A dynamic network loading model for anisotropic and congested pedestrian flows

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Unsteady, anisotropic and congested flow

Figure: Passageway in Central Station (MTR), Hong Kong
Aggregate pedestrian flow models

- graph-based models [CS94, Løv94]
  - interaction between streams entirely neglected
- cell transmission models [ASKT07, GHW11, HBFM14]
  - inherent assumption of isotropy
- continuum models [Hug02, HWZ+09, HvWKDD14]
  - expensive, particularly for multi-class applications

Scope: ‘cheap’ anisotropic macroscopic loading model
Decomposition of pedestrian flow into streams

- contiguous area $\xi$ of size $A_\xi$
- each stream $\lambda \in \Lambda_\xi$ characterized by
  - exogenous direction
  - accumulation $M_\lambda$
  - uni-directional speed $V_\lambda$

stream-based fundamental diagram $f(M)$ [WLC$^+$10, XW15, FL15]

- accumulation and speed vectors: $M_\xi = [M_\lambda]$, $V_\xi = [V_\lambda]$
- bounded velocity: $0 \leq V_\lambda \leq V_f$, $\forall \lambda \in \Lambda_\xi$
- monotonic density-speed relation: $\partial V_\lambda / \partial M_{\lambda'} \leq 0$, $\forall \lambda, \lambda' \in \Lambda_\xi$

$$V_\xi = V_f f_\xi(M_\xi; A_\xi)$$
Time, space and demand

- **time interval** $\tau \in \mathcal{T}$
  - choice of $\Delta \mathcal{T} = \lvert \tau \rvert$ crucial
- **area** $\xi \in \mathcal{X}$
  - no assumption regarding shape and size
- **route** $\rho \in \mathcal{R}$
  - origin/destination area: $\xi^o_\rho, \xi^d_\rho$
  - accessible network: $\mathcal{X}_\rho \subset \mathcal{X}$
- **pedestrian group** $\ell \in \mathcal{L}$
  - departure time interval $\tau_\ell$
  - group size $x_\ell$
  - route $\rho_\ell$
Pedestrian walking network

- $\mathcal{X}$: set of areas $\xi \in \mathcal{X}$
- $\mathcal{N}$: set of nodes $\nu \in \mathcal{N}$
- $\Lambda$: set of streams $\lambda \in \Lambda$, $\lambda : \nu^o_\lambda \to \nu^d_\lambda$
  - $L_\lambda > 0$: length of stream $\lambda$, $L_{\text{min}} = \min_{\lambda \in \Lambda} L_\lambda$
  - $\Lambda_\xi$: set of streams associated with area $\xi$

- area: range of interaction
- node: flow valve/splitter
- stream: uni-directional flow
State variables and hydrodynamic flow

• fragment size
  – $M_{\lambda,\tau}^\ell$: accumulation of group $\ell$ on stream $\lambda$ during interval $\tau$

• aggregated variables
  – stream accumulation: $M_{\lambda,\tau} = \sum_{\ell \in \mathcal{L}} M_{\lambda,\tau}^\ell$
  – area accumulation: $M_{\xi,\tau} = \sum_{\lambda \in \Lambda_{\xi}} M_{\lambda,\tau}$

• ‘hydrodynamic flow’ on stream $\lambda \in \Lambda$ during interval $\tau$
  – for uni-directional flow: flux = density $\times$ velocity
  – $\Delta Q_{\lambda,\tau} = L_{\min}/L_{\lambda} M_{\lambda,\tau} f_\lambda(M_{\xi,\tau})$ if $\Delta T = \Delta L_{\min}/V_f$ (CFL)
  – reaches maximum $\Delta Q_{\lambda,\tau}^{\text{opt}}$ at $M_{\lambda,\tau}^{\text{opt}}$
Hydrodynamic flow capacities

• hydrodynamic inflow capacity

\[ \Delta Q_{\lambda,\tau}^{\text{in}} = \begin{cases} \Delta Q_{\lambda,\tau}^{\text{opt}} & \text{if } M_{\lambda,\tau} \leq M_{\lambda,\tau}^{\text{opt}} \\ \Delta Q_{\lambda,\tau} & \text{otherwise} \end{cases} \]

• hydrodynamic outflow capacity

\[ \Delta Q_{\lambda,\tau}^{\text{out}} = \begin{cases} \Delta Q_{\lambda,\tau} & \text{if } M_{\lambda,\tau} \leq M_{\lambda,\tau}^{\text{opt}} \\ \Delta Q_{\lambda,\tau}^{\text{opt}} & \text{otherwise} \end{cases} \]
Sending capacity

- receiving capacity of stream $\lambda$ during interval $\tau$
  \[ R_{\lambda,\tau} = \Delta Q^\text{in}_{\lambda,\tau} \]

- sending capacity of group $\ell$ on stream $\lambda$ during interval $\tau$
  \[
  S^\ell_{\lambda \rightarrow \lambda',\tau} = \delta^\rho_{\lambda \rightarrow \lambda',\tau} \min \left\{ \frac{M^\ell_{\lambda,\tau}}{M_{\lambda,\tau}}, \frac{M^\ell_{\lambda,\tau}}{M_{\lambda,\tau}} \Delta Q^\text{out}_{\lambda,\tau} \right\}
  
  - $\delta^\rho_{\lambda \rightarrow \lambda',\tau}$: turning proportion
  - free-flow: full local group proceeds
  - congestion: demand-proportional supply distribution
Actual transition flow

• candidate inflow to stream $\lambda$ during interval $\tau$

$$S_{\lambda,\tau} = \sum_{\lambda' \in \Phi^\rho_{\lambda}} \sum_{\ell \in \mathcal{L}} S^\ell_{\lambda' \rightarrow \lambda,\tau}$$

- $\Phi^\rho_{\lambda}$, $\Theta^\rho_{\lambda}$: set of up-/downstream adjacent streams on route $\rho$

• actual transition flow

$$G^\ell_{\lambda \rightarrow \lambda',\tau} = \begin{cases} S^\ell_{\lambda \rightarrow \lambda',\tau} & \text{if } S^\ell_{\lambda',\tau} \leq R^\ell_{\lambda',\tau} \\ \zeta^\ell_{\lambda \rightarrow \lambda',\tau} R^\ell_{\lambda',\tau} & \text{otherwise} \end{cases}$$

- congestion: demand-proportional supply

$$\zeta^\ell_{\lambda \rightarrow \lambda',\tau} = \frac{S^\ell_{\lambda \rightarrow \lambda',\tau}}{S^\ell_{\lambda',\tau}}$$
Propagation model

- continuity equation \( \forall \tau \in \mathcal{T}, \forall \lambda \in \Lambda, \forall \ell \in \mathcal{L} \)

\[
M_{\lambda, \tau+1}^\ell = M_{\lambda, \tau}^\ell + \sum_{\lambda' \in \Phi^\rho_{\lambda}} G_{\lambda' \to \lambda, \tau}^\ell - \sum_{\lambda'' \in \Theta^\rho_{\lambda}} G_{\lambda \to \lambda'', \tau}^\ell + W_{\lambda, \tau}^\ell
\]

- source/sink term
Specification: Pedestrian fundamental diagram

- specification inspired by research at HKU [WLC+10, XW15]
- stream-based fundamental diagram (SbFD)

\[
V_\lambda = V_f \cdot \exp \left\{ -\vartheta \left( \frac{M_\xi}{A_\xi} \right)^2 \right\} \prod_{\lambda' \in \Lambda_\xi} \exp \left( -\beta \left( 1 - \cos \varphi_{\lambda,\lambda'} \right) \frac{M_{\lambda'}}{A_\xi} \right)
\]

- isotropic reduction (Drake, 1967)
- reduction due to pair-wise interaction of streams
  \( \varphi_{\lambda,\lambda'} \): intersection angle between streams \( \lambda, \lambda' \)


\[
V_\lambda = V_f \left\{ 1 - \exp \left[ -\gamma \left( \frac{A_\xi}{M_\xi} - \frac{1}{k_{jam}} \right) \right] \right\}
\]
Specification: Turning proportions

Potential field-based model [GHW11, HBFM14]

• route-specific potential $P_{\nu, \tau}^\rho$
  - e.g. $P_{\nu, \tau}^\rho \sim$ shortest path distance from node $\nu$ to area $\xi^d_{\rho}$ along route $\rho$ for traffic conditions prevalent during interval $\tau$

• turning proportions ($\lambda' \in \Theta_{\lambda}^\rho$)
  - logit-type model with weight $\mu$

\[
\delta_{\lambda \rightarrow \lambda', \tau} = \frac{\exp\{-\mu P_{\nu, \tau}^{d_{\lambda', \lambda}}\}}{\sum_{\lambda'' \in \Theta_{\lambda}^\rho} \exp\{-\mu P_{\nu, \tau}^{d_{\lambda''}}\}}
\]
Calibration

- maximum likelihood estimation
  - $\theta$: unknown parameter vector
  - pedestrian $i = \{1, \ldots, N\}$
    - $tt_i^{obs}$: observed travel time
    - $f_i^{est}(tt|X, \theta)$: estimated travel time probability density

$$\hat{\theta} = \arg \max \tilde{L}(tt_{obs}|X, \theta)$$

with

$$\tilde{L}(tt_{obs}|X, \theta) = \sum_{i=1}^{N} \log \left( f_i^{est}(tt_i^{obs}|X, \theta) \right)$$

- optimization algorithm: derivative-free trust-region method with random sampling of initial parameters [Pow09]
Counter-flow experiment (Wong et al., 2010)
## Counter-flow experiment: Observed speeds

<table>
<thead>
<tr>
<th>Exp.</th>
<th>major group</th>
<th>minor group</th>
</tr>
</thead>
<tbody>
<tr>
<td>#84</td>
<td>87 ped</td>
<td>1.08 ± 0.15 m/s</td>
</tr>
<tr>
<td>#85</td>
<td>79</td>
<td>1.19 ± 0.13</td>
</tr>
<tr>
<td>#86</td>
<td>68</td>
<td>0.90 ± 0.10</td>
</tr>
<tr>
<td>#87</td>
<td>61</td>
<td>0.82 ± 0.06</td>
</tr>
<tr>
<td>#88</td>
<td>53</td>
<td>0.83 ± 0.09</td>
</tr>
<tr>
<td>#89</td>
<td>44</td>
<td>0.79 ± 0.10</td>
</tr>
</tbody>
</table>

Extracted from Wong et al., 2010 [WLC+10]
Counter-flow experiment: Results

<table>
<thead>
<tr>
<th></th>
<th>Zero-Model</th>
<th>Drake</th>
<th>SbFD</th>
<th>Weidmann</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tilde{L}_{\text{calib}}^{85,87}$</td>
<td>-416.9</td>
<td>-374.0</td>
<td>-348.2</td>
<td>-360.7</td>
</tr>
<tr>
<td>$V_f$ [m/s]</td>
<td>1.166</td>
<td>1.170</td>
<td>1.115</td>
<td>1.169</td>
</tr>
<tr>
<td>$\mu$ [-]</td>
<td>1.43</td>
<td>12.15</td>
<td>10.18</td>
<td>14.84</td>
</tr>
<tr>
<td>$\vartheta$ [m$^4$]</td>
<td>0.078</td>
<td>0.001</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta$ [m$^2$]</td>
<td></td>
<td></td>
<td>0.210</td>
<td></td>
</tr>
<tr>
<td>$\gamma$ [m$^{-2}$]</td>
<td></td>
<td></td>
<td>4.92</td>
<td></td>
</tr>
<tr>
<td>$k_j$ [m$^{-2}$]</td>
<td></td>
<td></td>
<td></td>
<td>6.58</td>
</tr>
<tr>
<td>$\tilde{L}_{\text{valid}}^{84}$</td>
<td>-175.6</td>
<td>-166.2</td>
<td>-151.7</td>
<td>-170.1</td>
</tr>
<tr>
<td>$\tilde{L}_{\text{valid}}^{86}$</td>
<td>-188.9</td>
<td>-182.6</td>
<td>-173.7</td>
<td>-196.7</td>
</tr>
<tr>
<td>$\tilde{L}_{\text{valid}}^{88}$</td>
<td>-198.1</td>
<td>-189.3</td>
<td>-178.0</td>
<td>-213.7</td>
</tr>
<tr>
<td>$\tilde{L}_{\text{valid}}^{89}$</td>
<td>-227.1</td>
<td>-201.4</td>
<td>-194.4</td>
<td>-223.3</td>
</tr>
</tbody>
</table>

(SbFD also significantly better at aggregate level – not shown)
Cross-flow experiment (Plaue et al., 2014)
Cross-flow experiment: Results

<table>
<thead>
<tr>
<th></th>
<th>Zero-Model</th>
<th>Drake</th>
<th>SbFD</th>
<th>Weidmann</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \tilde{\mathcal{L}} )</td>
<td>-578.0</td>
<td>-547.5</td>
<td>-527.3</td>
<td>-545.4</td>
</tr>
<tr>
<td>( V_f ) [m/s]</td>
<td>1.307</td>
<td>1.308</td>
<td>1.308</td>
<td>1.332</td>
</tr>
<tr>
<td>( \mu ) [-]</td>
<td>1.16</td>
<td>1.39</td>
<td>2.64</td>
<td>2.05</td>
</tr>
<tr>
<td>( \vartheta ) [m^4]</td>
<td>0.139</td>
<td>0.143</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \beta ) [m^2]</td>
<td></td>
<td></td>
<td>0.300</td>
<td></td>
</tr>
<tr>
<td>( \gamma ) [m^{-2}]</td>
<td></td>
<td></td>
<td></td>
<td>1.76</td>
</tr>
<tr>
<td>( k_j ) [m^{-2}]</td>
<td></td>
<td></td>
<td></td>
<td>5.99</td>
</tr>
</tbody>
</table>

Aggregate route travel times:

<table>
<thead>
<tr>
<th></th>
<th>( N_{\text{ped}} )</th>
<th>( tt_{\text{obs}} )</th>
<th>( tt_{\text{zero}} )</th>
<th>( tt_{\text{drake}} )</th>
<th>( tt_{\text{sbfd}} )</th>
<th>( tt_{\text{weid}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>W \rightarrow E</td>
<td>118</td>
<td>12.4</td>
<td>10.8</td>
<td>13.3</td>
<td>12.6</td>
<td>14.0</td>
</tr>
<tr>
<td>N \rightarrow S</td>
<td>46</td>
<td>10.6</td>
<td>8.4</td>
<td>10.0</td>
<td>10.9</td>
<td>9.9</td>
</tr>
</tbody>
</table>
(a) Zero-Model (L²-error: 53.3 s)

(b) Drake (L²-error: 47.6 s)

(c) Weidmann (L²-error: 47.4 s)

(d) SbFD (L²-error: 39.2 s)
Illustration: Walking speed in counter-flow

\[ \lambda \in \Lambda_\xi: \]
\[ m_\lambda = \frac{M_\lambda}{A_\xi} \]
\[ v_\lambda = \frac{V_\lambda}{V_f} \]

Parameters:
\[ V_f = 1.308 \text{ m/s} \]
\[ \vartheta = 0.143 \text{ m}^4 \]
\[ \beta = 0.300 \text{ m}^2 \]

(Berlin data set)
Concluding remarks

• macroscopic model for congested, multi-directional flow

• explicit consideration of anisotropy
  – stream-based fundamental diagram

• calibration and validation using MLE
  – counter- and cross-flow experiments (Hong Kong and Berlin)

• future work
  – improvement in specification (e.g. fundamental diagram)
  – phenomena of self-organization
  – applications within DTA-framework, demand estimation
Thank you

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