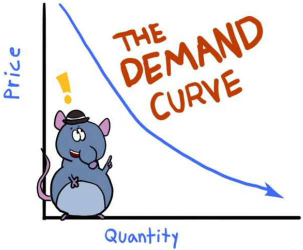
Attributes of alternative i: z_{in}

- Cost / price
- Travel time
- Waiting time
- Level of comfort
- Number of transfers
- Late/early arrival
- etc.

Characteristics of decision-maker n: s_n

- Income
- Age
- Sex
- Trip purpose
- Car ownership
- Education
- Profession
- etc.

Demand curve



Disaggregate model

$$P_n(i|c_{in}, z_{in}, s_n)$$

Total demand

$$D(i) = \sum_n P_n(i|c_{in}, z_{in}, s_n)$$

Difficulty

Non linear and non convex in c_{in} and z_{in}

Outline

Optimization problem

Given...

the demand

Find...

the best configuration of the transportation system.

Example: airline

Context

- An airline considers to propose various destinations $i = \{1, \ldots, J\}$ to its customers.
- Each potential destination i is served by an aircraft, with capacity c_i.
- The price of the ticket for destination i is p_i.
- The demand is known: W_i passengers want to travel to i.
- The fixed cost of operating a flight to destination i is F_i.
- The airline cannot invest more than a budget B.

Question

What destinations should the airline serve to maximize its revenues?

Example: airline

Decisions variables

$y_i \in \{0,1\}$: 1 if destination i is served, 0 otherwise.

Maximize revenues

$$\max \sum_{i=1}^{J} \min(W_i, c_i) p_i y_i$$

Constraints

$$\sum_{i=1}^{J} F_i y_i \leq B$$

Example: airline

Integer linear optimization problem

- Decision variables are integers.
- Objective function and constraints are linear.
- Here: knapsack problem.

Solving the problem

- Branch and bound
- Cutting planes

Example: airline

Pricing

- What price p_i should the airline propose?

$$\max \sum_{i=1}^{J} \min(W_i, c_i) p_i y_i$$

Issues

- Non linear objective
- Unbounded problem

Example: airline

Unbounded problem

- As demand is constant, the airline can make money with very high prices.
- We need to take into account the impact of price on demand.

Logit model

$$W_i = \sum_n P_n(i|p_i, z_{in}, s_n)$$

$$P_n(i|p_i, z_{in}, s_n) = \frac{y_i e^{V_{in}(p_i, z_{in}, s_n)}}{\sum_{j \in \mathcal{C}} y_j e^{V_{jn}(p_j, z_{jn}, s_n)}}.$$

The problem becomes highly non linear.

Outline

The main idea

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The main idea

Linearization

Hopeless to linearize the logit formula (we tried...)

First principles

Each customer solves an optimization problem

Solution

Use the utility and not the probability

A linear formulation

Utility function

$$U_{in} = V_{in} + \varepsilon_{in} = \sum_k \beta_k x_{ink} + f(z_{in}) + \varepsilon_{in}.$$

Simulation

- Assume a distribution for ε_{in}
- E.g. logit: i.i.d. extreme value
- Draw R realizations ξ_{inr}, $r = 1, \ldots, R$
- The choice problem becomes deterministic

Scenarios

Draws

- Draw R realizations ξ_{inr}, $r = 1, \ldots, R$
- We obtain R scenarios

$$U_{inr} = \sum_k \beta_k x_{ink} + f(z_{in}) + \xi_{inr}.$$

- For each scenario r, we can identify the largest utility.
- It corresponds to the chosen alternative.

Comparing utilities

Variables

$$\mu_{ijnr} = \begin{cases} 1 & \text{if } U_{inr} \geq U_{jnr}, \\ 0 & \text{if } U_{inr} < U_{jnr}. \end{cases}$$

Constraints

$$(\mu_{ijnr} - 1)M_{nr} \leq U_{inr} - U_{jnr} \leq \mu_{ijnr} M_{nr}, \forall i, j, n, r.$$

where

$$|U_{inr} - U_{jnr}| \leq M_{nr}, \forall i, j,$$

Comparing utilities

$$(\mu_{ijnr} - 1)M_{nr} \leq U_{inr} - U_{jnr} \leq \mu_{ijnr} M_{nr}, \forall i,j,n,r.$$

Constraints: $\mu_{ijnr} = 1$

$$0 \leq U_{inr} - U_{jnr} \leq M_{nr}, \forall i,j,n,r.$$

$$U_{jnr} \leq U_{inr}, \forall i,j,n,r.$$

Constraints: $\mu_{ijnr} = 0$

$$-M_{nr} \leq U_{inr} - U_{jnr} \leq 0, \forall i,j,n,r.$$

$$U_{inr} \leq U_{jnr}, \forall i,j,n,r.$$

Comparing utilities

$$(\mu_{ijnr} - 1)M_{nr} \leq U_{inr} - U_{jnr} \leq \mu_{ijnr} M_{nr}, \forall i, j, n, r.$$

Equivalence if no tie

$$\begin{aligned} \mu_{ijnr} = 1 &\Longrightarrow U_{inr} \geq U_{jnr} \\ \mu_{ijnr} = 0 &\Longrightarrow U_{inr} \leq U_{jnr} \\ U_{inr} > U_{jnr} &\Longrightarrow \mu_{ijnr} = 1 \\ U_{inr} < U_{jnr} &\Longrightarrow \mu_{ijnr} = 0 \end{aligned}$$

Accounting for availabilities

Motivation

- If $y_i = 0$, alternative i is not available.
- Its utility should not be involved in any constraint.

New variables: two alternatives are both available

$$\eta_{ij} = y_i y_j$$

Linearization:

$$y_i + y_j \leq 1 + \eta_{ij},$$
$$\eta_{ij} \leq y_i,$$
$$\eta_{ij} \leq y_j.$$

Comparing utilities of available alternatives

Constraints

$$M_{nr}\eta_{ij} - 2M_{nr} \leq U_{inr} - U_{jnr} - M_{nr}\mu_{ijnr} \leq (1-\eta_{ij})M_{nr}, \forall i,j,n,r.$$

$\eta_{ij} = 1$ and $\mu_{ijnr} = 1$

$$0 \leq U_{inr} - U_{jnr} \leq M_{nr}, \forall i,j,n,r.$$

$\eta_{ij} = 1$ and $\mu_{ijnr} = 0$

$$-M_{nr} \leq U_{inr} - U_{jnr} \leq 0, \forall i,j,n,r.$$

Comparing utilities of available alternatives

Constraints

$$M_{nr}\eta_{ij} - 2M_{nr} \leq U_{inr} - U_{jnr} - M_{nr}\mu_{ijnr} \leq (1-\eta_{ij})M_{nr}, \forall i,j,n,r.$$

$\eta_{ij} = 0$ and $\mu_{ijnr} = 1$

$$-M_{nr} \leq U_{inr} - U_{jnr} \leq 2M_{nr}, \forall i,j,n,r,$$

$\eta_{ij} = 0$ and $\mu_{ijnr} = 0$

$$-2M_{nr} \leq U_{inr} - U_{jnr} \leq M_{nr}, \forall i,j,n,r,$$

Comparing utilities of available alternatives

Valid inequalities

$$
\begin{aligned}
\mu_{ijnr} &\leq y_i, & \forall i, j, n, r, \\
\mu_{ijnr} + \mu_{jinr} &\leq 1, & \forall i, j, n, r.
\end{aligned}
$$

The choice

Variables

$$w_{inr} = \begin{cases} 1 & \text{if } n \text{ chooses } i \text{ in scenario } r, \\ 0 & \text{otherwise} \end{cases}$$

Maximum utility

$$w_{inr} \leq \mu_{ijnr}, \forall i, j, n, r.$$

Availability

$$w_{inr} \leq y_i, \forall i, n, r.$$

The choice

One choice

$$\sum_{i \in \mathcal{C}} w_{inr} = 1, \forall n, r.$$

Demand and revenues

Demand

$$W_i = \frac{1}{R} \sum_{n=1}^{n} \sum_{r=1}^{R} w_{inr}.$$

Revenues

$$R_i = \frac{1}{R} \sum_{n=1}^{N} p_i \sum_{r=1}^{R} w_{inr}.$$

Outline

A simple example

Data

- $\mathcal{C}$: set of movies
- Population of N individuals
- Utility function:
  $U_{in} = \beta_{in} p_{in} + f(z_{in}) + \varepsilon_{in}$

Decision variables

- What movies to propose? y_i
- What price? p_{in}

Demand model

Logit model

Probability that n chooses movie i:

$$P(i|y,p_n,z_n) = \frac{y_i e^{\beta_{in} p_{in} + f(z_{in})}}{\sum_j y_j e^{\beta_{jn} p_{jn} + f(z_{jn})}}$$

Total revenue:

$$\sum_{i \in C} y_i \sum_{n=1}^{N} p_{in} P(i|y,p_n,z_n)$$

Non linear and non convex in the decision variables

Example: programming movie theaters

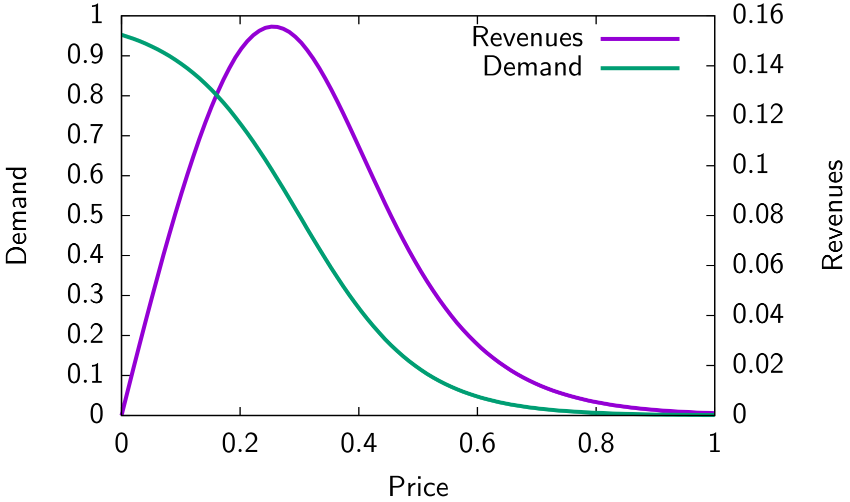
Data

- Two alternatives: my theater (m) and the competition (c)
- We assume an homogeneous population of N individuals

$$U_c = 0 + \varepsilon_c$$
$$U_m = \beta_c p_m + \varepsilon_m$$

- $\beta_c < 0$
- Logit model: ε_m i.i.d. EV

Demand and revenues



Optimization (with GLPK)

Data

- $N = 1$
- $R = 100$
- $U_m = -10p_m + 3$
- Prices: 0.10, 0.20, 0.30, 0.40, 0.50

Results

- Optimum price: 0.3
- Demand: 56%
- Revenues: 0.168

Heterogeneous population

Two groups in the population

$$U_{in} = \beta_n p_i + c_n$$

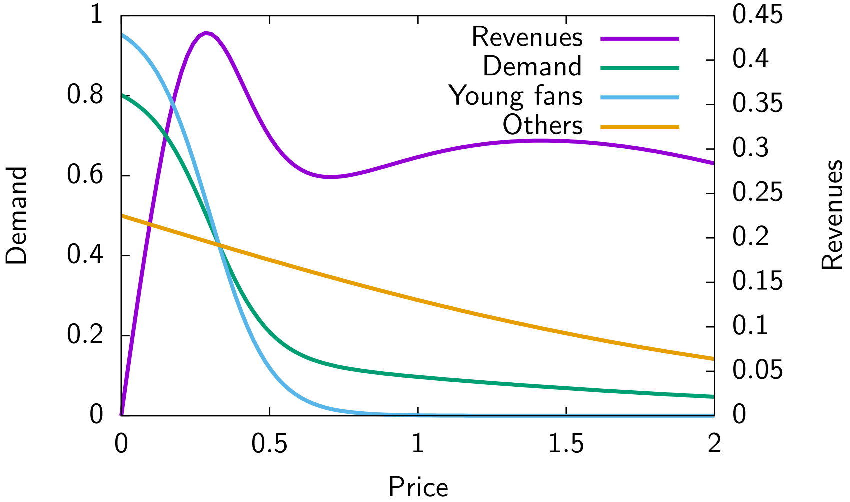
Young fans: 2/3

$\beta_1 = -10,\ c_1 = 3$

Others: 1/3

$\beta_1 = -0.9,\ c_1 = 0$

Demand and revenues



Optimization

Data

- $N = 3$
- $R = 100$
- $U_{m1} = -10p_m + 3$
- $U_{m2} = -0.9p_m$
- Prices: 0.3, 0.7, 1.1, 1.5, 1.9

Results

- Optimum price: 0.3
- Customer 1 (fan): 60% [theory: 50 %]
- Customer 2 (fan) : 49% [theory: 50 %]
- Customer 3 (other) : 45% [theory: 43 %]
- Demand: 1.54 (51%)
- Revenues: 0.48

Two theaters, different types of films

Two theaters, different types of films

Theater m

- Expensive
- Star Wars Episode VII

Theater k

- Cheap
- Tinker Tailor Soldier Spy

Heterogeneous demand

- Two third of the population is young (price sensitive)
- One third of the population is old (less price sensitive)

Two theaters, different types of films

Data

- Theaters m and k
- $N = 6$
- $R = 10$
- $U_{mn} = -10p_m + 4$, $n = 1,2,4,5$
- $U_{mn} = -0.9p_m$, $n = 3,6$
- $U_{kn} = -10p_k + 0$, $n = 1,2,4,5$
- $U_{kn} = -0.9p_k$, $n = 3,6$
- Prices m: 1.0, 1.2, 1.4, 1.6, 1.8
- Prices k: half price

Theater m

- Optimum price m: 1.6
- 4 young customers: 0
- 2 old customers: 0.5
- Demand: 0.5 (8.3%)
- Revenues: 0.8

Theater k

- Optimum price m: 0.5
- Young customers: 0.8
- Old customers: 1.5
- Demand: 2.3 (38%)
- Revenues: 1.15

Two theaters, same type of films

Theater m

- Expensive
- Star Wars Episode VII

Theater k

- Cheap
- Star Wars Episode VIII

Heterogeneous demand

- Two third of the population is young (price sensitive)
- One third of the population is old (less price sensitive)

Two theaters, same type of films

Data

- Theaters m and k
- $N = 6$
- $R = 10$
- $U_{mn} = -10p_m + \textcircled{4}$, $n = 1, 2, 4, 5$
- $U_{mn} = -0.9p_m$, $n = 3, 6$
- $U_{kn} = -10p_k + \textcircled{4}$, $n = 1, 2, 4, 5$
- $U_{kn} = -0.9p_k$, $n = 3, 6$
- Prices m: 1.0, 1.2, 1.4, 1.6, 1.8
- Prices k: half price

Theater m

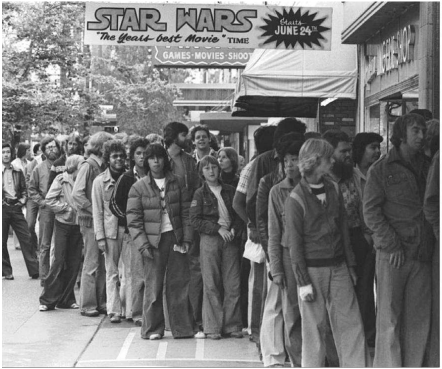
- Optimum price m: 1.8
- Young customers: 0
- Old customers: 1.9
- Demand: 1.9 (31.7%)
- Revenues: 3.42

Theater k

Closed

Extension: dealing with capacities

- Demand may exceed supply
- Not every choice can be accommodated
- Difficulty: who has access?
- Assumption: priority list is exogenous



Outline

Summary

Demand and supply

- Supply: prices and capacity
- Demand: choice of customers
- Interaction between the two

Discrete choice models

- Rich family of behavioral models
- Strong theoretical foundations
- Great deal of concrete applications
- Capture the heterogeneity of behavior
- Probabilistic models

Optimization

Discrete choice models

- Non linear and non convex
- Idea: use utility instead of probability
- Rely on simulation to capture stochasticity

Proposed formulation

- General: not designed for a specific application or context.
- Flexible: wide variety of demand and supply models.
- Scalable: the level of complexity can be adjusted.
- Integrated: not sequential.
- Operational: can be solved efficiently.

Ongoing research

Revenue management

Airlines, train operators, etc.

Decomposition methods

- Scenarios are (almost) independent from each other (except objective function)
- Individuals are also loosely coupled (except for capacity constraints)

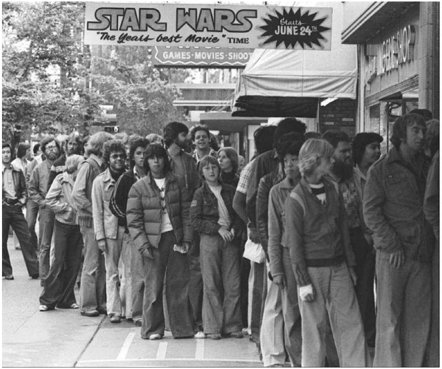
Thank you!

Questions?

Outline

Dealing with capacities

- Demand may exceed supply
- Not every choice can be accommodated
- Difficulty: who has access?
- Assumption: priority list is exogenous



Priority list

Application dependent

- First in, first out
- Frequent travelers
- Subscribers
- ...

In this framework

The list of customers must be sorted

Dealing with capacities

Variables

- y_{in}: decision of the operator
- y_{inr}: availability

Constraints

$$\sum_{n=1}^{N} w_{inr} \leq c_i$$

$$y_{inr} \leq y_{in}$$

$$y_{i(n+1)r} \leq y_{inr}$$

Constraints

$$c_i(1 - y_{inr}) \leq \sum_{m=1}^{n-1} w_{imr} + (1 - y_{in}) c_{\max}$$

$y_{in} = 1, \; y_{inr} = 1$

$$0 \leq \sum_{m=1}^{n-1} w_{imr}$$

$y_{in} = 1, \; y_{inr} = 0$

$$c_i \leq \sum_{m=1}^{n-1} w_{imr}$$

$y_{in} = 0, \; y_{inr} = 0$

$$c_i \leq \sum_{m=1}^{n-1} w_{imr} + c_{\max}$$

Constraints

$$\sum_{m=1}^{n-1} w_{imr} + (1 - y_{in})c_{\max} \leq (c_i - 1)y_{inr} + \max(n, c_{\max})(1 - y_{inr})$$

$y_{in} = 1,\ y_{inr} = 1$

$$1 + \sum_{m=1}^{n-1} w_{imr} \leq c_i$$

$y_{in} = 1,\ y_{inr} = 0$

$$\sum_{m=1}^{n-1} w_{imr} \leq \max(n, c_{\max})$$

$y_{in} = 0,\ y_{inr} = 0$

$$\sum_{m=1}^{n-1} w_{imr} + c_{\max} \leq \max(n, c_{\max})$$

Two theaters, different types of films

Data

- Theaters m and k
- Capacity: 2
- $N = 6$
- $R = 5$
- $U_{mn} = -10p_m + 4, \; n = 1, 2, 4, 5$
- $U_{mn} = -0.9p_m, \; n = 3, 6$
- $U_{kn} = -10p_k + 0, \; n = 1, 2, 4, 5$
- $U_{kn} = -0.9p_k, \; n = 3, 6$
- Prices m: 1.0, 1.2, 1.4, 1.6, 1.8
- Prices k: half price

Theater m

- Optimum price m: 1.8
- Demand: 0.2 (3.3%)
- Revenues: 0.36

Theater k

- Optimum price m: 0.5
- Demand: 2 (33.3%)
- Revenues: 1.15

Example of two scenarios

Customer	Choice	Capacity m	Capacity k
1	0	2	2
2	0	2	2
3	k	2	1
4	0	2	1
5	0	2	1
6	k	2	0

Customer	Choice	Capacity m	Capacity k
1	0	2	2
2	k	2	1
3	0	2	1
4	k	2	0
5	0	2	0
6	0	2	0

Two theaters: all prices divided by 2

Data

- Theaters m and k
- Capacity: 2
- $N = 6$
- $R = 5$
- $U_{mn} = -10p_m + 4, \; n = 1, 2, 4, 5$
- $U_{mn} = -0.9p_m, \; n = 3, 6$
- $U_{kn} = -10p_k + 0, \; n = 1, 2, 4, 5$
- $U_{kn} = -0.9p_k, \; n = 3, 6$
- Prices m: 0.5, 0.6, 0.7, 0.8, 0.9
- Prices k: half price

Theater m

- Optimum price m: 0.5
- Demand: 1.4
- Revenues: 0.7

Theater k

- Optimum price m: 0.45
- Demand: 1.6
- Revenues: 0.72

Example of two scenarios

Customer	Choice	Capacity m	Capacity k
1	0	2	2
2	0	2	2
3	0	2	2
4	k	2	1
5	k	2	0
6	0	2	0

Customer	Choice	Capacity m	Capacity k
1	k	2	1
2	k	2	0
3	0	2	0
4	m	1	0
5	0	1	0
6	m	0	0