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ESTIMATION OF HARD X-RAY PRODUCTION FROM RUNAWAY ELECTRONS IN TCV

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SUMMARY

A full flux TCV disruption, 1MAmp and 0.9 Volt·sec, causes a dose of approximately 300REM in an area of $1/2m^2$ at a distance of 8m. A 1.0 meter thick Bartyes Concrete Wall (ρ=3.5gm/cm^3) reduces this to 0.01REM/disruption.

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I ESTIMATED RADIATION FROM TCV

We use the general formula derived in Appendix K, based on the assumptions that 2% of stored magnetic energy is converted to a monoenergetic electron beam at a plasma disruption, which slows down in the limiter, producing gamma rays with spatial and energy distribution, which are integrated to give :

$$\text{DOSE (REM)} = 2.2 \times 10^3\, I_{MAMP} \phi_{VS}{}^3\, ((\eta/\rho)/.031)\, 1/R_{MAJ}{}^2 \text{meters}$$

In TCV, R_{MAJ}=0.87, and for high energy runaways, $\eta/\rho \approx 0.016$. We obtain the current and flux from Appendix A. The maximum value of $I\phi^3$ is found for 2.5/1 elongated plasmas with ø (in vessel)=0.9VS and I=1 MAmp, giving $I\phi^3$=.73.

The Dose is then 1093 REMS. The value of γ_{max} is $\gamma_{max}=70(\phi/R_{maj})=72$ ($\Rightarrow E_{max}$=37MeV). The size of this beam at L=8 meters is 0.83m high (from mag field angle) by .13m wide (=$9.6/\gamma_{max}$). From Appendix F, we see that because of large γ, the effect of perpendicular temperature is important. For γ=72, $\gamma\theta_g$=1.7, θ_g=.024, width=L(2)(.024)=0.38m, and beam height =.83 + .38 = 1.2m, increasing the beam area from .11m^2 to .46m^2, reducing the DOSE (which is inversely proportional to area) to

$$\text{DOSE} = 260\text{REMS at } L=8\text{m, Area}=0.46\text{m}^2$$

The area is equal to the area of a human body, so even if the perpendicular temperature is less, the same dose is obtained by averaging over several shots, or by averaging over the human body, for the whole body dose.

With shielding of 1 meter thick Barytes concrete of ρ=3.5gm/cm^3, the dose per disruption for someone standing in the beam is 260 exp(-0.1 x 100) = 0.012REM/disruption. At a rate of 400 disruptions at full parameters per year (≈4% x 10'000 shots), this gives a DOSE of 4.7REMS/YEAR for a person standing in the path of all 400 disruptions, which is unlikely.

APPENDIX A - *Inductance and stored Volt-seconds*

A.1) Analytic estimates

The stored magnetic energy in the plasma and the space between the plasma and vessel wall is available for being dissipated and driving runaway currents during disruptions. For a circular plasma of radius a in a shell radius b with internal inductance li, the inductance inside the vessel is

$$L = \mu_0 R\,(\ln b/a + li/2)$$

In TCV, both b/a and li/2 are hard to estimate, since a very wide range of shapes and current profiles are available in a rectangular vessel. For 1.2MA, in a racetrack, 1.2→1.5 Volt-seconds were required from the OH coil to inductively establish the plasma at 1.2MA. For Dees with peaked profiles and large open spaces in the vessel, L may be larger. For the above examples, L=1μHenry and LI=1.2 Volt-seconds seems a good choice. An L of 1μH is obtained from the circular formula if li=1 (peaked) and b/a=1.5 (that is, an average 12cm from plasma to wall on average in the 56cm wide vessel.

In JET, with li=1 and $b/a \approx 1.2$ for elongated plasma, $L \approx 2.6\mu H$.

A.2) Flux and inductance from MHD calculations in TCV

A large range of possible equilibria in TCV have been calculated by F. Hofmann, where the plasma current and flux from the limiter surface to the magnetic axis is calculated. It is difficult to calculate exactly the flux trapped inside the vessel, since the vessel is not a flux surface. A "typical" plasma plus vacuum flux inside the vessel is found by counting an "average" number of flux surfaces outside the vessel. An example is case S115, where 5 extra surfaces are taken, thereby multiplying plasma flux only by (10+5)/10, giving ø=0.9 Volt sec at 994kA, $L_{int+vac}$=0.9μHenry, for q_L/q_0=2.5.

Looking at a range of racetracks, dees, triplets, beans, we find the inductance decreases as the elongation K increases, giving the following table of typical values.

Elongation K	Plasma current I_p(MA)	Flux inside vessel ø(VS)	Inductance ø/I=L(μH)
2	0.8	0.9	1.1
2.5	1.0	0.9	0.9
3.0	1.2	0.7	0.6

APPENDIX AA - *Maximum electron energy from stored volt-seconds*

Jarvis' model [O.N. Jarvis, G. Sadler, J.L. Thompson, private communication, to be published] is that during a disruption, the voltage goes high for a short time (kilovolts for milliseconds) and turns 2% of stored magnetic plasma energy into runaways. Runaway electrons accelerate to the speed of light, accelerate in energy up to a maximum, and then strike a material surface when the plasma is lost. This factor of 2% can vary by ± a factor of 3 per shot, and is an average.

AA.1) Direction of electric field

The direction of the resistive electric field $E=\eta J$ is always in the same direction as the plasma current. When the external applied voltage is created (during current rampup), the extra Volt-seconds are stored as magnetic energy. During a disruption, when the plasma resistivity η greatly increases but J is still near its maximum, E greatly increases to satisfy $E=\eta J$, and the stored energy is taken from the $\partial/\partial t\ (LI^2/2)$ stored inductive energy. In other words, the electric field becomes much stronger to drive the same electron current (by sign convention opposite to J) against a higher resistance. The stored Volt-second flux is ø=LI, and the resistive voltage uses this up such that

$$ø = LI = \int_0^t V_{res} dt$$

AA.2) Energy gain of an electron in free fall

A runaway electron has an initial energy such that the electric field accelerates it more than the drag of neighboring electrons. Starting with a 1keV plasma, with electron tail distribution up to several KeV, such runaways are generated when the resistive electric field rapidly increases.

We first ask, how many volt-seconds are required to accelerate an electron from nearly at rest to the speed of light $c=3x10^8$m/sec, where $m_{elec}c^2=0.511$MeV. The acceleration on a runaway electron is constant at eE/mass=Force/mass. Since $E=V/2\pi R$, V=Volts/turn (resistive), R=Major Radius=0.87m for TCV, the velocity after time t is $eVt/2\pi Rm$.

Setting the velocity equal to the speed of light,

$$Vt = 2\pi Rcm/e = 0.01 \text{Volt·sec in TCV.}$$

Thus we see that only the voltage-time product is involved, and only 1% of the 1.2 Volt-seconds stored in TCV (L=1μHenry), (I=1.2MAmp) is needed to reach light speed.

We now ask, what energy can a runaway electron gain? Since the electron travels at the speed of light, gaining the loop voltage for each turn around the circumference during the time the disruption lasts, the maximum energy is simply

$$E_{max} = \frac{\text{(stored volt sec) (velocity of light)}}{\text{(circumference)}} = \frac{(LI)(c)}{2\pi R}$$

For TCV, with LI=1.2 volt sec, E_{max}=66MeV.

In JET, up to 100MeV was inferred from photoneutron measurements. With R=3m, this implies that LI=6.3 Volt·seconds (Vs) were consumed. With an inductance of the plasma of ≈ 2.6μH, this corresponds to 2.4MA. 6.3 Vs is also about 18% of the total Vs available. In TCV, the 1.2 Vs represents about 30% of the Vs available. Thus the 66MeV maximum energy may be an overestimate of the maximum energy. If so, then 2% of the magnetic energy would give proportionally more photons at lower maximum energy, which will in any case be several 10's of MeV.

The freefall acceleration was experimentally observed in ORMAK, [Knoepfel et al., 7th Europ. Conf., Lausanne, 1975] during a normal discharge, with the 10MeV energy limited by the discharge duration. The numbers of runaways produced at the beginning and accelerated are 10^{13} - 10^{14}, approximately 1 - 2 orders of magnitude less than produced by 2% of magnetic energy in a TCV disruption.

APPENDIX B - *Energy of electrons containable by the magnetic field*

The outward shift of high energy electrons in a tokamak which can lead to their loss is discussed in "Dynamics of high energy runaway electrons in ORMAK" by [Knoepfel et al., 7th Europ. Conf., Lausanne, 1975] . For a uniform current profile, which gives the lowest internal magnetic field and thus the greatest shift, the shift of the electron drift surface relative to its flux surface is

$$d_\gamma \approx r_L^2\ I_A/2RI$$

where r_L is the limiter radius, R is the major radius, I the plasma current, and I_A the Alfvén current, $I_A=17\sqrt{\gamma^2-1}$ (kAmp), kinetic energy $W_K=0.51(\gamma-1)$ MeV. Rewriting, the ratio of the shift to the minor radius is

$$\left[\frac{d_\gamma}{r_L}\right] = \left(\frac{r_L}{2R}\right)\left[\frac{17\left[\frac{W_k}{0.51}+1\right]}{I_{kAMPS}}\right]$$

Since the important parameter is the gyroradius in the poloidal field, for noncircular tokamaks like TCV and JET with elongation K, we normalize the current to equivalent circular poloidal field by dividing the I_{kAMPS} by $\sqrt{(1+k^2)/2}$ and defining the Aspect ratio as $A=R/r_L$, giving

$$\left[\frac{d}{r_L}\right] = \left(\frac{1}{2A}\right)\left[17\left(\frac{W_k^{MEV}}{0.51}+1\right)\right]\left[\frac{1}{I_{kAMP}}\right]\left[\sqrt{\frac{1+k^2}{2}}\right]$$

FOR ORMAK : A=3.8, W_k=10MEV, I=100kAMP, K=1
⇒Fractional shift d_γ/r_L = 46% of minor radius.
Note : 10MeV limited by normal pulse length only.

FOR JET : A=2.4, W_k=100MEV, I_{kAMP}=3500, K=1.6
⇒ Fractional shift d_γ/r_L=27% of minor radius.

As an example for TCV, if I_{kAMP}=1200 and A=3.6, W_k=66MEV, K=2.5

⇒ Fractional shift d_γ/r_L=49% of minor radius.

In TCV, the shift fraction is comparable to ORMAK, but higher than in JET. The fraction indicates that 66MeV electrons can be efficiently contained in TCV at 1200kA. During a disruption, the current decreases, reducing the maximum energy.

We consider the limit imposed by the current decrease during a disruption. If we take the JET fractional shift of 27% relative to the initial current, this implies a maximum energy in TCV of 27%/49%(66MeV)=36MeV.

In TCV, when E=36MeV, there will remain 66-36/66=45% of the original volt seconds and current.

The fractional shift at 36MeV and 45% of the current is taken at I=45%(1200)=540kA. However, K is reduced at this current to perhaps 1.4. We obtain

$$\frac{d_\gamma}{r_L} = \frac{1}{2(3.6)}[17(\frac{36}{0.51}+1)]\frac{1}{540}[\sqrt{\frac{1+1.4^2}{2}}] = 0.38 \text{ at } 36 \text{ MEV}$$

This fractional shift is below the maximum allowed of 1 and is even below the ORMAK shift of 46% when the energy was limited by pulse length, not disruption.

We now consider the shift for E=50MeV in TCV, representing 3/4 of the maximum 66MeV, which could occur at 1/4 the current, =300kA, which we consider for K=1. The relative shift become

$$\frac{d_\gamma}{r_L} = \frac{1}{2(3.6)}[17[\frac{50}{.51}+1]]\ \frac{1}{300}\ [\sqrt{\frac{1+1}{2}}] = 78\ \% \text{ at } 50\text{MeV}$$

We conclude that even allowing for the decrease in current during a disruption, electrons up to 36 - 50MeV can be contained by the remaining current.

We therefore consider the maximum energy to be 3/4 of the maximum energy due to flux available only. Using $\gamma_{max} \approx E_{MEV}/.511$, we obtain

$$\gamma_{max} = (3/4)(\phi c)/(2\pi R \times 10^6)(.511) = 70\phi/R_{maj}.$$

APPENDIX C - *Electron energy sustainable against synchrotron radiation*

Runaway electrons in a plasma have their velocity mostly parallel to the magnetic field, and therefore describe a nearly circular orbit with major radius R of the tokamak. The synchrotron radiation associated with this must be compensated by the loop voltage, or the electron will stop gaining energy. Formulas for synchrotron radiation loss are given in ["Atomic and molecular radiation physics by L.G. Christophorou, p. 87 - 88, based on Tomboulian and Hartman, 1956). The power per electron per wavelength (ergs $sec^{-1}cm^{-1}$) is given by

$$P(\lambda) = \left[\frac{3^{5/2} e^2 c}{16\pi^2 R^3} \left(\frac{E}{mc^2}\right)^7\right] G\left(\frac{\lambda}{\lambda_c}\right)$$

$$\lambda_c = \frac{4\pi R}{3} \left(\frac{mc^2}{E}\right)^3$$

An example of this was given for 120MeV and 180MeV electron radiation from the R=83.4cm NBS synchrotron. The symbols are $e=4.8\text{x}10^{-10}$ esu, $c=3\text{x}10^{10}$cm/sec, R=major radius(cm), E in MeV, mc^2=0.511MeV, and $G(\lambda/\lambda_c)$ is the universal spontaneous power distribution. It is graphed from $0=(\lambda/\lambda_c)=4$, has a maximum of G=1.2 at λ/λ_c =0.42, and is nearly 0 at λ/λ_c=4. We approximate it for the integrals to get total power.

$$P\left(\frac{ergs}{sec}\right) = [\ ....] \int_0^{\lambda_{max}} G\left(\frac{\lambda}{\lambda_c}\right) \partial\lambda$$

$$G\left(\frac{\lambda}{\lambda_c}\right) \approx 1.2\frac{\lambda}{\lambda_c};\ 0 \le \frac{\lambda}{\lambda_c} \le .42$$

$$\approx 1.2 \left(\frac{.42}{\lambda/\lambda_c}\right) \quad .42 \le \frac{\lambda}{\lambda_c} \le 4$$

$$\lambda_{max} = 4\lambda_c, \quad z \equiv \lambda/\lambda_c$$

$$P = [....]\, 1.2\lambda_c \left[\int_0^{.42} z\,dz + \int_{.42}^{4} \frac{.42}{z}\,dz\right] = [\qquad](1.63)\,\lambda_c$$

$$P\left(\frac{\text{ergs}}{\text{sec}}\right) = 6.9 \times 10^{-8} \frac{E^4_{Mev}}{R^2_{cm}} \qquad \text{per electron}$$

changing units,

$$P = \left(.043 \frac{\text{MeV}}{\text{sec}}\right) \frac{E^4_{MEV}}{R^2_{cm}}$$

For TCV, with E=66MeV and R=87cm. P_{rad}=108MeV/sec/elec.

The power gained per second is

$$\frac{P_{OH}}{\text{turn}} = \frac{(\text{volts/turn})\,(\text{speed of light})}{2\pi R}$$

To sustain the radiation loss, we set the powers equal and obtain volts/turn=2. volts to sustain radiation.

During a disruption, we may have hundreds of volts, so the synchrotron losses are easily maintained. However, during a normal discharge with 1 - 2volts per turn, 66MeV represents the maximum energy.

In JET, with R=300cm and E=100MeV, we obtain 0.48x10^8eV/sec, requiring 3volts/turn. Note that with the E^4 scaling, an energy of 400MeV would require 3x4^4volts=768volts/turn during a disruption, indicating a limit to runaway energy.

APPENDIX D - *Fraction of electron energy going into radiation*

D.I. From [The handbook of physics by Vavorsky and Detlaf, p. 861], for a high energy electron slowing down in a material with effective charge Z [the case of a high energy runaway electron striking a limiter, eg a Z=6 carbon tile], the ratio of radiation loss to the ionization loss of an electron of energy E_c (in MeV) is :

$$R \equiv \frac{(\partial E_c/\partial x)\ \mathrm{rad}}{(\partial E_0/\partial x)\ \mathrm{ion}} = \frac{E_e Z}{800}$$

For Z=6 and E_e=66MeV, this ratio is 0.5, so that 0.5/(1+0.5) = 1/3 of the energy is radiated at 66MeV. This fraction decreases as the electron slows down. The fraction of the total energy radiated F is given by

$$F \equiv \frac{\int_0^{E_{max}} \frac{R}{1+R}\, \partial E}{\int_0^{E_{max}} \partial E} = 1 - \left[\frac{\ln\left(1 + \frac{E_{max} Z}{800}\right)}{\left(\frac{E_{max} Z}{800}\right)}\right]$$

In TCV, for Z=6 and E_{max}^{MEV}=66MeV, as an example we obtain F=0.19.

For JET, with Z=6 and E_{max}^{MeV}=100, we obtain F=0.25.

To obtain the total hard X-ray energy, we multiply the runaway energy by this ratio F.

Stopping range and direction

The range of multi-MeV electron for a 1/e energy loss in carbon is ≈ 10-20cm. This means that electrons may be scattered inside the carbon in direction, and that they may even emerge on the other side of the limiter. This may result in changes in radiation direction, and deviated electrons may even strike a nearby vessel wall, their orbits still being dominated by the toroidal and poloidal magnetic field.

However, this should effect the most intense, most dangerous radiation level very little. The highest energy, most directed photons can only be emitted by high energy electrons, which have their initial velocity direction.

APPENDIX E - *Spectrum of hard X-rays resulting from slowing down of runaway electrons in the limiter*

The HXR spectrum on JET was not directly measured. The maximum electron energy was inferred from photoneutron energy. Mike Jarvis suggested that the spectrum of photons should go like 1/E, i.e. $\partial N/\partial E = K/E$. the measured spectrum of forward HXR in ORMAK up to 10MeV, has approximately this form, with the spectrum roughly flat to 2MeV, or slightly peaked at 2MeV and falling to zero at 10MeV [Kneopfel et al., Lausanne 1975]. The spectrum is discussed in [Principles of modern physics, Leighton, P. 414 - 415]. The thin target momentum distribution of X-ray quanta from a high energy electron accelerator is :

$$\frac{\partial N}{\partial \rho} = \frac{A}{\rho}, \; \rho_{min} < \rho < \rho_{max}$$

ρ is the momentum of the photon, A a constant. Since $\rho \sim \sqrt{E}$, we equally obtain $\partial N/\partial E = A/E$, as discussed above. For a thick target, the relationship looks more linear, $\partial E/\partial \nu \sim A - B\nu$ up to ν_{max}. Also, the slowing down fraction F should be folded in, etc.

However, we should have a fairly good approximation to the spectrum by using $\partial N/\partial E = K/E$, with the integral from $E_{max}/50$ to E_{max} normalized to give a fraction F of the incident electron energy.

APPENDIX F - *Spatial distribution of hard X-ray radiation*

F.1) <u>Pattern</u>

From [Classical electrodynamics by Jackson, p. 473], the radiation pattern is given for a highly relativistic electron accelerated in its direction of motion. At very high energy, the angular distribution is tipped forward more and more and increases in magnitude.

The angle θ_{max} for which the intensity is a maximum is given by :

$$\theta_{max} \rightarrow \frac{1}{2\gamma}, \text{ where } \gamma = \frac{E_{MeV}}{.511\text{MeV}}, \quad \gamma \gg 1$$

The total radiation pattern for the complete slowing down of an electron will be very complicated. since the highest energy photons can only be generated by an equally high energy electron, we make the approximation that the angular distribution of the photons follows the above θ_{max}.

From Jackson p. 474, the angular distribution of the radiation versus θ is :

$$\frac{\partial P}{\partial A} = K \frac{\gamma^2\theta^2}{(1+\gamma^2\theta^2)^5} \quad \text{, K a constant to be determined}$$

This function is 0 at θ=0, maximum at γθ=0.5, and has fallen by an order of magnitude at γθ=1.5. Using the energy distribution from Appendix E, we have

$$\frac{\partial^2 P}{\partial E\, \partial A} = \left[\frac{\frac{\text{Photons}}{\text{MeV}}}{E}\right] K \left[\frac{\gamma^2\theta^2}{(1+\gamma^2\theta^2)^5}\right] \text{, where } E_{MeV} = 0.511\gamma$$

The element of area is $\partial A=2\pi L^2\theta\partial\theta$, where L is the distance from the limiter.

Normalizing so that the integral over the angular part gives 1, we obtain $K=24\gamma^2/2\pi L^2$.

To find the integrated radiation due to all energies at a given angle θ, we integrate from $\gamma_{min}=E_{min}/.511$ to $\gamma_{max}=E_{max}/.511$, giving

$$\text{Photon flux } \left(\frac{\text{Photons}}{m^2}\right) = \int_{\gamma_{min}}^{\gamma_{max}} \left(\frac{\text{Photons}}{\text{MeV}}\right) \left(\frac{12}{\pi L^2}\right) \gamma\partial\gamma \left[\frac{\gamma^2\theta^2}{(1+\gamma^2\theta^2)^5}\right]$$

F.2) Spatial averaging due to magnetic field angle change during disruption

The above distribution [especially after multiplying by the biological dose factor (Appendix G) which increases with energy] gives a very peaked flux around $\theta=0.5/\gamma_{max}$. The average dose during a disruption is strongly reduced because the magnetic field angle of the runaway electrons is not the same for all electrons in a discharge. In terms of the safety factor q which is function of minor radius r, the average angle of the field line on a surface with major radius R and elongation K is

$$\theta = \tan^{-1}\left(\frac{B_\rho}{B_{t0}}\right) \approx \frac{B_\rho}{B_{t0}} \approx \frac{r}{qR_0}\sqrt{\frac{1+K^2}{2}}$$

For example, for runaways inside the q=1 surface in a circular tokamak, θ varies from 0 to r/R, so at a distance of L meters, the runaway strike zone spatially averages the dose from a disruption over the distance $L\theta=Lr/R$, which is, for example (8)(.12)(.87)=0.83 meter. The sweep is upwards or downwards on the inside or outside limiter. In TCV, the maximum angle is for largest r and K, and smallest q. The angle becomes

$$\theta = \tan^{-1}(B_\rho/B_T) = \tan^{-1}\left[\frac{a(a+R_0)}{qR_0^2}\sqrt{\frac{1+K^2}{2}}\right]$$

For a=.24, R_0=0.87, K=3, and q=2, we get $\tan^{-1}(.39)=21^0$ and $8m(\tan\theta)$=3.14meters.

The precise averaging would be very complicated. However, it is certain that the runaway strike zone will be vertically swept a certain distance.

If the annulus ring has a width Δ on the top and Δ on the bottom, and it has an outer width W, the area is approximately $\pi W\Delta$. The sweeping distributes the radiation over the area HW. The intensity is therefore reduced by the factor $S = HW/\pi W\Delta = H/\pi\Delta$. Since the aspect ratio of TCA and

TCV are similar, we take for both the smallest value of H=0.83meters at L=8meters, so the reduction factor is

$$s=(0.26 \text{ meters} / \Delta_{meters}).$$

F.3) <u>Spatial averaging due to perpendicular velocity of runaway electrons in a magnetic field</u>

Up to now, it has been assumed that the runaway electron has all parallel energy, which is only approximately true. For a relativistic (and non-relativistic) electron, even with relativistic increased mass, the ratio of perpendicular to parallel velocity is $\sqrt{E_\perp/E_{||}}$. The corresponding angle due to perp. gyration is

$$\theta_g = \sqrt{\frac{E_\perp}{E_{||}}} = \frac{\sqrt{E_\perp}}{\sqrt{.511\gamma}} \quad ,$$

so

$$\gamma\theta_g = \sqrt{\gamma}\sqrt{\frac{E_\perp}{.511\text{MeV}}}$$

This effect will increase the beam width if

$$\gamma\theta_g \geq \gamma\theta_{max\ inter} \approx 0.5$$

or equivalently if

$$E_\perp > (0.13\ \text{MeV})/\gamma$$

The value of $E_\perp$ is hard to judge. A discussion of "Perpendicular Bremsstrahlung emission of suprathermal electrons" is given by [S. Von Goeler et al., Rev. Sci. Instr. <u>57</u>(8), August 1986, p. 2130], where measurements indicate perpendicular temperatures in the range of 20-50keV during lower hybrid or ECH or runaway experiments. [These measurements may also be related to observations of perpendicular HXR in JET.] In TCV, before a disruption, T_e may be 1keV, so there will be electrons with higher energy, which will all cool during a disruption.

<u>IF</u> we assume $T_\perp \approx E_\perp$ = 20keV = .020MeV, then $\gamma\theta_g = \sqrt{\gamma}(0.2)$.

For example, if γ=100, then $\gamma\theta_g$=2, which increases the beam width by a factor of 4 over $\gamma\theta=0.5$.

APPENDIX G - *Biological dose calculations*

We need to calculate the biological dose outside the shield wall at L=8m.

From [Physics Vade Mecum, Anderson, AIP], we obtain (p. 219) the following yearly allowed doses :

Combined whole-body occupational exposure	5 REM
Public, or occasionally exposed individuals	0.5 REM

The conversion from photon flux to REM=R is given from data in Swiss Army Handbooks and References on Radiological Protection.

The dose from a point source is $D=\Gamma(A/r^2)$, with Γ in $(R/h)(m^2/c)$ where $\Gamma=19.5E_\gamma(\eta/\rho)$, E_γ in MeV.

For A=1 Curie at r=1m for E=1MeV gamma rays in water, $\eta/\rho=.0310$, giving $\Gamma=.6045$ and D=0.6045R/h=0.6045REM/hour.

For a dose lasting 1 sec, the dose is 1.68×10^{-4}REM. The flux of photons from a 1 Curie source (isotropic) at 1m distance is $(3.7\times10^{10}\text{photons/sec})/4\pi(1\,\text{m})^2 = (2.9\times10^{9}\text{photons/sec})/\text{m}^2$.

Therefore, the dose is given by (at 1MeV)

$$\text{DOSE(REM)} = \text{Flux}\,(5.8\times10^{-14}\text{REM/(Photon/sec)}).$$

Therefore, a dose of 1 REM is obtained by a flux of 1.7×10^{13}(photons at 1MeV)/m^2, [or 1.7×10^{9}/cm^2]. This compares with the statement [Vade Mecum p. 45]. "Fluxes (per cm^2) to liberate 1 rad in Carbon 1.9×10^{9}photons of 1MeV (these fluxes are correct to within a factor of two for all materials).

At any energy, the dose becomes

$$DOSE(REM) = [1REM]\left[\frac{\text{Photon flux}}{1.7\text{x}10^{13}\frac{\text{photons}}{m^2}}\right]\left[\frac{\eta/\rho}{0.031}\right]\left[\frac{\text{Energy}_{MEV}\text{ of }\gamma\text{ ray}}{1MeV}\right]$$

Values for η/ρ are :

Energy (MeV)	η/ρ
0.1	.025
1	.031
2	.026
4	.021
8	.016
10	.016

Note that above 4MeV, η/ρ becomes constant and the dose at a given flux increases linearly with energy.

Taking the formula from Appendix F, the dose in REM at a given angle θ at distance L obtained by integrating over all photon energies and reducing the dose by the magnetic field sweeping s, we obtain

$$DOSE(REM) = \left[\left(\frac{\Delta_{meter}}{0.26m}\right)\left(\frac{\text{Photon}}{\text{MeV}}\right)\left(\frac{12}{\pi L^2}\right)\left(\frac{\eta/\rho}{0.031}\right)(.511MeV)\left(\frac{1}{1.7\text{x}10^{13}\frac{\text{Photons}}{m^2}}\right)\right]$$

$$\text{x}\int_{\gamma_{min}}^{\gamma_{max}}\gamma^2\partial\gamma\left[\frac{\gamma^2\theta^2}{[1+\gamma^2\theta^2]^5}\right]$$

Let $x\equiv\gamma\theta$

Then, DOSE becomes

$$DOSE(REM) = \left[\left(\frac{\Delta_{meter}}{0.26m}\right)\left(\frac{\text{Photon}}{\text{MeV}}\right)\left(\frac{12}{\pi L^2}\right)\left(\frac{\eta/\rho}{0.031}\right)(.511MeV)\left(\frac{1}{1.7\text{x}10^{13}\frac{\text{Photons}}{m^2}}\right)\right]$$

$$x \; \frac{1}{\theta^3} \int_{\gamma_{min}\theta}^{\gamma_{max}\theta} \frac{x^4 dx}{(1+x^2)^5}$$

For L=8 meters

$$\text{DOSE(REM)} = [(\frac{\Delta_{meters}}{0.026m})(\frac{\eta/\rho}{0.031})(\frac{\text{Photons}}{\text{MeV}})(1.8\text{x}10^{-15})](\frac{1}{\theta^3})\, F(x) \,\Big|_{\gamma_{min}\theta}^{\gamma_{max}\theta}$$

$$\text{where } F(x) = \int \frac{x^4 dx}{(1+x^2)^5}$$

$$= -.146 \frac{x}{(1+x^2)^3} + .076 \frac{x}{(1+x^2)^2} + .034 \frac{x}{2(1+x^2)} + 0.34 \frac{\tan^{-1} x}{2} + \frac{1}{8} \frac{x}{(1+x^2)^4}$$

Since $F(x) \rightarrow 0$ as $x \rightarrow 0$, we can usually ignore the lower limit in the integration.

The function F was calculated for x = 0.0, 0.4, 0.5, 0.6, 1, 2, 10. For all values of

$x \geq 0.5$,	$F = 2.8 \times 10^{-2}$
$x = 0.4$,	$F = 4.3 \times 10^{-3}$
$x = 0.$	$F = 0.$

For larger values of x=10, the lower limit is still unimportant.

The function $(1/x^3)F(x)$ has the following values :

x	$(1/x^3)F(x)$
0.4	.067
0.5	.224
0.6	.130, where $x \equiv \gamma\theta$

$$\text{We rewrite } \frac{1}{\theta^3} F(x) \,\Big|_{\gamma_{min}\theta}^{\gamma_{max}\theta} \text{ as } \frac{\gamma^3_{max}}{\gamma^3_{max}\theta^3} F(\gamma_{max}\theta)$$

The region of maximum dose occurs in

$0.4 \le \gamma_{max}\theta \le 0.6$
or $0.4/\gamma_{max} \le \theta \le 0.6/\gamma_{max}$

The average value in this region is about 3/4 of the peak value, so $(1/(\gamma_{max}{}^3\theta^3))F(\gamma_{max}\theta) \approx 0.17$ so $((\gamma_{max}{}^3)/(\gamma_{max}\theta)^3)F(\gamma_{max}\theta) \approx 0.17\gamma_{max}{}^3$. The thickness $\cdot\Delta_{meter}$ of the ring annulus is (L=8m)

$$\Delta_{meter} = L\tan(\Delta\theta) \approx L(\frac{0.2}{\gamma_{max}}) \approx \frac{1.6\text{ meters}}{\gamma_{max}}$$

The total annulus width W is

$$W = L\tan(2\theta) \approx L(\frac{1.2}{\gamma_{max}}) = \frac{9.6\text{meters}}{\gamma_{max}}$$

The dose becomes

$$\text{DOSE(REM)} = (\frac{6.2}{\gamma_{max}})(\frac{\eta/\rho}{.031})(\frac{\text{Photons}}{\text{MeV}})(1.8 \times 10^{-15})\gamma_{max}^3(0.17)$$

$$\text{DOSE(REM)} = (\frac{\text{Photons}}{\text{MeV}})(\frac{\eta/\rho}{.031})(1.9 \times 10^{-15})\gamma_{max}^2$$

This dose is delivered into a region at L=8 meters with width $W=9.6\text{meters}/\gamma_{max}$ and height of (0.83 meters plus width W).

APPENDIX H - *Shielding attenuation and compton scattering*

[Reference : AIP Vade Mecum + Swiss Army Handbooks] For a concrete shield wall of "Barybeton", Barytes concrete, $\rho=3.5\text{gm/cm}^3$, the following table gives, as a function of photon energy, the thickness required to reduce radiation by a factor of 10, and the linear attenuation coefficient A for shield thickness T, such that $\text{Flux}=\text{Flux}_0 \exp(-AT)$.

Photon Energy (MeV)	Thickness for factor 10 decrease (cm)	Attenuation Coef. A (cm^{-1})
1.	11.	.213
3.	18.	.127
6.	21.	.110
10.	23.	.100

Shielding depends on the product of shield density and thickness, so that thickness could be nearly halved by using Iron-Portland Cement, with ρ=6gm/cm^3, and a composition by weight of (% weight) : hydrogen 1.0; oxygen 52.9; silicon 33.7; aluminum 3.4; iron 1.4; calcium 4.4; magnesium 0.2; carbon 0.1; sodium 1.6; potassium 1.3.

The stopping cross section of hard X-rays in materials is determined by three effects [Ref. Handbook of Physics, Yavorsky, p. 866]. The energy range and dominant effects are

Energy	Effect
0 - 1 MeV	Photoeffect
1 - 12 MeV	Compton scattering
> 12 MeV	Pair production

The cross section has a broad minimum in the range of 3 to 20MeV, and has the same value at 2 and 50MeV, about 1.6 times the minimum.

An important point about Compton scattering is that it can cause nearly isotropic scattering with only 1MeV loss per scatter. Thus the shielding wall becomes a new source of 1 -12 MeV radiation. Shield wall heights and adjacent office buildings should be studied accordingly. However, because the scattered radiation is relatively isotropic, the intensity is reduced by the area ratio $(4\pi 16^2)/(.5)$=6000. Since the scattered radiation is a fraction of incident radiation, the dose is reduced by more than 20000.

APPENDIX J - *Model of why a constant fraction of magnetic energy becomes runaway energy after a disruption, and discussion of Dreicer limit*

For a high speed electron slowing down a slower electron, the acceleration is

$$\frac{\partial V_e}{\partial t} = -V_e \; [n_e \ln \Lambda_{ee} (7.7 \times 10^{-6}) E_{eV}^{-3/2}]$$

units cm, sec, eV, test particle velocity V_e in cm/sec and energy E in eV. For a runaway to occur, this deceleration must be equalled or exceeded by the applied resistive electric field.

$eE = m(\partial V_e/\partial t)$
$(\partial V/\partial t)|_{cm/sec} = 1.76 \times 10^{15} \varepsilon_{volts/cm}$

combining these, $n_e(cm^{-3})/E_{run} = 2.6 \times 10^{11}\ \varepsilon_{volts/cm}$

The resistive electric field is : $\varepsilon = \eta J$

For $\eta = 2 \times 10^{-2}\ z \ln \Lambda\ T_{eV}^{-3/2}$ (ohm-cm), T_e in eV, we obtain :

$$\frac{E_{run}}{T_e} = \frac{\sqrt{T_e}\ (n_e/J)}{7.8 \times 10^{10}\ Z_{eff}}$$

Note : During plasma startup, J is determined by plasma resistivity and starting electric field. At a disruption, both n_e and J are established, and are proportional to the Murakami number, where $n_e \sim J$. We see that all tokamaks operating at similar Murakami numbers should have similar runaway generation at disruption.

From the Maxwell-Boltzman distribution, $n \sim n_0 \exp(-(E/T))$. Therefore, the fraction of runaways depends on the Murakami number.

Let us call this constant $K \sim \exp(-(E_{run}/T_e)$.

Let the ratio of runaway electron density to initial electron density be $K = n_{run}/n_{e0}$. The total number of runaways is (Volume)Kn_{e0}. The runaway energy is $VKn_{e0}E_{run}$. The ratio of runaway energy to stored magnetic energy (≈2% in JET) is

$$R \equiv \frac{W_{run}}{W_{mag}} = \frac{VKn_{eo}E_{run}}{LI^2/2}$$

But, since the initial flux LI determines the runaway energy as E_{run}=(e(LI)c)/2πR, then R becomes (using J≡I/(Volume/2πR))

$$R = (2ec)(K)\left(\frac{n_{eo}}{J}\right)$$

Since K is a function of (n_{eo}/J), then the value of R depends on n_{eo}/J.

We assume that during a disruption the current stays unchanged, but T_e drops rapidly due to energy loss and impurities.

For the volt seconds to dissipate in milliseconds, T_e must drop to about 10eV, where there is also a radiation barrier to ionize oxygen and carbon. In JET, if we take T_e=10eV, Z_{eff}=2.5 (partially ionized impurities), J=3.5x10^{6}A/(π)(125)^2(1.4) = 50A/cm^2, n_e=4x10^{13}cm^{-3}, we obtain E_{run}/T_e=12.8, E_{run}=128eV, and exp(-12.8)=2.6x10^{-6}. With K=2.6x10^{-6}, we obtain R=0.02. (We note that K and therefore R vary strongly with n_{eo}/J.)

In JET, we have used (n_{eo}/j)=8x10^{11}/Amp·cm).

In TCV, take J=(1.2x10^6)/(π)(.24)^2(2.5)(10^4)=265A/cm^2. If n_e=5x10^{13}, then n_{eo}/J=(2x10^{11})/(Amp·cm). If n_e=2x10^{14}, then n_{eo}/J=(8x10^{11})/Amp·cm. Thus a disruption at 5x10^{13} would create many more runaways.

APPENDIX K - *Unified formula for biological dose (REM) versus plasma current and magnetic flux*

The expression for E_{max} and γ_{max} is (Appendix AA)

$$.511\,\gamma_{max} = E_{max}(\text{MeV}) = \left(\frac{1}{10^6}\right)\left(\frac{3}{4}\right)\left(\frac{LI(c)}{2\pi R_{MAJ}}\right) = \frac{70\emptyset}{R_{MAJ}}$$

The total runaway electron energy is 2% (LI^2/2). We assume this energy is present in a mono-energetic electron beam with energy E_{max}. If there is an energy spread, then more electrons would be present at lower energies.

The photon energy is 2% $(LI^2/2)F$, where (Appendix D)

$$F = 1 - \frac{\ln(1+\frac{E_{max}Z}{800})}{(\frac{E_{max}Z}{800})}$$

For carbon, z=6, $E_{max}Z/800=.0038\gamma_{max}$,
For $.0038\gamma_{max} \ll 1$ ($\Rightarrow \gamma_{max} \ll 263$) $F \approx .0019\gamma_{max}$.

The spectrum of hard X-rays (Appendix E) is $\partial N/\partial E = K/E$, where K is now determined by

$$W_{photon}(MeV) = \int_0^{E_{max}} E(\frac{K}{E})\partial E,$$

giving

$$\frac{2\%(LI^2/2)\,(.0019\gamma_{max})}{(10^6)\,(1.6x10^{-19})} = K{=}0.511\gamma_{max})$$

$$(\frac{Photons}{MeV}) \equiv K = 2.3 \times 10^8\, LI^2$$

From Appendix G, having used the spatial distribution from Appendix F, the dose is :

$$DOSE\ (REM) = (\frac{Photons}{MeV})\,(\frac{\eta/\rho}{.031})\,(1.9 \times 10^{-15})\,\gamma_{max}^2$$

Using the above expressions for (Photons/MeV) and γ_{max}, we obtain, using $\varnothing \equiv LI$ and expressing I in MAmps as I_{MA}

$$DOSE(REM) = (2.2 \times 10^3 REMS)\, I_{MAMP}\, \phi^3_{volt\ sec} \left(\frac{\eta/\rho}{.031}\right) \frac{1}{R^2_{MAJ,\ meters}}$$

into an area of 0.83 meters high by $9.6m\gamma_{max}$ wide at L=8 meters.