

An Economic Analysis of Pervasive, Incentive-Based Demand Response

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Abstract—Demand response (DR) emerges as one of the cheapest and greenest solutions to match supply and demand in the electricity sector. While DR has been focused on large and industrial consumers, pervasive implementation (by including residential consumers) is needed to maximize its potential. This paper presents theoretical analysis of pervasive, incentive-based DR from the economics perspective. Our analysis consider cases whether (1) DR is used to encourage consumers to decrease or increase their demand, and (2) utility companies have access to a single or multiple energy sources. We determine the necessary conditions and derive the optimal incentives to benefit from DR events.

I. INTRODUCTION

While there are two types of DR, i.e., price- and incentive-based [1]–[3], this paper focuses on incentive-based DR, where under flat-rate price, consumers are offered some incentives to alter their energy consumption. The incentives can be, for example, movie tickets, bill rebates, redeemable vouchers, or virtual currencies which can be converted into real products or other benefits (as in the concept of *miles* in airline marketing). In addition, messages sent to communicate upcoming DR events can also be seen as triggers as to *when to act* [4].

DR has long been focused on large, industrial consumers. To maximize its potential, however, DR implementation needs to be pervasive by attracting residential consumers as well. To this end, the vision of smart grids and deployment of smart meters provided a wide-open opportunity to include the participation of residential consumers. Consequently, deep insight about pervasive DR is necessary. For example, *what are the necessary conditions to benefit from DR? How much incentives should be given to consumers?*

However, these tasks are very challenging due to at least three reasons. First, utility companies acquire their energy supply from various sources. Some of them buy energy from the market, some own coal power plant, some use natural gas (or other sources), and some own several generators powered by different sources, which contribute differently to companies' profit. Second, DR can be used not only to induce lower energy consumption, but also to encourage higher consumption, especially in case of energy surplus in the grid. The surplus can happen, for example, when the sun shines (for area with high penetration of solar power) or when the wind blows strongly (for area with high penetration of wind power) but the demand is low. Third, there are a lot of uncertainties in consumer responses, since they are influenced by numerous factors, such as the incentives, time of day, day of the week, outdoor temperature, weather, holidays, or guests at home.

Overview of Contributions. This paper provides theoret-

ical analysis of pervasive, incentive-based DR. Our analysis focuses more on the economic subsystem rather than the physical subsystem.¹ We consider cases where DR is used as a mechanism to reduce and increase energy consumption. We determine the lower bound of the generation cost and *consumer reduction (or increase) rate*² such that DR is still beneficial. Additionally, we derive the optimal incentives to maximize gains from DR events. We present our analysis in cases where utility companies have access to both, single and multiple energy sources. This paper provides a formalization and base model to analyze/understand DR incentive schemes further.

II. KEY ASSUMPTIONS

A. Load Generation

Let us assume that a utility company has access to a set of generators or energy sources G . For a particular time period, let:

- L_{g_i} be the load assigned to generator $g_i \in G$, and $L = \sum_{g_i \in G} L_{g_i}$,
- $cap(g_i)$ be the capacity of generator g_i , and
- $C_{g_i}(L')$ be the total cost of meeting load demand L' using generator g_i for the discussed period.

To meet the load demand L , a utility company assigns L to one or more generators³ depending on the capacity and generation cost. We assume that the utility company assigns (or, the market implicitly assigns) the load to the cheapest generator first, up to its capacity, before using the more expensive ones. Thus, for simplicity, we assume that the set of generators G is ordered by its generation cost, i.e., $C_{g_i}(L') \leq C_{g_j}(L')$ for $i \leq j$. Thus, for $j = i + 1$, we have $L_{g_j} > 0$ iff $L_{g_i} = cap(g_i)$ and $L > \sum_{1 \leq k \leq i} L_{g_k}$.⁴

We also assume that there are no temporal constraints⁵ and

¹While the physical subsystem considers hardware that physically produces and transmits electricity, the economic subsystem considers the actors that are involved in the production, trade, or consumption of electricity and their mutual relationships [5].

²It indicates how sensitive a consumer is to incentives (see also Section II-B). The higher the consumer reduction (or increase) rate, the more sensitive she is.

³We use the terms *energy sources* and *generators* interchangeably.

⁴Note that, we consider the ordering here only for the discussed time period. The ordering could change, of course, from one period to another depending on the generation cost.

⁵For example, generator ramp constraints and start-up constraints. Note that, although large power plants (coal/nuclear) have non-negligible ramping constraints or startup cost, small-scale distribution utility companies, who also serves consumers, typically use smaller generation units (e.g., gas-powered) with small startup costs and higher ramping capabilities.

no appropriate energy storage solutions available to the utility company.

B. Consumer Responses

One of the main challenges faced by utility companies to carry out DR events for residential consumers is that only little is known about how consumers will respond to DR signals. Consumer responses can be affected by many factors, such as the amount of incentives, time of the day, day of the week, weather, outdoor temperature, holidays, or guests at home. Understanding the influence of these factors requires real implementation. However, utility companies need to have an understanding and holistic view about DR before starting any real implementation. Hence, we have a *chicken and egg* problem here.

Inspired by prior research on consumer response to dynamic pricing, which confirmed that consumer energy reduction increases as the electricity prices increases [6], we assume that consumer response grows proportionally with the amount of incentives, given other factors are held constant. For simplicity, below we specify consumer responses as demand reduction (see Section IV for consumer responses in the context of DR to increase demand). Let $\{x_1, x_2, \dots, x_n\}$ be the set of factors that (possibly) influence consumer responses, and $x_1 = \mathcal{I}$ be the incentives (unit: \$).⁶ We define consumer responses (or the demand reduction) during a DR event as:

$$r(x_1, x_2, \dots, x_n) = r(\mathcal{I}, x_2, \dots, x_n) = m\mathcal{I} + c, \quad (1)$$

where m is the *consumer reduction rate* (unit: $kWh/\$$), and c is a constant. We assume $m \geq 0$ and $\mathcal{I} \geq 0$. Note that, m may vary depending on other factors x_2, \dots, x_n (e.g., time of day, day of week, temperature). Here, m is the consumer reduction rate in the discussed period.⁷ We also assume that any reduction during a DR event requires some incentives, i.e., $c = 0$. Thus, we have:

$$r(\mathcal{I}, x_2, \dots, x_n) = m\mathcal{I}. \quad (2)$$

Intuitively, consumer reduction rate m is the amount of demand reduction that can be obtained using a unit of incentive. It can also be thought of as $\frac{dr}{d\mathcal{I}}$, the rate of reduction per unit incentive. In the sequel, we often use R instead of $r(\mathcal{I}, f_1, \dots, f_n)$ to simplify notation.

III. DR TO REDUCE ENERGY CONSUMPTION

A. Without DR (Business As Usual)

Cost We define the cost to meet load demand L as $\sum_{g_i \in G} C_{g_i}(L_{g_i})$, where $L = \sum_{g_i \in G} L_{g_i}$.

Revenue Let P_{ret} be the retail price (unit: $\$/kWh$) paid by the consumers, and L be the total demand. Thus, company revenue = $P_{ret}L$.

Profit We define company's profit as its revenue minus cost:

$$P_{ret}L - \sum_{g_i \in G} C_{g_i}(L_{g_i}) \quad (3)$$

⁶In practice, the incentives can be of form movie tickets, bill rebates, redeemable vouchers, etc. For simplicity, we quantify \mathcal{I} with its monetary value (unit: \$) – company's cost to provide the incentives.

⁷While identifying how m varies depending on other factors is interesting by itself, but it is beyond the scope of this paper. In practice, it can be estimated by the demand reduction for a unit of incentive.

B. With DR

Let \mathcal{I} be the incentives given to the consumers, and R denotes the consumers' demand reduction in a particular DR event. Thus, the total demand in the presence of the event is $L^{DR} = L - R$.

Cost The cost of meeting the load demand L^{DR} is $\sum_{g_i \in G} C_{g_i}(L_{g_i}^{DR})$, where $L^{DR} = \sum_{g_i \in G} L_{g_i}^{DR}$.

Revenue Company's revenue = $P_{ret}L^{DR}$.

Profit Company's profit = revenue - cost - incentives:

$$P_{ret}L^{DR} - \sum_{g_i \in G} C_{g_i}(L_{g_i}^{DR}) - \mathcal{I} \quad (4)$$

Gain We define utility company's gain from a DR event as the difference between its profit with and without the DR event, i.e., by subtracting Eq. 3 from Eq. 4:

$$\begin{aligned} gain &= P_{ret}L^{DR} - \sum_{g_i \in G} C_{g_i}(L_{g_i}^{DR}) - \mathcal{I} - P_{ret}L + \\ &\quad \sum_{g_i \in G} C_{g_i}(L_{g_i}) \\ &= \sum_{g_i \in G} (C_{g_i}(L_{g_i}) - C_{g_i}(L_{g_i}^{DR})) - \\ &\quad P_{ret}(L - L^{DR}) - \mathcal{I} \\ &= \sum_{g_i \in G} (C_{g_i}(L_{g_i}) - C_{g_i}(L_{g_i}^{DR})) - P_{ret}R - \mathcal{I}. \end{aligned} \quad (5)$$

If $L^{DR} \leq L$, then there are some generators that are unused or do not run at their full capacity. Let us denote this set of generators as G_{red} , i.e., the *reduced* generators. And, we define the rest of the generators as $G_{base} = G \setminus G_{red}$. Formally:

- $g_i \in G_{red}$, iff $L_{g_i}^{DR} < L_{g_i}$, and
- $g_i \in G_{base}$, iff $L_{g_i}^{DR} = L_{g_i}$.

For instance, when there is a demand reduction during a DR event, then by definition, the most expensive generator belongs to G_{red} . We can rewrite the gain computation in Eq. 5 by separating the set of generators in G_{base} and G_{red} as:

$$\begin{aligned} gain &= \sum_{g_i \in G} (C_{g_i}(L_{g_i}) - C_{g_i}(L_{g_i}^{DR})) - P_{ret}R - \mathcal{I} \\ &= \sum_{g_i \in G_{base}} (C_{g_i}(L_{g_i}) - C_{g_i}(L_{g_i}^{DR})) + \\ &\quad \sum_{g_i \in G_{red}} (C_{g_i}(L_{g_i}) - C_{g_i}(L_{g_i}^{DR})) - P_{ret}R - \mathcal{I} \\ &= \sum_{g_i \in G_{red}} (C_{g_i}(L_{g_i}) - C_{g_i}(L_{g_i}^{DR})) - P_{ret}R - \mathcal{I} \end{aligned} \quad (6)$$

Thus, company's gain from a DR event depends on the (i) difference in the total cost of meeting the load demand, (ii) reduction in the revenue, and (iii) total incentives.

The next challenge is to determine just the right incentives to obtain positive gain (or benefit) from DR events. After identifying conditions to obtain positive gain, we aim to find the optimal incentives to maximize gain. To this end, we divide the problem into two cases, i.e., first, where a utility company has access to a single energy source, and second, in a more general setting, where a utility company has access to multiple energy sources.

C. Single Energy Source

In this section, we study more deeply the setting where a utility company has access to only one energy source, i.e., $|G| = 1$. The energy source can be, for example, an energy market. Let P_{mkt} be the unit cost to meet the load demand for the period under discussion, or the market price if the energy

source is an energy market. Then, we can rewrite the gain computation in Eq. 6:

$$\begin{aligned} \text{gain} &= \sum_{g_i \in G_{\text{red}}} (C_{g_i}(L_{g_i}) - C_{g_i}(L_{g_i}^{DR})) - P_{\text{ret}}R - \mathcal{I} \\ &= P_{\text{mkt}}L - P_{\text{mkt}}L^{DR} - P_{\text{ret}}R - \mathcal{I} \\ &= P_{\text{mkt}}R - P_{\text{ret}}R - \mathcal{I} = (P_{\text{mkt}} - P_{\text{ret}})R - \mathcal{I}. \end{aligned} \quad (7)$$

Note that, when the energy source considered is indeed an energy market, then the market price before and after DR events might be different due to the changes in the demand. In this case, however, we assume that demand reduction R does not influence the market price, i.e., it is much smaller compared to the overall demand in the market. Next, without loss of generality, we assume that the energy source is the market.

1) *The lower bound of market price:* By substituting R from Eq. 2 into Eq. 7, we obtain:

$$\begin{aligned} \text{gain} &= (P_{\text{mkt}} - P_{\text{ret}})m\mathcal{I} - \mathcal{I} \\ &= ((P_{\text{mkt}} - P_{\text{ret}})m - 1)\mathcal{I}. \end{aligned} \quad (8)$$

To have positive gain, $(P_{\text{mkt}} - P_{\text{ret}})m - 1$ should be positive. Thus:

$$P_{\text{mkt}} > \frac{1}{m} + P_{\text{ret}}. \quad (9)$$

Recall that the consumer reduction rate, m (unit: $kWh/\$$), expresses the amount of energy a consumer willing to sacrifice for a unit of incentive during a DR event. In economics, m is also known as consumer's *willingness to pay*, that is, the maximum amount of energy a consumer is willing to sacrifice from her normal consumption level for a unit incentive. On the other hand, $\frac{1}{m}$ (unit: $\$/kWh$) is consumer *willingness to accept*, i.e., the minimum amount of incentive a consumer is willing to accept for a unit of energy she sacrifices from her normal consumption during a DR event.

Inequality in Eq. 9 states that DR can bring an advantage to utility companies when the market price is greater than the retail price plus the consumer *willingness to accept*. It provides deeper insight to the common belief that DR should be carried out when $P_{\text{mkt}} > P_{\text{ret}}$. The customer willingness to accept should be taken into account as well, i.e., $P_{\text{mkt}} > \frac{1}{m} + P_{\text{ret}}$.

2) *The lower bound of consumer reduction rate:* From Eq. 9, we can also derive the lowest consumer reduction rate required to obtain positive gain:

$$m > \frac{1}{P_{\text{mkt}} - P_{\text{ret}}}. \quad (10)$$

That is, the higher the difference between the market and the retail price, the less sensitive the consumers that a company needs in its portfolio for a DR event, i.e., having consumers with low m works just fine. However, when there is only a marginal difference between the market and the retail price, the company should prefer consumers that are highly sensitive to incentives.

3) *The optimal incentives:* When Eq. 9 or 10 is satisfied, setting larger \mathcal{I} leads to higher gain (see Eq. 8). However, a larger \mathcal{I} also causes a larger R . Thus, when $R > L$, it means that consumers give some energy back to the grid (or producing energy, i.e., becoming *prosumers*). Then, the (positive) gain is due to consumers' energy price, $\frac{1}{m}$, is cheaper than $P_{\text{mkt}} - P_{\text{ret}}$. However, demand reduction is typically limited. This can be, for example, because the grids accept only

limited bidirectional energy flow, or consumers (distributed) energy generation capacity are limited. Let R_{max} denotes the consumers' maximum reduction, where $R \leq R_{\text{max}}$:

$$R = \begin{cases} m\mathcal{I}, & \text{if } \mathcal{I} \leq \frac{R_{\text{max}}}{m}, \\ R_{\text{max}}, & \text{if } \mathcal{I} > \frac{R_{\text{max}}}{m}. \end{cases} \quad (11)$$

In most cases, it is easier to estimate the base load L_{min} , the minimum amount of electricity the consumers cannot live without. Thus, one can estimate R_{max} by computing $R_{\text{max}} = L - L_{\text{min}}$.

Theorem 1: In case of single energy source, when consumer reduction is bounded by R_{max} and Eq. 9 or 10 is satisfied, then the incentives that maximize company's gain is:

$$\mathcal{I}_{\text{opt}} = \frac{R_{\text{max}}}{m}.$$

Proof: Let $\text{gain}(\star)$ be the company's gain using incentives \star . Then, there are two cases:

Case 1: The company gives incentives $\mathcal{I}' < \mathcal{I}_{\text{opt}}$. Let $R' = m\mathcal{I}'$. Below, we show that $\text{gain}(\mathcal{I}') < \text{gain}(\mathcal{I}_{\text{opt}})$.

$$\begin{aligned} \text{gain}(\mathcal{I}') &= (P_{\text{mkt}} - P_{\text{ret}})R' - \mathcal{I}' \\ &= (P_{\text{mkt}} - P_{\text{ret}})m\mathcal{I}' - \mathcal{I}' \\ &< ((P_{\text{mkt}} - P_{\text{ret}})m\mathcal{I}' - \mathcal{I}') \frac{\mathcal{I}_{\text{opt}}}{\mathcal{I}'} \\ &= (P_{\text{mkt}} - P_{\text{ret}})m\mathcal{I}_{\text{opt}} - \mathcal{I}_{\text{opt}} \\ &= \text{gain}(\mathcal{I}_{\text{opt}}). \end{aligned}$$

Case 2: The company gives incentives $\mathcal{I}' > \mathcal{I}_{\text{opt}}$. Since $\mathcal{I}' > \frac{R_{\text{max}}}{m}$, then according to Eq. 11, the demand reduction $R' = R_{\text{max}}$. Below, we show that $\text{gain}(\mathcal{I}') < \text{gain}(\mathcal{I}_{\text{opt}})$.

$$\begin{aligned} \text{gain}(\mathcal{I}') &= (P_{\text{mkt}} - P_{\text{ret}})R' - \mathcal{I}' \\ &= (P_{\text{mkt}} - P_{\text{ret}})R_{\text{max}} - \mathcal{I}' \\ &< (P_{\text{mkt}} - P_{\text{ret}})R_{\text{max}} - \mathcal{I}_{\text{opt}} \\ &= (P_{\text{mkt}} - P_{\text{ret}})m\mathcal{I}_{\text{opt}} - \mathcal{I}_{\text{opt}} \\ &= \text{gain}(\mathcal{I}_{\text{opt}}). \end{aligned}$$

4) *The upper bound of incentives:* To obtain the highest incentive which still gives us positive gain, we require the gain to be greater than 0 while considering the maximum reduction R_{max} . We replace R in Eq. 7 with R_{max} :

$$\begin{aligned} (P_{\text{mkt}} - P_{\text{ret}})R_{\text{max}} - \mathcal{I} &> 0 \\ (P_{\text{mkt}} - P_{\text{ret}})R_{\text{max}} &> \mathcal{I}. \end{aligned} \quad (12)$$

Example 1: While typically the market price is lower than the retail price, we consider in this example a period where the market price is higher than the retail price. Let the market price $P_{\text{mkt}} = 1\$/kWh$, the retail price $P_{\text{ret}} = 0.2\$/kWh$, and $m = 2kWh/\$$. Thus, inequalities in Eq. 9 and 10 are met. Suppose that the total load demand is $1000kWh$ and $L_{\text{min}} = 200kWh$. Then, we have $R_{\text{max}} = 1000 - 200 = 800kWh$. Figure 1 illustrates company's gain for different amount of incentives given to customers. Maximum gain is obtained where the incentive is equal to $\mathcal{I}_{\text{opt}} = R_{\text{max}}/m = 400\$$. And, the upper bound of the incentives such that the company still experiences positive gain is $(P_{\text{mkt}} - P_{\text{ret}})R_{\text{max}} = 640\$$.

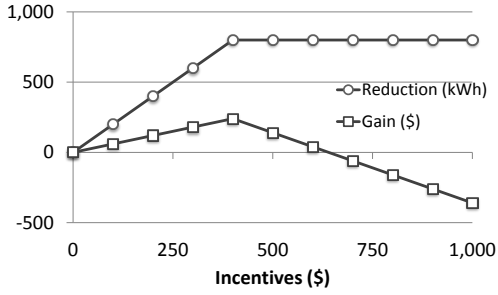


Fig. 1: Company's gain for different incentives, where $P_{ret} = 0.2\$/kWh$, $P_{mkt} = 1\$/kWh$, $m = 2kWh/\$$, and $R_{max} = 800kWh$. The optimal incentive is 400\$, while the highest incentive such that the company still experiences positive gain is 640\$.

D. Multiple Energy Sources

In this section, we discuss the case of utility companies who have access to multiple energy sources.⁸ Let us define R_{g_i} as the demand reduction for generator g_i , that is, $R_{g_i} = L_{g_i} - L_{g_i}^{DR}$, and $\mathcal{I}_{g_i} = R_{g_i}/m$. Recall that, using the definition of G_{red} and G_{base} , we can rewrite the demand reduction as $R = \sum_{g_i \in G} R_{g_i} = \sum_{g_i \in G_{red}} R_{g_i}$. Similarly, we can also rewrite the incentives as:

$$\begin{aligned} R &= \sum_{g_i \in G_{red}} R_{g_i} \\ m\mathcal{I} &= \sum_{g_i \in G_{red}} (m\mathcal{I}_{g_i}) \\ \mathcal{I} &= \sum_{g_i \in G_{red}} \mathcal{I}_{g_i}. \end{aligned} \quad (13)$$

Suppose that the unit cost to meet load demand for the discussed period using generator g_i is P_{g_i} , or $C_{g_i}(L_{g_i}) = P_{g_i}L_{g_i}$. Note that, this assumption makes the cost function linear. However, sometimes we also would like to express the cost function as a step function. In this case, we could formulate the step function by considering each step as a distinct generator. Some studies also consider quadratic cost function [7]–[9]. Since a quadratic function can be approximated by a step function, our formulation above also allows us to approximate quadratic cost functions as well. We could then rewrite the gain computation in Eq. 6 as:

$$\begin{aligned} gain &= \sum_{g_i \in G_{red}} (P_{g_i}L_{g_i} - P_{g_i}L_{g_i}^{DR}) - P_{ret}R - \mathcal{I} \\ &= \sum_{g_i \in G_{red}} (P_{g_i}R_{g_i}) - P_{ret}R - \mathcal{I} \\ &= \sum_{g_i \in G_{red}} (P_{g_i}R_{g_i}) - \sum_{g_i \in G_{red}} (P_{ret}R_{g_i}) - \sum_{g_i \in G_{red}} \mathcal{I}_{g_i} \\ &= \sum_{g_i \in G_{red}} ((P_{g_i} - P_{ret})R_{g_i} - \mathcal{I}_{g_i}) \end{aligned} \quad (14)$$

Further, we denote $gain_{g_i} = (P_{g_i} - P_{ret})R_{g_i} - \mathcal{I}_{g_i}$ as the gain contributed by generator g_i .

1) *Meaningful DR events*: We define *meaningful* demand reduction as the reduction that yields positive gain, whereas demand reduction that yields zero or negative gain is *meaningless*. Additionally, a DR event is *meaningful* if and only if the demand reduction of each generator is meaningful.⁹ Thus, it implies that any effort and incentive given to customers should

⁸Note that, an energy market could also be one of them.

⁹It is also possible to define meaningful DR events in a *weaker* sense, i.e., if only if the event yield a positive gain. If we define it that way, however, it means that there could be a set of generators G_{red}^+ where $gain_{g_i} > 0$ for all $g_i \in G_{red}^+$ and a set of generators G_{red}^- where $gain_{g_i} \leq 0$ for all $g_i \in G_{red}^-$. Since our goal is maximizing the gain, there is no point in considering (to reduce the load of) G_{red}^- . This is the reason we define meaningful DR events in its *stronger* sense, considering only G_{red}^+ .

contribute to company's gain:

$$\begin{aligned} \forall g_i \in G_{red}, \quad gain_{g_i} &> 0, \\ (P_{g_i} - P_{ret})R_{g_i} - \mathcal{I}_{g_i} &> 0, \\ (P_{g_i} - P_{ret})m\mathcal{I}_{g_i} - \mathcal{I}_{g_i} &> 0, \\ ((P_{g_i} - P_{ret})m - 1)\mathcal{I}_{g_i} &> 0. \end{aligned} \quad (15)$$

To have positive gain, $((P_{g_i} - P_{ret})m - 1)$ should be positive. Thus:

$$\forall g_i \in G_{red}, \quad P_{g_i} > \frac{1}{m} + P_{ret}, \text{ or } m > \frac{1}{P_{g_i} - P_{ret}}. \quad (16)$$

2) *The optimal incentives*: Providing that the inequalities in Eq. 16 are satisfied, we can derive the optimal incentive.

Lemma 2: In case of a utility company with multiple energy sources, if demand reduction R is unbounded and Eq. 16 is satisfied, then higher incentives \mathcal{I} leads to higher demand reduction R , and eventually higher gain.

Proof: Let $gain(\mathcal{I})$ be the gain using incentives \mathcal{I} and $gain_{g_i}(\mathcal{I}_{g_i})$ be the gain contributed by generator g_i using incentives \mathcal{I}_{g_i} . Therefore, $gain(\mathcal{I}) = \sum_{g_i \in G_{red}} gain_{g_i}(\mathcal{I}_{g_i})$ (see also Eq. 14).

We need to show that whenever $\mathcal{I} > \mathcal{I}'$, then $gain(\mathcal{I}) > gain(\mathcal{I}')$. We assume that whether the company gives \mathcal{I} or \mathcal{I}' , Eq. 16 is satisfied. Let $R = m\mathcal{I}$ and $R' = m\mathcal{I}'$ be the demand reduction and G_{red} and G'_{red} be the set of *reduced* generators when the utility company gives incentives \mathcal{I} and \mathcal{I}' , respectively. Because $R > R'$, then $G_{red} \supseteq G'_{red}$. Then, there are two cases:

Case 1: $G_{red} = G'_{red}$. This means that we have the same amount of reduction in every generators, in G_{red} and G'_{red} , up to their capacity, except for the cheapest one (since $R > R'$). Let g_{ω} be the cheapest generator in G_{red} (and in G'_{red}). Then, $\forall g_i \in G_{red} \setminus g_{\omega}$, we have $R_{g_i} = R'_{g_i} = cap(g_i)$. Consequently, since $R > R'$, we have $R_{g_{\omega}} > R'_{g_{\omega}}$. And since $\mathcal{I}_{g_i} = R_{g_i}/m$ and $\mathcal{I}'_{g_i} = R'_{g_i}/m$, we have $\mathcal{I}_{g_{\omega}} > \mathcal{I}'_{g_{\omega}}$ and $gain_{g_{\omega}}(\mathcal{I}_{g_{\omega}}) > gain_{g_{\omega}}(\mathcal{I}'_{g_{\omega}})$. Thus:

$$\begin{aligned} gain(\mathcal{I}) &= \sum_{g_i \in G_{red} \setminus g_{\omega}} gain_{g_i}(\mathcal{I}_{g_i}) + gain_{g_{\omega}}(\mathcal{I}_{g_{\omega}}) \\ &> \sum_{g_i \in G_{red} \setminus g_{\omega}} gain_{g_i}(\mathcal{I}_{g_i}) + gain_{g_{\omega}}(\mathcal{I}'_{g_{\omega}}) \\ &= \sum_{g_i \in G'_{red} \setminus g_{\omega}} gain_{g_i}(\mathcal{I}'_{g_i}) + gain_{g_{\omega}}(\mathcal{I}'_{g_{\omega}}) \\ &= gain(\mathcal{I}') \end{aligned}$$

Case 2: $G_{red} \supset G'_{red}$. Since the inequalities in Eq. 16 is satisfied, $\forall g_i \in G_{red} \setminus G'_{red}$, we have $gain_{g_i}(\mathcal{I}_{g_i}) > 0$. Thus:

$$\begin{aligned} gain(\mathcal{I}) &= \sum_{g_i \in G_{red}} gain_{g_i}(\mathcal{I}_{g_i}) \\ &= \sum_{g_i \in G'_{red}} gain_{g_i}(\mathcal{I}_{g_i}) + \sum_{g_i \in G_{red} \setminus G'_{red}} gain_{g_i}(\mathcal{I}_{g_i}) \\ &> \sum_{g_i \in G'_{red}} gain_{g_i}(\mathcal{I}_{g_i}) \\ &= gain(\mathcal{I}') \end{aligned}$$

Similar to the case of single energy source, however, consumer demand reduction is typically not unbounded. Let R_{max} denotes the upper bound of the reduction (see also Eq. 11).

Theorem 3: When demand reduction is bounded by R_{max} and Eq. 16 is satisfied, then the optimal incentives that maximize company's gain is: $\mathcal{I}_{opt} = R_{max}/m$.

Proof: Let $gain(\mathcal{I})$ denotes the gain using incentives \mathcal{I} . We show that when a utility company gives incentives $\mathcal{I}' \neq \mathcal{I}_{opt}$, then $gain(\mathcal{I}') < gain(\mathcal{I}_{opt})$. Let $R' = m\mathcal{I}'$ and $R_{opt} = m\mathcal{I}_{opt}$ be the demand reduction and G'_{red} and G_{red} be the set of reduced generators when the company gives incentives \mathcal{I}' and \mathcal{I}_{opt} , respectively. Then, there are two cases:

Case 1: The company gives incentives $\mathcal{I}_{opt} > \mathcal{I}'$. We need to show that $gain(\mathcal{I}_{opt}) > gain(\mathcal{I}')$. Note that, in this case, we have $R > R'$, which also implies $G_{red} \supseteq G'_{red}$. Then, we proceed as in the proof of Lemma 2, *Case 1* and *Case 2*, by substituting \mathcal{I} with \mathcal{I}_{opt} and R with R_{opt} .

Case 2: The company gives incentives $\mathcal{I}_{opt} < \mathcal{I}'$. We need show that $gain(\mathcal{I}_{opt}) > gain(\mathcal{I}')$. Since $\mathcal{I}' > \mathcal{I}_{opt} = R_{max}/m$, in case of bounded reduction (see Eq. 11), we have $R' = R_{max} = m\mathcal{I}_{opt} = R_{opt}$. This also implies that $G'_{red} = G_{red}$.

$$\begin{aligned} gain(\mathcal{I}') &= \sum_{g_i \in G'_{red}} (P_{g_i} - P_{ret})R' - \mathcal{I}' \\ &< \sum_{g_i \in G'_{red}} (P_{g_i} - P_{ret})R' - \mathcal{I}_{opt} \\ &= \sum_{g_i \in G_{red}} (P_{g_i} - P_{ret})R_{opt} - \mathcal{I}_{opt} \\ &= gain(\mathcal{I}_{opt}). \end{aligned}$$

3) *The upper bound of incentives:* When the demand reduction is maximum, i.e., $R_{max} = L - L^{DR}$, we denote $R_{g_i}^{max} = L_{g_i} - L_{g_i}^{DR}$ as the demand reduction for generator g_i . Consequently, we have $R_{max} = \sum_{g_i \in G_{red}} R_{g_i}^{max}$. Then, by replacing R_{g_i} in Eq. 14 with $R_{g_i}^{max}$, we can derive the highest incentives a company could provide while maintaining positive gain:

$$\begin{aligned} \sum_{g_i \in G_{red}} [(P_{g_i} - P_{ret})R_{g_i}^{max} - \mathcal{I}_{g_i}] &> 0 \\ \sum_{g_i \in G_{red}} [(P_{g_i} - P_{ret})R_{g_i}^{max}] - \mathcal{I} &> 0 \quad (17) \\ \sum_{g_i \in G_{red}} (P_{g_i} - P_{ret})R_{g_i}^{max} &> \mathcal{I}. \end{aligned}$$

Example 2: Suppose that a utility company has access to two different energy sources, g_1 and g_2 , where $cap(g_1) = 5000kW$ and $cap(g_2) = 2000kW$. Assume that for a specific time period, we have load demand $L = 6500kWh$, $P_{g_1} = 0.1\$/kWh$ (base generator), and $P_{g_2} = 1\$/kWh$ (peak generator). Additionally, let $P_{ret} = 0.2\$/kWh$, $m = 2kWh/\$$, and $R_{max} = 1200kWh$. Figure 2 shows company's gain for different amount of incentives given to the customers. Maximum gain is obtained when the incentive is equal to $\mathcal{I}_{opt} = R_{max}/m = 600\%$. The highest incentive such that the company still experiences positive gain is $(P_{g_2} - P_{ret})R_{g_2}^{max} = (1 - 0.2) \cdot 1200 = 960\%$.¹⁰

IV. DR TO INCREASE ENERGY CONSUMPTION

In the previous section, we discuss about DR for energy reduction. However, DR could also be used to incentivize consumers to increase their energy consumption, especially when there is a surplus energy and balancing the surplus is costly. This can be, for example, when (the output of) some generators cannot be turned off (or reduced) easily without additional cost. Or, when the demand is surprisingly low, much less than the amount of energy bought in the day-ahead

¹⁰Note that, to compute $R_{g_2}^{max}$, we need to first compute L^{DR} as if the reduction is R_{max} . Thus, $L^{DR} = L - R_{max} = 5300kWh$. Using the load generation assumption in Section II-A, we have $L_{g_1} = 5000kWh$, $L_{g_2} = 1500kWh$, $L_{g_1}^{DR} = 5000kWh$, and $L_{g_2}^{DR} = 300kWh$. Thus, $R_{g_1}^{max} = 0kWh$ and $R_{g_2}^{max} = 1200kWh$.

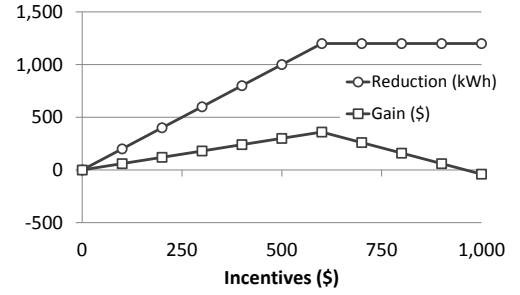


Fig. 2: Company's gain for different incentives where the company has two energy sources, g_1 and g_2 , with $cap(g_1) = 5000kW$, $cap(g_2) = 2000kW$. For this time period, we have $L = 2000kWh$, $P_{g_1} = 0.1\$/kWh$, $P_{g_2} = 1\$/kWh$, $P_{ret} = 0.2\$/kWh$, $m = 2kWh/\$$, and $R_{max} = 1200kWh$. The optimal incentive is 600\$, while the highest incentive such that company still experiences positive gain is 960\$.

market. If the grid operator has to balance the surplus, then the company needs to pay the penalty for the imbalances.

Together with its ability to reduce demand, DR's ability to increase it forms a framework for a load shaping strategy. In this section, we discuss the case where DR is used as a mechanism to encourage demand increase. We present only the case where utility companies have access to *multiple* energy sources, since the case of single energy source can be regarded as a special case of that.

Let L^+ be the total energy produced in the surplus/abundance period, where $L^+ > L$, and $S_{max} = L^+ - L$ be the excess energy. Additionally, let $L_{g_i}^+$ be the energy produced by generator g_i . We define $C_B(S_{max})$ as the balancing cost, e.g., the cost to balance the grid due to the excess energy (it is generally performed in a balancing market). Note that, in Section III, the role of the balancing market to generate additional electricity when the supply is short can also be seen as one of (typically more expensive) energy sources.

A. Without DR (Business As Usual)

Cost We define company's total cost as the sum of generation and balancing cost: $\sum_{g_i \in G} C_{g_i}(L_{g_i}^+) + C_B(S_{max})$.

Revenue Company's revenue: $P_{ret}L$.

Profit Company's profit = revenue - cost:

$$P_{ret}L - \sum_{g_i \in G} C_{g_i}(L_{g_i}^+) - C_B(S_{max}). \quad (18)$$

B. With DR

Note that, in this case there is a surplus of energy, S_{max} , that is ready to be consumed. Encouraging customers to consume this surplus energy not only results in increasing company's revenue (due to higher customers' energy bill and lower balancing cost), but also acts as an energy balancing mechanism, if there would be a shortage of energy in the future. It might be easier for people to consume less energy in the future (when the supply is short), if some activities have been performed or shifted to the previous (energy abundance) period. Therefore, we aim to use DR for incentivizing consumers to increase their consumption during this surplus period. Let us assume that the consumers increase their demand from L to $L + S$, where $0 \leq S \leq S_{max}$.

Cost Compared to the previous case (without DR), company's cost is lower due to the possible decrease in the balancing cost: $\sum_{g_i \in G} C_{g_i}(L_{g_i}^+) + C_B(S_{max} - S)$.

Revenue Company's revenue increases due to the possible increase in consumers' demand: $P_{ret}(L + S)$.

Profit Company's profit = revenue - cost - incentive:

$$P_{ret}(L + S) - \sum_{g_i \in G} C_{g_i}(L_{g_i}^+) - C_B(S_{max} - S) - \mathcal{I}. \quad (19)$$

Gain We define the gain of a utility company during a DR event in this abundance period as the difference between its profit with and without the DR event, i.e., Eq. 19 - Eq 18:

$$\begin{aligned} gain &= P_{ret}(L + S) - \sum_{g_i \in G} C_{g_i}(L_{g_i}^+) - C_B(S_{max} - S) - \\ &\quad \mathcal{I} - P_{ret}L + \sum_{g_i \in G} C_{g_i}(L_{g_i}^+) + C_B(S_{max}) \\ &= P_{ret}S - \mathcal{I} - C_B(S_{max} - S) + C_B(S_{max}) \end{aligned} \quad (20)$$

As in the case of the generation cost (see also Section III-D), let us assume that the balancing cost has a unit cost P_B . Then, we have:

$$\begin{aligned} gain &= P_{ret}S - \mathcal{I} - P_B(S_{max} - S) + P_B \cdot S_{max} \\ &= P_{ret}S - \mathcal{I} + P_B S \\ &= (P_{ret} + P_B)S - \mathcal{I} \end{aligned} \quad (21)$$

Similar to the case of DR to reduce energy consumption, we assume that consumer responses grow linearly with the incentives offered, i.e., $S = m^+ \cdot \mathcal{I}$, where m^+ is the consumer increase rate (unit: $kWh/\$$). The higher the consumer increase rate, the more sensitive she is to incentives.¹¹ Since S is bounded by S_{max} , consequently,

$$S = \begin{cases} m^+ \cdot \mathcal{I}, & \text{if } \mathcal{I} \leq \frac{S_{max}}{m^+} \\ S_{max}, & \text{if } \mathcal{I} > \frac{S_{max}}{m^+}. \end{cases} \quad (22)$$

1) *The lower bound of consumer increase rate:* In order to derive the lowest consumer increase rate required to obtain positive gain:

$$\begin{aligned} (P_{ret} + P_B)S - \mathcal{I} &> 0 \\ (P_{ret} + P_B)m^+ \cdot \mathcal{I} - \mathcal{I} &> 0 \\ ((P_{ret} + P_B)m^+ - 1)\mathcal{I} &> 0 \end{aligned} \quad (23)$$

To have positive gain, the term $(P_{ret} + P_B)m^+ - 1$ should be positive. Thus:

$$m^+ > \frac{1}{P_{ret} + P_B}, \quad (24)$$

If we assume that the retail price P_{ret} is fixed, then the sensitivity of the consumers required for a DR event is inversely related with the balancing prices. While highly sensitive consumers are generally preferred for DR, having less sensitive consumers (with low m^+) is also fine when the balancing price is high enough.

¹¹In practice, consumers can increase their demand by shifting their later activities to the DR event period. If they do not have any activities to be shifted, however, rational consumers would have $1/m^+ > P_{ret}$, i.e., they respond (or increase their demand) iff the incentive per unit energy is greater than the retail price.

2) *The lower bound of the balancing price:* Using Eq. 24, we can also derive the lower bound of the balancing price:

$$P_B > \frac{1}{m^+} - P_{ret} \quad (25)$$

3) *The optimal incentives:* When Eq. 24 or 25 is satisfied, we can derive the optimal incentives that maximize company's gain.

Theorem 4: In the case of DR to increase energy consumption, given that Eq. 24 or 25 is satisfied, the optimal incentives that maximize company's gain is: $\mathcal{I}_{opt} = S_{max}/m^+$.

Proof: Let $gain(\mathcal{I})$ denotes company's gain by providing incentives \mathcal{I} . If Eq. 24 or 25 is satisfied, then for $\mathcal{I}' \neq \mathcal{I}_{opt}$, we show that $gain(\mathcal{I}') < gain(\mathcal{I}_{opt})$. Let $S' = m^+ \cdot \mathcal{I}'$ be the increase in energy consumption due to incentives \mathcal{I}' . Then, there are two cases:

Case 1: $\mathcal{I}' < \mathcal{I}_{opt}$

$$\begin{aligned} gain(\mathcal{I}') &= (P_{ret} + P_B)S' - \mathcal{I}' \\ &= (P_{ret} + P_B)m^+ \cdot \mathcal{I}' - \mathcal{I}' \\ &< ((P_{ret} + P_B)m^+ \cdot \mathcal{I}' - \mathcal{I}') \frac{\mathcal{I}_{opt}}{\mathcal{I}'} \\ &= (P_{ret} + P_B)m^+ \cdot \mathcal{I}_{opt} - \mathcal{I}_{opt} \\ &= gain(\mathcal{I}_{opt}). \end{aligned}$$

Case 2: $\mathcal{I}' > \mathcal{I}_{opt}$. Since $\mathcal{I}' > S_{max}/m^+$, we have $S' = S_{max}$ (see Eq. 22). Then,

$$\begin{aligned} gain(\mathcal{I}') &= (P_{ret} + P_B)S' - \mathcal{I}' \\ &= (P_{ret} + P_B)S_{max} - \mathcal{I}' \\ &< (P_{ret} + P_B)S_{max} - \mathcal{I}_{opt} \\ &= (P_{ret} + P_B)m^+ \cdot \mathcal{I}_{opt} - \mathcal{I}_{opt} \\ &= gain(\mathcal{I}_{opt}). \end{aligned}$$

4) *The upper bound of incentives:* The highest incentives that a utility company can give to consumers while still experiences positive gain can be obtained by assuming the maximum increase in the consumption:

$$\begin{aligned} (P_{ret} + P_B)S_{max} - \mathcal{I} &> 0 \\ (P_{ret} + P_B)S_{max} &> \mathcal{I}, \end{aligned} \quad (26)$$

Example 3: Let us consider a case where the load demand $L = 1000kWh$ and the total energy generated in the abundance period $L^+ = 1500kWh$. In addition, let us assume that $P_{ret} = 0.2\$/kWh$, $P_B = 1\$/kWh$, and $m^+ = 1kWh/\$$. Figure 3 shows company's gain for different amount of incentives provided to consumers. Note that, we have $S_{max} = 1500kWh - 1000kWh = 500kWh$. Maximum gain is obtained when the incentive is equal to $\mathcal{I}_{opt} = S_{max}/m^+ = 500\$$. The highest incentives such that the company still experiences positive gain is $(P_{ret} + P_B)S_{max} = 600\$$.

V. RELATED WORK

Borenstein and Holland analyzed the economics of real-time pricing (RTP) [10]. In particular, they assessed the market efficiency where both, RTP and flat-pricing scheme, coexist in the consumer base. They showed that increasing the number of consumers who adopt RTP scheme might harm the consumers who are already on RTP, but could bring advantage to consumers who stay in the flat-rate. There are also a number of studies that aim to foster the emergence of pervasive DR. However, most of them focused on price-

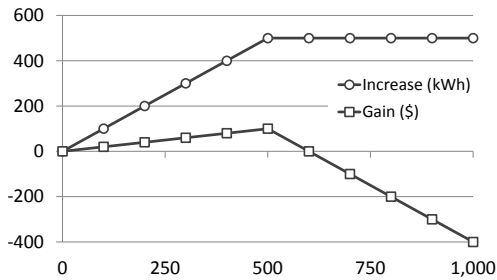


Fig. 3: Increase in energy consumption and gain for different incentives given to consumers in the case of DR to increase energy consumption, where $L = 1000kWh$, $L^+ = 1500kWh$, $P_{ret} = 0.2\$/kWh$, $P_B = 1\$/kWh$, and $m^+ = 1kWh/\$$. The optimal incentive is 500\$, while the highest incentive for which the company still experiences positive gain is 600\$.

based mechanisms [7], [11]–[16]. These mechanisms require consumers to tirelessly track price fluctuation and adjust their consumption schedule accordingly to maximize their benefit (paying electricity bill as low as possible with the least inconvenience). To this end, some studies propose the use of software agents (or energy management systems) for automatic price monitoring and schedule optimization, and making the entire process seamless from the consumers’ perspective [7], [13]–[16].

While this paper focuses on a more holistic view of incentive-based DR, there are also some studies dedicated to consumer baselines. In incentive-based DR, *consumer baselines* (or DR baselines) are estimates of what consumers would have consumed in the absence of DR events. It also plays an important role to determine the amount of incentives that consumers should receive [17]. Analyses of consumer baselines applied to large and commercial consumers have been discussed in [18]–[21], while its application to residential consumers has been discussed in [22].

VI. CONCLUSION AND FUTURE WORK

We have identified the conditions to benefit from pervasive DR, i.e., the lower bound of the consumer reduction/increase rate, m , the lower bound of the generation costs and market prices, the lower bound of the balancing prices, and the upper bound of the incentives. Furthermore, we have determined the optimal incentives to maximize gain. Although we model a single time period, the approach can be extended to multiple time periods by performing it iteratively.

It would certainly be interesting to find out how m varies depending on various factors, or to accurately model the customer response function $r(\cdot)$. Additionally, to complicate matters, both could be noisy. Previous work has defined several customer models, but they are analytical models, are not based on real observations [22]. Accurately modeling both functions would require real project deployment, and it should be a large one so that the outcome is useful and generalizable. To start the project, however, utility companies would need the first insight on what to expect and how much incentives to provide. This paper aims to provide that first insight and a common ground/formalization to study the problem and enable one to craft better solutions.

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