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CALCULATION OF MATRICES FOR SHAPING AND CONTROL OF ELONGATED PLASMAS

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ABSTRACT

A method for shaping and controlling non-circular plasmas has been proposed by Hofmann (16.6.88) based on methods used to find equilibria in the FBT code. This method is used here to calculate the matrices which relate required shaping coil voltages to magnetic measurements. The results are usable for the TCV plasma control hardware under construction, and for tests with a simulation code, for example TSC by Jardin.

A second method proposed by Marcus, Jardin, Hofmann (18.11.85), using a few multipole moments, extended to include thyristor PID control, and tested in the TSC code, is also used here to calculate another set of matrix elements.

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INTRODUCTION TO METHOD BY F. HOFMANN

There are many possible ways of controlling and shaping elongated plasmas. In TCV, with 16 independent shaping coils, the ohmic coils, and a continuous, thick, current carrying vacuum vessel, and with fast internal coils, there are many more possibilities than in most tokamaks, which have hard-wired moment control.

In this memo, we discuss two possible methods. The first, proposed by F. Hofmann, uses methods developed for finding and optimising equilibria in his FBT code (F. Hofmann, Computer Physics Comm. 48 (1988) 207-221).

The FBT code allows the computation of arbitrarily shaped tokamak equilibria, with external or internal separatrices and multiple magnetic axes. It is shown that bifurcations can be controlled by an iterative procedure using shape feedaback. Three specific applications are discussed: (1) the problem of shape accuracy, (2) the calculation of a startup evolution for a highly elongated tokamak, and (3) methods to control saddle points.

In this memo, the method of control proposed by Hofmann (16.6.88) is adapted to be compatible with the TCV hardware for plasma control currently under design and construction, which multiplies input measurements by a time-varying matrix. The derivation of the matrix elements is described in detail.

In this method, complete circuit equations, including self and mutual inductances and resistances, are written for the plasma, vessel, shaping and ohmic coils, in terms of current increments during the following time step. The vessel current increments are solved for. A cost function Q is minimized so as to minimize flux errors and current magnitudes according to weighting functions. We then solve for shaping and vessel currents, and finally shaping and ohmic voltages to be applied to achieve shaping and control.

The following assumptions are made:

1) The plasma may be treated as having (varying with elongation) one current; one self-inductance; one mutual with each system element; a resistive loop voltage known in advance, and each current step much less than the total current.
2) The vessel may be represented by a few elements, and its local currents may be accurately measured or deduced.
3) The boundary flux values are measured directly in simulations or deduced from magnetic measurements by another matrix.
4) The ohmic coil is here treated as an ideal voltage source on all other elements.

DETERMINATION OF MATRIX ELEMENTS FOR METHOD BY HOFMANN

The following definitions are used to put the equations into FORTRAN-like notation, with max dimension in parentheses after =.

ALPV(I=IV)	MUTUAL INDUCTANCE BETWEEN PLASMA AND Ith VESSEL ELEMENT
ALPP	SELF INDUCTANCE OF THE PLASMA
AVRCP(I=IV)	= -VRESDT +(ALPV(I) - ALPP)*CPP
ADIV(I=IV,J=IV)	DERIVED MATRIX
ADIS(I=IV,JJ=IS)	DERIVED MATRIX
ARVDT(I=IV)	=RESISTANCE OF VESSEL ELEMENT TIMES DT
ALPHA	LEAST SQUARES WEIGHT FOR FLUXES
ALSS(MM=IS,JJ=IS)	MUTUAL INDUCTANCE MATRIX BETWEEN SHAPING COILS
BETA	LEAST SQUARES WEIGHT FOR SHAPING CURRENTS
BDIS(J=IV,JJ=IS)	DERIVED MATRIX
BRVDT(J=IV)	DERIVED VECTOR
BVRCP(J=IV)	DERIVED VECTOR
BLPVPP(J=IV)	DERIVED VECTOR
CPP	PROGRAMMED CURRENT FOR PLASMA AT TIME T PLUS DT
CPM	MEASURED PLASMA CURRENTS AT TIME T
CV(L=IV)	MEASURED VESSEL ELEMENT CURRENT
CCPM(K=ICB)	DERIVED VECTOR
CURSM(LE=IS)	MEASURED CURRENT IN SHAPING COILS

CDIS(K=ICB,JJ=IS)	DERIVED VECTOR
DT	CHOSEN TIME INTERVAL TO NEXT FEEDBACK CALCULATION STEP. IN SIMULATION CODE USE, NOT EQUAL TO TIME STEP
DIV(L=IV)	CHANGE IN CURRENT IN Lth VESSEL ELEMENT DESIRED AFTER TIME DT
DIS(M=IS+1)	CHANGE IN CURRENT IN Mth SHAPING COIL DESIRED AFTER TIME DT. IS+1th ELEMENT IS PHI
DIP	=CPP-CPM , CHANGE IN PLASMA CURRENT DESIRED AFTER TIME DT
DPSIB(K=ICB)	IDEAL DESIRED FLUX CHANGE AT Kth BOUNDARY POINT ON THE DESIRED PLASMA SURFACE
FDIS(LL=IS+1,MM=IS+1)	DERIVED MATRIX
FCON(LL=IS+1)	DERIVED VECTOR
FCPM(LL=IS+1)	DERIVED VECTOR
FCV(LL=IS+1,J=IV)	DERIVED MATRIX
FPSIM(LL=IS+1,K=ICB)	DERIVED MATRIX
FCURSM(LL=IS+1)	DERIVED VECTOR
FMEAS(NN=NNXX,LL=IS+1)	DERIVED MATRIX
FINDIS(MM=IS+1,LL=IS+1)	INVERSE MATRIX OF FDIS
FFMEAS(MM=IS+1,NN=NNXX)	DERIVED MATRIX
FDIV(J=IV,NN=NNXX)	DERIVED MATRIX
FVOH(NN=NNXX)	DERIVED VECTOR

FVSH(MM=IS,NN=NNXX)	DERIVED MATRIX
FINELM(NN=NNXX,NNX=NNXM)	MATRIX GIVEN BY F. HOFFMAN FINITE ELEMENT METHOD (OR ANOTHER METHOD) TO RELATE PROCESSED TO RAW MAGNETIC MEASUREMENTS
GPB(K=ICB)	MUTUAL INDUCTANCE BETWEEN PLASMA AND Kth BOUNDARY POINT
GVB(K=ICB,J=IV)	MUTUAL INDUCTANCE BETWEEN Jth VESSEL ELEMENT AND Kth BOUNDARY POINT
GSB(K=ICB,JJ=IS)	MUTUAL INDUCTANCE BETWEEN JJth SHAPING COIL AND Kth BOUNDARY POINT
IV	NUMBER OF VACUUM VESSEL ELEMENTS
ICB	NUMBER OF FLUX MEASUREMENT POINTS
IS	NUMBER OF SHAPING COILS
NNXM	NUMBER OF RAW MEASUREMENTS
NNXX	= 1 + 1 + IV + ICB +IS NUMBER OF PROCESSED MEASUREMENTS
PHI	DESIRED VALUE OF LIMITER FLUX AT TIME T + DT. DIS(IS+1) IS SET EQUAL TO PHI.
RDTVV(I=IV)	=0.5*RV*DT FOR EACH VESSEL ELEMENT
RV(I=IV)	RESISTANCE OF EACH VESSEL ELEMENT
RESSH(MM=IS)	RESISTANCE OF EACH SHAPING COIL
TRANSM(NNX=NNXM,MM=IS)	DERIVED MATRIX
TRANOH(NNX=NNXM)	DERIVED VECTOR
TRANSF(NNX=NNXM,MMX=IS+1)	DERIVED MATRIX
T	TIME WHEN MEASUREMENTS ARE TAKEN

VMEAS(NN=NNXX)	VECTOR OF PROCESSED MEASUREMENTS
VMEASX(NNX=NNXM)	VECTOR OF RAW MEASUREMENTS
VRESDT	TIME-VARYING CONSTANT ESTIMATED IN ADVANCE, EQUAL TO PLASMA LOOP RESISTIVE VOLTAGE TIMES DT.
VOH	RESULT FOR APPLIED, INDUCED VOLTAGE REQUIRED FROM OHMIC COIL
VSH(MM=IS)	RESULT FOR VOLTAGES TO APPLY TO SHAPING COILS
VSHOH(MMX=IS+1)	RESULT FOR VOLTAGES TO APPLY TO IS SHAPING COILS, AND OH VOLTAGE, DERIVED FROM RAW MEASUREMENT VECTOR
XADIV(J=IV,I=IV)	INVERSE MATRIX OF ADIV
ZMUTVV(I=IV,K=IV)	MUTUAL INDUCTANCE MATRIX BETWEEN VESSEL ELEMENTS
ZMUTVP(I=IV)	MUTUAL INDUCTANCE VECTOR BETWEEN VESSEL ELEMENTS AND PLASMA
ZMUTSV(I=IV,L=IS)	MUTUAL INDUCTANCE MATRIX BETWEEN Ith VESSEL ELEMENT AND Lth SHAPING COIL
ZMUTSP(L=IS)	MUTUAL INDUCTANCE BETWEEN PLASMA AND Lth SHAPING COIL

MATRIX ELEMENT CALCULATION

Full circuit equations are written for the plasma, vessel elements, shaping coils, and OH coil voltage. The VOH equation is substituted into the vessel equations to obtain the vessel current changes DIV(J). In these equations, we have the following matrix multiplying DIV(J),

$$\begin{aligned} ADIV(I,J) \equiv\ & RDTVV(I) && \text{when } J=I \\ & + ZMUTVV(I,J) && \text{for all } J \\ & - ZMUTVP(I) && \text{when } J=I \end{aligned}$$

and the following matrix multiplying DIS(JJ), shaping current changes.

$$ADIS(I,JJ) \equiv ZMUTSV(I,JJ) - ZMUTSP(JJ)$$

We then solve for DIV(J) in terms of other variables with XADIV(J,I), the inverse matrix of ADIV(I,J). to obtain the following equations for DIV(J)

$$\begin{aligned} DIV(J) = \sum_{JJ=1}^{IS} & BDIS(J,JJ) * DIS(JJ) + BRVDT(J) * CV(J) \\ & + BVRCP(J) + BLPVPP(J) * CPM \end{aligned}$$

where we have defined the following as:

$$BDIS(J,JJ) \equiv - \sum_{I=1}^{IV} XADIV(J,I) * ADIS(I,JJ)$$

$$BRVDT(J) \equiv - \sum_{I=1}^{IV} XADIV(J,I) * ARVDT(I)$$

$$BVRCP(J) \equiv \sum_{I=1}^{IV} XADIV(J,I) * AVRCP(I)$$

$$BLPVPP(J) \equiv - \sum_{I=1}^{IV} XADIV(J,I) * (ALPV(I) - ALPP)$$

The real flux changes at the plasma boundary points are:

$$DPSIB(K) = CCPM(K) * CPM + \sum_{JJ=1}^{IS} CDIS(K,JJ) * DIS(JJ) + GPB(K) * CPP + \sum_{J=1}^{IV} GVB(K,J) * \{BRVDT(J) * CV(J) + BVRCP(J)\}$$

where we have defined the following as:

$$CCPM(K) \equiv -GPB(K) + \sum_{J=1}^{IV} GVB(K,J) * BLPVPP(J)$$

$$CDIS(K,JJ) \equiv GSB(K,JJ) + \sum_{J=1}^{IV} GVB(K,J) * BDIS(JJ)$$

The cost function Q to be minimized with regards to flux and shaping currents is given by

$$Q = ALPHA * \sum_{K=1}^{ICB} (DPSIB(K) - PHI - PSIM(K))^2 + BETA * \sum_{JJJ=1}^{IS} (CURSM(JJJ) + DIS(JJJ))^2$$

Redefining PHI to be DIS(IS+1), the equations resulting from $\partial Q/\partial(DIS)$ and $\partial Q/\partial(PHI) = 0$ are:

$$\sum_{MM=1}^{IS+1} FDIS(LL,MM) * DIS(MM) = FCON(LL) + FCPM(LL) * CPM + \sum_{J=1}^{IV} FCV(LL,J) * CV(J) + \sum_{K=1}^{ICB} FPSIM(LL,K) * PSIM(K) + FCURSM(LL) * CURSM(LL)$$

(Note that CURSM(IS+1) = 0.)

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where we have defined the following:

FDIS(LL, MM) $\equiv$ (For LL = 1 to IS)

$-2. * ALPHA * \sum_{K=1}^{ICB} CDIS(K,LL) * CDIS(K,MM)$ for MM=1 to IS

$-2. * BETA$ for MM=LL only

$+2. * ALPHA * \sum_{K=1}^{ICB} CDIS(K,LL)$ for MM= IS+1 only

FDIS(LL, MM) $\equiv$ (For LL = IS+1)

$2. * ALPHA * \sum_{K=1}^{ICB} CDIS(K,MM)$ for all MM

$-2. * ICB * ALPHA$ for MM=IS+1 only

and where we have also defined:

$$FCON(LL) \equiv 2.*ALPHA*\sum_{K=1}^{ICB} CDIS(K,LL)*\left[GPB(K)*CPP + \sum_{J=1}^{IV} GVB(K,J)*BVRCP(J)\right] \quad (\text{For } LL=1 \text{ to } IS)$$

$$FCON(LL) \equiv -2.*ALPHA*\sum_{K=1}^{ICB} \left[\sum_{J=1}^{IV} GVB(K,J)*BVRCP(J) + CPB(K)*CPP\right] \quad (\text{For } LL=IS+1)$$

$$FCPM(LL) \equiv 2.*ALPHA*\sum_{K=1}^{ICB} CDIS(K,LL)*CCPM(K) \quad (\text{For } LL=1 \text{ to } IS)$$

$$FCPM(LL) \equiv -2.*ALPHA*\sum_{K=1}^{ICB} CCPM(K) \quad (\text{For } LL=IS+1)$$

$$FCV(LL,J) \equiv 2.*ALPHA*\sum_{K=1}^{ICB} CDIS(K,LL)*GVB(K,J)*BRVDT(J) \quad (\text{For } LL=1 \text{ to } IS)$$

$$FCV(LL,J) \equiv -2.*ALPHA*\sum_{K=1}^{ICB} GVB(K,J)*BRVDT(J) \quad (\text{For } LL=IS+1)$$

$$FPSIM(LL,K) \equiv 2.*ALPHA*CDIS(K,LL) \quad (\text{For } LL=1 \text{ to } IS)$$

$$FPSIM(LL,K) \equiv 2.*ALPHA \quad (\text{For } LL=IS+1)$$

$$FCURSM(LL) \equiv 2.*BETA \quad (\text{For } LL=1 \text{ to } IS)$$

$$FCURSM(LL) \equiv 0. \quad (\text{For } LL=IS+1)$$

Combining all measurements, plus a constant term, into one vector, we obtain the following equations for DIS(MM), with LL=1 to IS+1

$$\sum_{MM=1}^{IS+1} FDIS(LL,MM) * DIS(MM) = \sum_{NN=1}^{1+1+IV+ICB+IS} FMEAS(NN,LL) * VMEAS(NN)$$

where the measurement vector contains the following.

VMEAS(NN)	≡	1.0	NN = 1
	≡	CPM	NN = 1+1
	≡	CV(J)	NN = 1+1+J, J = 1 to IV
	≡	PSIM(K)	NN = 1+1+IV+K, K = 1 to ICB
	≡	CURSM(LE)	NN = 1+1+IV+ICB+LE, LE = 1 to IS

Note that PSIM(K) are derived boundary fluxes from other measurements. The matrix FMEAS(NN, LL) is defined as:

FMEAS(NN, LL)	≡	FCON(LL)	NN = 1
	≡	FCPM(LL)	NN = 1+1
	≡	FCV(LL, J)	NN = 1+1+J, J = 1 to IV
	≡	FPSIM(LL, K)	NN = 1+1+IV+K, K = 1 to ICB
	≡	FCURSM(LL)	NN = 1+1+IV+ICB+LE, LE = 1 to IS

We solve this equation by taking FINDIS(MM, LL) to be the inverse matrix of FDIS(LL, MM) and we obtain, for MM = 1 to IS+1,

$$DIS(MM) = \sum_{NN=1}^{1+1+IV+ICB+IS} FFMEAS(MM,NN) * VMEAS(NN)$$

where we have defined

$$FFMEAS(MM,NN) \equiv \sum_{LL=1}^{IS+1} FINDIS(MM,LL) * FMEAS(NN,LL)$$

Using the result for DIS(MM), we solve for DIV(J), J=1 to IV

$$DIV(J) = \sum_{NN=1}^{1+1+IV+ICB+IS} FDIV(J,NN) * VMEAS(NN)$$

where we have defined

$$\begin{aligned} FDIV(J,NN) \equiv & \sum_{JJ=1}^{IS} BDIS(J,JJ) * FFMEAS(JJ,NN) \\ & + BRVDT(J) \qquad \text{(ONLY for } NN=1) \\ & + BVRCP(J) \qquad \text{(ONLY for } NN=1+1+J) \\ & + BLPVPP(J) \qquad \text{(ONLY for } NN=1+1) \end{aligned}$$

Substituting, we obtain the inductive voltage which we wish the ohmic coil to produce at the other coils.

$$VOH = \sum_{NN=1}^{1+1+IV+ICB+IS} FVOH(NN) * VMEAS(NN)$$

where we have defined

$$FVOH(NN) \equiv -\frac{1}{DT} * \left\{ \begin{aligned} & \sum_{J=1}^{IV} ALPV(J) * FDIV(J,NN) \\ & + \sum_{I=1}^{IS} ZMUTSP(I) * FFMEAS(I,NN) \\ & + VRESDT \qquad \text{(ONLY FOR } NN=1) \\ & + ALPP * CPP \qquad \text{(ONLY FOR } NN=1) \\ & - ALPP \qquad \text{(ONLY FOR } NN=1+1) \end{aligned} \right\}$$

Substituting, we obtain the desired MM=1 to IS results for the shaping voltages VSH(MM) to be applied during the following DT time interval as:

$$VSH(MM) = \sum_{NN=1}^{1+1+IV+ICB+IS} FVSH(MM,NN) * VMEAS(NN)$$

We have defined the matrix FVSH (MM, NN) as

FVSH (MM, NN) ≡

$$\frac{1}{DT} * \sum_{KK=1}^{IV} ZMUTSV(KK,MM) * FDIV(KK,NN)$$

$$+ \frac{1}{DT} * \sum_{JJ=1}^{IS} ALSS(MM,JJ) * FFMEAS(JJ,NN)$$

$$- \frac{1}{DT} * ZMUTSP(MM) \qquad \text{(ONLY FOR } NN=2\text{)}$$

$$+ \frac{1}{DT} * ZMUTSP(MM) * CPP \qquad \text{(ONLY FOR } NN=1\text{)}$$

$$+ RESSH(MM) \qquad \text{(ONLY FOR } NN=1+1+IV+ICB+MM\text{)}$$

$$+ FVOH(NN)$$

INCLUDE TRANSFORMATION FROM RAW MEASUREMENTS

We have assumed that the measurement vector VMEAS(NN) contains, among other quantities, the measured flux on the plasma surface and the currents in the vacuum vessel elements, which in fact cannot be measured directly. F. Hofmann has developed a method using finite elements whereby the flux at the plasma boundary and the currents flowing in the finite elements in the plasma are a linear combination of measurements. Other methods using filaments, etc. have also been developed elsewhere. Vessel currents may be indirectly measured from the signals and/or their time-derivatives from magnetic probes near the vessel elements. In TSC simulations, VMEAS(NN) may be used directly. In an actual experiment, or a more complicated TSC simulation, we may find VMEAS(NN) as follows:

$$\text{VMEAS(NN)} = \sum_{\text{NNX}=1}^{\text{NNXM}} \text{FINELM(NN,NNX)} * \text{VMEASX(NNX)}$$

where VMEAS(NNX) is the measurement vector of all required measured quantities (including one constant term = 1.0) containing NNXM terms, and FINELM(NN,NNX) is the transformation matrix developed from the finite element (or another) method to produce boundary flux values from measured magnetic signals. Since VMEAS(NN) contains not only flux values, but also some input measurements such as coil currents, some matrix elements will be 1.0 in a row or column of zeroes to give a 1-to-1 transfer. The shaping voltages then become:

$$\text{VSH(MM)} = \sum_{\text{NNX}=1}^{\text{NNXM}} \text{TRANSM(NNX,MM)} * \text{VMEASX(NNX)}$$

where

$$\text{TRANSM(NNX,MM)} = \sum_{\text{NN}=1}^{1+1+\text{IV}+\text{ICB}+\text{IS}} \text{FVSH(MM,NN)} * \text{FINELM(NN,NNX)}$$

Similarly, VOH is given by:

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                        NNXM
VOH =                   (SUM)  TRANOH(NNX) * VMEASX(NNX)
                        NNX=1
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where

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                        NNNX
TRANOH(NNX) =           (SUM)  FVOH(NN) * FINELM(NN,NNX)
                        NN=1
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If VOH is taken to be the IS+1 th shaping voltage of the vector VSHOH(MM) containing the voltages to be applied to the shaping coils, then the final result, giving the transfer function between all measurements and the voltages to be applied, for MMX = 1 to IS + 1, is:

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                        NNXM
VSHOH(MMX) =            (SUM)  TRANSF(NNX,MMX) * VMEASX(NNX)
                        NNX=1
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where we have defined:

TRANSF(NNX,MMX) = TRANSM(NNX,MM)   for MMX=MM , MM=1 to IS

TRANSF(NNX,MMX) = TRANOH(NNX)   for MMX= IS + 1

We have therefore reduced the entire calculation to a single matrix multiplication.

INTRODUCTION TO
METHOD OF F. MARCUS, S. JARDIN, F. HOFMANN

This method was published in Phys. Rev. Lett. 55 (1985) 2289. It is based on using a few moments, variable in time, to control the plasma. It does not rely on explicitly calculating the response of each element in the vacuum vessel, nor on the mutual interaction of all the coils and vessel and plasma. The method has been extended in more recent publications to include a PID current control loop for thyristor supply control. (F. Marcus, F. Hofmann, G. Tonetti, S.C. Jardin, CRPP Report LRP 342/88, presented DPP-APS, San Diego, 1987.) The coefficients are consistent with recent calculations by R. Keller (1988).

This control method has been demonstrated to work in TSC simulations of the TCV tokamak. In this report, we show how the elements of a transfer function matrix using this method may be derived.

The model uses many simplifications to avoid deriving many difficult quantities to measure, especially the nonlinear response of a deformable plasma with distributed currents. The interaction of all coils is not included. The plasma response is considered only in terms of the response of flux measurements at the plasma boundary. Vessel response is not measured. The few shaping moments are produced with the correct distribution in the 16 independent shaping coils by controlling each coil current with a PID on the current loop giving the command voltage to the thyristor power supplies.

Recent simulations have been made with PD voltage control on fast coils inside the vacuum vessel for vertical instability control, where a voltage proportional to the fast coil current with a radial field moment distribution is applied to the slower coils outside the vessel (F. Marcus et al., 15th EPS conference, Dubrovnik, 1988). Further studies are required on the compatibility of this lastest research and the method being described in this section. One method would be to send the fast coil current as a desired current to the slower coils outside the vessel. The current in the fast coils inside the vessel would be limited as the current in the slower coils increased.

CALCULATION OF MATRIX ELEMENTS FOR METHOD OF MARCUS, JARDIN, HOFMANN

Referring to the Phys. Rev. Lett. article, we first need to find the interpolated flux measurement vector. This may be done in one of two ways. Both methods initially require a means, as discussed previously, of transforming raw measurements to flux values at desired points on the plasma boundary. In method (A), the flux values at the points desired are directly calculated for each time step. This has the disadvantage that the transformation matrix from raw to processed measurements has to be recalculated for each time step. It may therefore be simpler and require much less calculation between shots to use the method (B), as described in the Phys. Rev. Lett., of interpolating between sets of flux points, corresponding to precalculated equilibria with preprogrammed shaping coil currents.

In both methods, we use the equation (Note that here, all variables end with 2 to distinguish them from the method by F. Hofmann) :

$$\mathrm{PSIM2(NN2)} = \sum_{\mathrm{NNX2}=1}^{\mathrm{NNXM2}} \mathrm{FINELM2(NN2,NNX2)} * \mathrm{VMEASX2(NNX2)}$$

In method (A), the matrix FINELM2 is calculated for each time point. In method (B), the matrix is derived as a linear interpolation in programmed time between two matrices corresponding to the two sets of time points.

PSIM2(NN2) has NN2 = 1 to ICB2 elements.

VMEASX2(NNX2) has NNX2 = 1 to NNXM2 elements, contains all measurements, including a first element equal to 1.0 .

The position and shape error moments are then given by J = 1 to JMOM2 equations:

$$\mathrm{PMOM2(J)} = \sum_{\mathrm{I}=1}^{\mathrm{ICB2}} \mathrm{TMOM2(J,I)} * \mathrm{PSIM2(I)}$$

where TMOM2(J,I) has J = 1 to JMOM2 times I = 1 to ICB2 elements, and contains the flux moments which determine the errors.

Corresponding to each moment, there is a current distribution in the coils which corrects each position or shape error. The desired currents in the shaping coils are given by CSD2(K), for K = 1 to IS2 coils, where

$$\text{CSD2(K)} = \sum_{\text{J=1}}^{\text{JMOM2}} \text{CMOM2(K,J)} * \text{PMOM2(J)} + \text{CPREP2(K)}$$

where CMOM2(K,J) has K = 1 to IS times J = 1 to JMOM2 elements with the current moments for feedback control in the coils. Note that CMOM2(K,J) contains an implicit gain factor multiplying each moment, which can be varied. In this method, the values of the PID coefs in the regulators stay fixed, and this implicit gain factor allows dynamic adjustment. CPREP2(K) has K = 1 to IS2 preprogrammed currents interpolated from a few MHD equilibria, which should be close to the actual plasma equilibria.

To convert these desired currents into actual coil currents, the method adopted here is to send the desired current and the measured current to a PID regulator for each coil with delivers a command voltage to the thyristor supply based on the current error. This PID controller may reside either in the plasma control processor or in the power supply controller. The ohmic coil induced voltage is taken as proportional to the error between the desired and actual plasma current.

A simplified form for CSD2(K), with K=1 to IS2, which gives the final result for the required matrix elements is:

$$\text{CSD2(K)} = \sum_{\text{NNX2=1}}^{\text{NNXM2}} \text{TRANS2(K,NNX2)} * \text{VMEASX2(NNX2)}$$

where TRANS2(K,NNX2) has K = 1 to IS2 times NNX2 = 1 to NNXM2 elements and is given by:

$$\text{TRANS2(K,NNX2)} = \sum_{\text{J=1}}^{\text{JMOM2}} \text{CMOM2(K,J)}* \sum_{\text{I=1}}^{\text{ICB2}} \text{TMOM2(J,I)*FINELM2(K,NNX2)}$$

(for K = 1 to IS2 and NNX2 = 1 to NNXM2)

$$+\text{CPREP2(K)}$$

(for K = 1 to IS2 and NNX2 = 1)