INT 160/89 AVRIL 1989

SHEAR STRESS IN THE INSULATION AT THE CONDUCTOR ENDS OF A COIL

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1) INTRODUCTION

The tensile stress in a coil resulting from the hoop force is transferred through the conductor at the coil ends (connections) where it induces a shear stress in the insulation layer. A 1-D model is used to predict this shear and to derive equations to design the coil ends so as to maintain the shear within tolerable limits. With the design planned for the TCV coils, the maximum shear should not be larger than 15 MPa.

2) 1-D MODEL

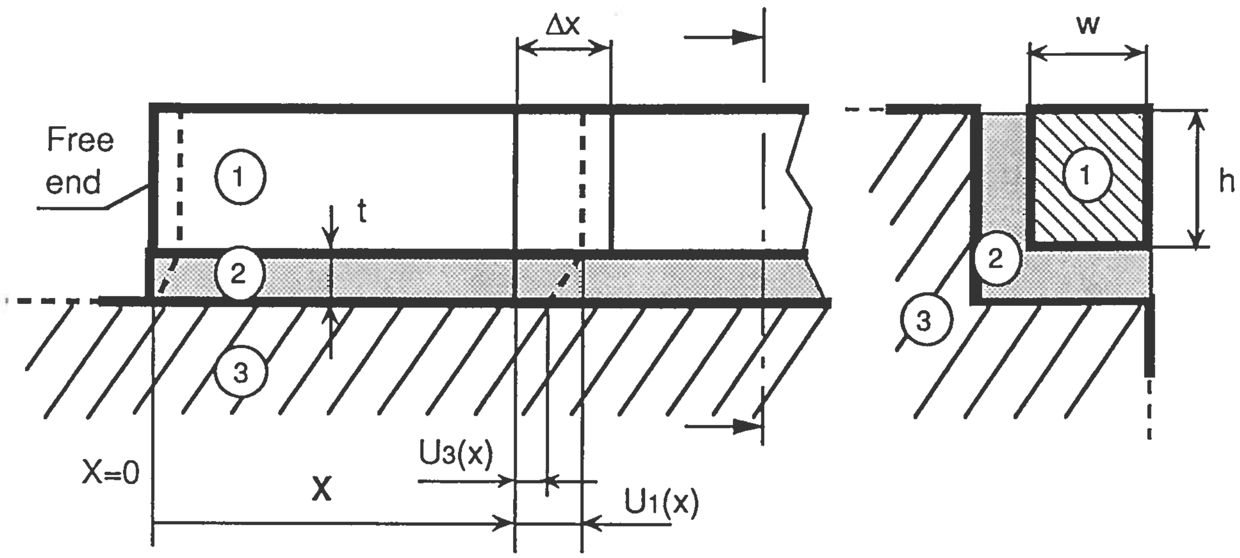

Fig. 1 1-D model of the conductor end of the coil.

1 : Copper conductor of X-section = w h
2 : Epoxy-fiberglass insulation layer of thickness = t
3 : Main body of the coil

The main body of the coil (region 3) is under a constant tensile stress σ_3. The conductor end is glued over a contour of length p (usually p = w+h) of constant thickness t . The stress σ_3 induces a stress $\sigma_1(x)$ in the conductor end (region 1) and a shear $\tau_2(x)$ in the insulation (region 2) near the end.
$u_1(x)$ and $u_3(x)$ are the deformation of the conductor end and of the coil.

By definition, the stresses and the shear are given by :

$$\sigma_1 = Y_c \cdot \frac{du_1}{dx} \qquad 1)$$

$$\sigma_3 = Y_c \cdot \frac{du_3}{dx} \qquad 2)$$

$$\tau_2 = G_e \cdot \frac{u_1 - u_3}{t} \qquad 3)$$

The tensile stress σ_3 is constant, therefore :

$$u_3 = \frac{\sigma_3}{Y_c} \cdot x = \alpha \cdot x \qquad 4)$$

For a conductor with a constant X-section S, the force balance on the element Δx gives :

$$\frac{d\sigma_1}{dx} - \frac{\tau_2 \cdot p}{S} = 0 \qquad 5)$$

and from 3 the equation for τ_2 is :

$$\frac{d\tau_2}{dx} - \frac{G_e}{Y_c} \cdot \frac{(\sigma_1 - \sigma_3)}{t} = 0 \qquad 6)$$

The combination of the equations 1,3,4,5 gives for $u_1(x)$:

$$\frac{d^2u_1}{dx^2} - k^2 \cdot u_1 = - k^2 \cdot \alpha \cdot x \qquad 7)$$

where $$k^2 = \frac{G_e}{Y_c} \cdot \frac{p}{t} \cdot \frac{1}{S} \qquad 8)$$

The independent equations for σ_1 and τ_2 are derived in the same way :

$$\frac{d^2\sigma_1}{dx^2} - k^2 \cdot \sigma_1 = - k^2 \cdot \sigma_3 \qquad 9)$$

$$\frac{d^2\tau_2}{dx^2} - k^2 \cdot \tau_2 = 0 \qquad 10)$$

The solution of the equation 9 with the boundary conditions :

$$\sigma_1(x = 0) = \sigma_0 \quad ; \quad \sigma_1(x = \infty) = \sigma_3 \qquad 11)$$

is : $$\sigma_1 = \sigma_3 - (\sigma_3 - \sigma_0) \cdot e^{-k \cdot x} \qquad 12)$$

where σ_0 is the residual stress at the end of the conductor. This stress is determined by the volume and geometry of the epoxy filling at the end.

From equation 5, τ_2 is :

$$\tau_2 = \frac{k \cdot S}{p} \cdot (\sigma_3 - \sigma_0) \cdot e^{-k \cdot x} \qquad 13)$$

The deformation u1 is obtained from equations 3 and 4.

$$u_1 = \frac{1}{k} \cdot \frac{\sigma_3 - \sigma_0}{Y_c} \cdot e^{-k \cdot x} + \frac{\sigma_3}{Y_c} \cdot x \qquad 14)$$

The maximum shear in the coil insulation is obtained from the equation 13 for x = 0.

$$\tau_{max} = \frac{k \cdot S}{p} \cdot (\sigma_3 - \sigma_0) = \sqrt{\frac{G_e}{Y_c} \cdot \frac{S}{p \cdot t}} \cdot (\sigma_3 - \sigma_0) \qquad 15)$$

This simple formula gives a criterion ($\tau_{max} < 15$ MPa)to find out whether a coil needs a specific design of its connections to withstand the shear stress in the insulation.

2) APPLICATION TO THE TCV CENTRAL COLUMN

COIL A

$S = 491\ mm^2$
$h = 19.8$ mm $\qquad$ $w = 28.5$ mm
$p \approx 50$ mm $\qquad$ $t \approx 2$ mm
$Y_c = 1{,}2\ 10^{11}$ Pa $\qquad$ $G_e = 3\ 10^9$ Pa
$\sigma_3 \approx 85\ 10^6$ Pa $\qquad$ $\sigma_0 = 0$ (This gives the maximum τ)

$$k = \sqrt{\frac{G_e}{Y_c} \cdot \frac{p}{t \cdot S}} = 36\ m^{-1} \quad \text{and} \quad \tau_{max} = 0.35\,\sigma_3 = 30\ \text{MPa} \quad !$$

The region with a shear ≥ 10 MPa is only for $x \leq x_c$ with

$$x_c = \frac{1}{k} \cdot \ln\frac{\tau_{max}}{\tau} = 3\ \text{cm} \quad !$$

COIL E

$S = 168 \ mm^2$
$h = 13 \ mm$ $w = 15 \ mm$
$p \approx 28 \ mm$ $t \approx 2 \ mm$
$Y_c = 1{,}2 \ 10^{11} \ Pa$ $G_e = 3 \ 10^9 \ Pa$
$\sigma_3 \approx 80 \ 10^6 \ Pa$ $\sigma_0 = 0$ (This gives the maximum τ)

$$k = \sqrt{\frac{G_e}{Y_c} \cdot \frac{p}{t \cdot S}} = 46 \ m^{-1} \quad \text{and} \quad \tau_{max} = 0.28 \ \sigma_3 = 22 \ MPa \ !$$

The region with a shear ≥ 10 MPa is only for $x \leq x_c$ with

$$x_c = \frac{1}{k} \cdot \ln \frac{\tau_{max}}{\tau} = 1.7 \ cm \ !$$

3) EQUATIONS WITH VARIABLE CROSS SECTION.

If the cross section S, the cemented contour length p, and the thickness of insulation t are variable along the coil ends $x \leq \lambda$ then the equations 1 to 4 are still valid and equations 5 and 6 become :

$$\frac{d\sigma_1}{dx} + \sigma_1 \cdot \frac{1}{S} \cdot \frac{dS}{dx} - \frac{p}{S} \cdot \tau_2 = 0 \qquad 16)$$

$$\frac{d\tau_2}{dx} + \tau_2 \cdot \frac{1}{t} \cdot \frac{dt}{dx} - \frac{G_e}{Y_c} \cdot \frac{(\sigma_1 - \sigma_3)}{t} = 0 \qquad 17)$$

where S, p, and t are functions of x in the region $x \leq \lambda$. t is constant over the contour length p to avoid stress concentration.

In general, these equations have to be solved numerically for $x \leq \lambda$ with the boundary conditions :

$$\sigma_1(x = 0) = \sigma_0 \ ; \ \sigma_1(x = \lambda) = \sigma_\lambda \qquad 18)$$

The solution in the region $x > \lambda$ are those of equations 9 and 10 with the boundary conditions :

$$\sigma_1(x = \lambda) = \sigma_\lambda \ ; \ \sigma_1(x = \infty) = \sigma_3 \qquad 19)$$

which gives :

$$\sigma_1(x) = \sigma_3 - (\sigma_3 - \sigma_\lambda) \cdot e^{-k(x-\lambda)} \tag{20}$$

$$\tau_2(x) = \frac{k \cdot S}{p} \cdot (\sigma_3 - \sigma_\lambda) \cdot e^{-k(x-\lambda)} \tag{21}$$

The boundary value σ_λ is related to τ_λ by :

$$\sigma_\lambda = \sigma_3 - \frac{p}{k \cdot S} \cdot \tau_\lambda \tag{22}$$

where τ_λ should be set to a value of the order of 10 MPa.

Remarks :

- For manufacturing reasons, it is preferable to look for solutions where t, the insulation thickness, is constant along the conductor ends.
- The tensile stress in the insulation at the conductor ends (σ_0) must also be kept at a reasonable level. This is particularly important if the insulation thickness increases toward the end.
- To include σ_0 in the solution of the equations 16 and 17 it is necessary to proceed by iteration. First the equations are solved with $\sigma_0 = 0$ and $u_1(0)$ is calculated with the equations 3 and 4. From this deformation σ_0 is evaluated taking into account the geometry of the epoxy filling at the end. The equations are then solved again with this new σ_0. This procedure is repeated until a stable solution is found.
- To avoid large electric fields, the conductor ends must not have sharp edges.

4) CONCLUSION FOR THE TCV CENTRAL COLUMN

At the maximum tensile stresses evaluated for the TCV central column, the shear stresses reach high values along the last few centimeters at the ends of the conductors. It is therefore necessary to design the coil feeds so as to keep these shears below 10 to 15 MPa. This can be achieved by a specific tailoring of the copper conductor cross section and or the insulation thickness at the coil feeds.