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Comments on

"Relation of fluid drift to oscillation center drift"

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ABSTRACT

The fluid theory and single particle theory of the ponderomotive force can yield different results. By considering the two possible interpretations of a recent paper,[1] we conclude either that this apparent paradox has been examined incorrectly or that a completely different problem has been addressed.

That the fluid theory and single particle theory of the ponderomotive force in the presence of a magnetostatic field yield different results has been noticed in the past by several authors (see e.g. Refs. 2,3). Whereas in the single particle approach the ponderomotive force can be expressed as the gradient of the ponderomotive potential[4], it has been shown[2,5,6] that in the fluid approach this is, in general, not possible. In fact, in the fluid description of the ponderomotive force perpendicular to the magnetostatic field, an extra term arises due to the interaction of the rf induced magnetization current with the magnetostatic field.

In the discussion of this apparent paradox, it is important to define unambiguously the various concepts needed in order to avoid confusion.

By single particle theory we mean the theory that describes the motion of a single particle or of a collection of non-interacting particles. It is essential to distinguish the concept of a single particle from that of a "fluid particle", the latter referring to a fluid element comprised of a collection of interacting particles. The description of the motion of a fluid element is provided by a fluid theory. Such a macroscopic description treats ensemble averaged quantities, for example, the density, velocity, and temperature of the fluid element. Of course, the procedure of ensemble averaging is meaningful only for a system of interacting particles that exhibit collective behavior (either the Debye length or the particle mean free path being much smaller than characteristic scale lengths of the system).

It is well known that the behavior of a fluid element may mathematically be described using either Langrangian or Eulerian variables. From a physical point of view, it has long been established that these two descriptions are equivalent and can be related by a transformation between the two reference frames. It must be stressed that a fluid element in the Lagrangian description should not be confused with a single particle, even though under some conditions these physically different quantities obey equations of motion having the same form.

Finally, the term "oscillation center theory" should be used with caution. The oscillation center approximation is a mathematical technique which has been used in conjunction with single particle[4], fluid[2,5,6] and kinetic[7] theories. Thus, the use of the term "oscillation center theory" by itself does not provide a priori knowledge of the type of physical quantity to be considered.

Reference 1 addresses the problem of the different results for the effect of the ponderomotive force in the presence of a magnetostatic field obtained from different theories. Both fluid theory and an "oscillation center theory" have been considered. Unfortunately, the latter term is ambiguous for the reasons discussed above. In the context of Ref. 1 this term can be associated either with single particle theory or with fluid theory. We shall show that if this term is interpreted as pertaining to a single particle, Ref. 1 contains mathematical and physical flaws. Conversely, if this term refers to the analysis of a fluid element in the Lagrangian variables, the problem of the apparent paradox has not been addressed.

In the first interpretation, a reader assumes that Eqs. (3)-(6) of Ref. 1 describe the motion of a single particle. This assumption is based on the text contained between these equations, the nomenclature defined in the first paragraph of Ref. 1, and the reference to the classical single particle theory of Motz and Watson.[4] Equation (7) attempts to relate the time average of a fluid velocity $\langle\vec{v}_{2D}(x)\rangle$ to the time average of the velocity $\langle\dot{y}_f\rangle$ of a single particle. Mathematically, the only average performed in Eq. (7) is the time average: there is no ensemble average despite the possible allusion to one in the sentence directly preceding this equation. As written, Eq. (7) states that the time average of the rapidly oscillating quantity $\langle\dot{y}_f\rangle$ is non-zero. However, this is in contradiction with the central assumption of the oscillation center approximation that fast and slow time scales can be separated.[8] The non-zero time average of $\dot{y}_f$ was obtained by erroneously identifying y_f as a field quantity, allowing the Taylor expansion around x.

The first sentence of the last paragraph of Ref. 1 implies, in this interpretation, that the fluid result can be obtained from the oscillation center approximation of single particle theory. That this is not possible can be shown most convincingly by comparing the fluid result with the solution of the exact equation of motion for a single particle.[9]

A reader of Ref. 1 may adopt the second interpretation mentioned above by assuming that the term "particle" refers to a "fluid particle" or fluid element.[10] The phrase "a cold particle" appearing in Ref. 1 may then have a meaning, even though it clearly does not if referring to a single particle. Equations (3) - (6), in this interpretation of Ref. 1, describe not the motion of a single particle, but that of a fluid element in the Lagrangian variables. (Note that this important distinction is not made in Ref. 10.) Using this interpretation, therefore, the only conclusion to be drawn from Ref. 1 is simply that of the equivalence of the Lagrangian and Eulerian descriptions of the ponderomotive force exerted on the fluid element. Since in both of these descriptions a fluid theory is used, it is obvious that the results obtained must be equivalent and related by a transformation between reference frames. Of course, no ensemble average is required to compare the results since in both descriptions only ensemble averaged quantities are treated. However, it is then clear that the problem of the apparent paradox has not been addressed.

As has recently been shown,[8,11] this problem can only be addressed by considering an appropriate ensemble average over the oscillation centers of a collection of interacting single particles. By this means it is possible to resolve the apparent paradox that the fluid theory and single particle theory of the ponderomotive force can yield different results. It should be stressed that the ponderomotive forces calculated from these two theories are fundamentally different since they act on fundamentally different physical objects.

Acknowledgment

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