Characterization of multidirectional pedestrian flows based on three-dimensional Voronoi tessellations

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hEART 2015
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September 9, 2015
Traffic characteristics

- **Density**: the number of pedestrians present in an area at a certain time instance [\#ped/m²]
- **Flow**: the number of pedestrians passing a line segment in a unit of time [\#ped/ms]
- **Velocity**: the average of the velocities of pedestrians present in an area at a certain time instance / passing a line segment in a unit of time [m/s]

- LOS indicators
- Fundamental diagram specification
- Models of pedestrian dynamics
Pedestrian flow characterization

- Several approaches proposed in the literature [Duives, 2012; Zhang, 2012]
- Arbitrary discretization
- Inconsistent results
- Multi-directional flow composition neglected
Pedestrian flow characterization - Issues

Arbitrary discretization

- It may generate noise in the data [Openshaw, 1983]
- Results may be highly sensitive to minor changes of discretization
Pedestrian flow characterization - Issues

Inconsistent results in observations and modeling

- Averaging over different degrees of freedom may lead to incomparable results [Seyfried et al., 2005]

Multi-directional nature of pedestrian flows

- Definitions may not result in the desired outcome if pedestrians do not walk in the same direction [van Wageningen-Kessels et al., 2014]
Voronoi-based spatial discretization

• Assigns a personal region $A_i$ to each pedestrian $i$: each point in the personal region is closer to $i$ than to any other pedestrian, with respect of the Euclidean distance

$$A_i = \{ p | d_E(p, p_i) \leq d_E(p, p_j), \forall j \}$$
Voronoi-based characterization

**Steffen and Seyfried, 2010**

- Density and speed are defined per unit of space via Voronoi diagrams

\[
k = \frac{\int \int \rho_{xy} \, dx \, dy}{\Delta x \Delta y}, \quad v = \frac{\int \int \nu_{xy} \, dx \, dy}{\Delta x \Delta y}
\]

\[\rho_{xy} = \frac{1}{A_i}, \quad \rho_{xy} \text{ - density distribution, } A_i \text{ - area of Voronoi cell associated to pedestrian } i\]

\[\nu_{xy} \text{ - instantaneous speed of pedestrian } i\]
Characterization based on Edie’s definitions

van Wageningen-Kessels et al., 2014

Density: \( k(V) = \frac{\sum_{i} t_i}{dx \times dy \times dt} \)

Flow: \( \bar{q}(V) = \left( \begin{array}{c} \sum_{i} x_i \\ \sum_{i} y_i \\ \end{array} \right) \frac{\sum_{i} t_i}{dx \times dy \times dt} \)

Velocity: \( \bar{v}(V) = \frac{\bar{q}(V)}{k(V)} = \left( \begin{array}{c} \sum_{i} x_i \\ \sum_{i} y_i \\ \end{array} \right) \frac{1}{\sum_{i} t_i} \)
Pedestrian trajectories

- The trajectory of pedestrian $i$ is a curve in space and time

$$p_i(t) = (x_i(t), y_i(t), t)$$

- Voronoi diagram associated with trajectories

- A point $p(t)$ belongs to the set $V_i(t)$ if

$$d(p(t), p_i(t)) \leq d(p(t), p_j(t)), \forall j$$

- Each pedestrian $i$ is associated with a Voronoi tube $V_i$
Voronoï-based traffic indicators

The set of all points in $V_i$ corresponding to a specific time $t$

$$V_i(t) = \{(x, y, t) \in V_i\} \sim [m^2]$$

Density indicator

$$k_i(x, y, t) = \frac{1}{V_i(t)}$$
Voronoi-based traffic indicators

The set of all points in $V_i$ corresponding to a given location $x$ and $y$

$$V_i(x) = \{(x, y, t) \in V_i\} \sim [\text{ms}]$$

$$V_i(y) = \{(x, y, t) \in V_i\} \sim [\text{ms}]$$

Flow indicator

$$\vec{q}_i(x, y, t) = \left( \frac{1}{V_i(x)} \right)$$

Velocity indicator

$$\vec{v}_i(x, y, t) = \frac{\vec{q}_i(x, y, t)}{k_i(x, y, t)}$$
Pedestrian trajectory data

- In practice, collected through an appropriate tracking technology [Daamen and Hoogendoorn, 2003, Alahi et al., 2011]
- Time is discretized: \( t_s = [t_0, t_1, ..., t_f] \)
- The trajectory is described as a finite collection of triplets

\[ p_{is} = (x_{is}, y_{is}, t_s) \]
Characterization based on the sample of points

- Interpolation
  - Introduces errors

- Voronoi diagrams at $t_s$
  - Needs data that are synchronized
  - Otherwise, the density is underestimated

- 3D Voronoi diagrams for the sample of points
  - The points at $t_s$ are the only available data
  - Spatio-temporal distance (assignment rule)
Data-driven discretization

- Voronoi diagram associated with the points $p_{is}$
- Each point $p_{is}$ is associated with a Voronoi cell $V_{is}$
- A point $p$ belongs to the set $V_{is}$ if

$$d_*(p, p_{is}) \leq d_*(p, p_{js}), \forall j$$

- $d_*(p, p_{is})$ - spatio-temporal distance
Voronoi-based traffic indicators

- The set of all points in $V_{is}$ corresponding to a given location $(x, y)$

$$V_{is}(x, y) = \{(x, y, t) \in V_{is}\} \sim [s]$$

Density indicator

$$k_i(x, y, t) = \frac{V_{is}(x_{is}, y_{is})}{Vol(V_{is})}$$
Voronoi-based traffic indicators

- The set of all points in $V_{is}$ corresponding to a specific time $t$
  
  $V_{is}(t) = \{(x, y, t) \in V_{is}\} \sim [m^2]$

Flow indicator

$$\vec{q}_i(x, y, t) = \left( \begin{array}{c} \frac{x_i}{V_{is}} \\ \frac{y_i}{V_{is}} \end{array} \right)$$

$x_i$ - a maximum distance in $x$ direction in $V_{is}(t_{is})$

$y_i$ - a maximum distance in $y$ direction in $V_{is}(t_{is})$

Velocity indicator

$$\vec{v}_i(x, y, t) = \frac{\vec{q}_i(x, y, t)}{k_i(x, y, t)}$$
Spatio-temporal distances

**Euclidean distance**

\[ d_E(p, p_{is}) = \sqrt{(p - p_{is})^T(p - p_{is})} \]

**Mahalanobis distance**

\[ d_M(p, p_{is}) = \sqrt{(p - p_{is})^T M_{is} (p - p_{is})} \]

- \( M_{is} \) - symmetric, positive-definite matrix
- \( M_{is} \) - defines how distances are measured from the perspective of pedestrian \( i \)
Spatio-temporal distances

**Euclidean distance**

\[ d_E(p, p_{is}) = \sqrt{(p - p_{is})^T (p - p_{is})} \]

**Mahalanobis distance**

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3D Voronoi discretization

Euclidean distance
Voronoi-based density maps

Euclidean distance

Reproduces settings with uniform and non-uniform movement
Delft case study

Bidirectional flow [Daamen and Hoogendoorn, 2003]

- Trajectories extracted from the digital video sequences
- The position of each individual is available every 0.1s
- Total number of trajectories: 1,123
- The average length of the trajectories: 10 meters
- The average time of the trajectories: 10 seconds.
Voronoi-based velocity maps

Lane formation

Allows to correlate the momentary speed of an individual pedestrian (or a group of pedestrians) with the availability of space
3D Voronoi vs. grid-based method

Density sequences

Voronoi-based approach leads to smooth transitions in measured characteristics
Spatio-temporal distances

Euclidean distance

\[ d_E(p, p_{is}) = \sqrt{(p - p_{is})^T(p - p_{is})} \]

Mahalanobis distance

\[ d_M(p, p_{is}) = \sqrt{(p - p_{is})^T M_{is} (p - p_{is})} \]

- \( M_{is} \) - symmetric, positive-definite matrix
- \( M_{is} \) - defines how distances are measured from the perspective of pedestrian \( i \)
**Mahalanobis distance**

**Directions of interest**

\[ p_{is} = (x_{is}, y_{is}, t_s), \quad v_i(t_s) = \frac{1}{t_{(s+1)}-t_s} \begin{pmatrix} x_i(s+1) - x_{is} \\ y_i(s+1) - y_{is} \\ 1 \end{pmatrix} \]

\[ d^1(t_s) = \frac{v_i(t_s)}{||v_i(t_s)||}, \quad ||d^1(t_s)|| = 1 \]

\[ d^2(t_s) = \begin{pmatrix} d^1_x(t_s) \\ d^1_y(t_s) \\ 0 \end{pmatrix}, \quad d^1(t_s)^T d^2(t_s) = 0, \quad ||d^2(t_s)|| = 1 \]

\[ d^3(t_s) = \begin{pmatrix} 0 \\ 0 \\ t_{(s+1)} - t_s \end{pmatrix}, \quad ||d^3(t_s)|| = t_{(s+1)} - t_s \]
Mahalanobis distance

Change of coordinates

\[ S_1(t_s, \delta) = p_{is} + (t_{(s+1)} - t_s)v_i(t_s) + \delta d^1(t_s) \]
\[ S_2(t_s, \delta) = p_{is} - (t_{(s+1)} - t_s)v_i(t_s) - \delta d^1(t_s) \]
\[ S_3(t_s, \delta) = p_{is} + \delta d^2(t_s) \]
\[ S_4(t_s, \delta) = p_{is} - \delta d^2(t_s) \]
\[ S_5(t_s, \delta) = p_{is} + \delta d^3(t_s) \]
\[ S_6(t_s, \delta) = p_{is} - \delta d^3(t_s) \]

\[ d_M = \sqrt{(S_j(t_s, \delta) - p_{is})^T M_{is}(S_j(t_s, \delta) - p_{is})} = \delta, j = 1, \ldots, 6 \]
3D Voronoi discretization

Mahalanobis distance
Numerical analysis

**Scenarios**

**Benchmark**
- Synthetic pedestrian trajectories
- Voronoi-based method for trajectories

**Sample of points from trajectories**
- Different sampling frequency
- Method
  - Voronoi diagrams with $d_E$
  - Voronoi diagrams with $d_M$
Numerical analysis

Scenarios

Benchmark
- Synthetic pedestrian trajectories
- Voronoi-based method for trajectories

Sample of points from trajectories
- Different sampling frequency
- Method
  - Voronoi diagrams with $d_E$
  - Voronoi diagrams with $d_M$
Density indicator
Speed indicator
More numerical analysis

- Different scenarios
- Importance sampling
- Comparison with interpolation
- Different assignment rules - anticipation of the forward movement of pedestrians
Conclusions

- The framework for pedestrian-oriented flow characterization
- Edie’s definitions adapted through a data-driven discretization
- Reproduces the settings with uniform and non-uniform movement
- Reflects the self-organization phenomena
- Leads to smooth transitions in measured traffic characteristics
- Sampling frequency affects the accuracy of 3D Voronoi results
Future research

- More numerical analysis
- Real case study: train stations in Lausanne and Basel [Alahi et al., 2014]
- Stream-based definitions of indicators and their interaction [Nikolić and Bierlaire, 2014]
- Stream-based fundamental relationships for pedestrians
Thank you for your attention