December 1985 INT 123/85

# TCV: FEEDBACK STABILISATION OF THE VERTICAL MOTION

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## 1. THE ELECTRO-MECHANICAL MODEL

The strongly elongated plasma is simulated by two perfectly conducting rings which coincide with the centre of curvature of the top and the bottom of the racetrack. A further simplification is the assumption of a high aspect ration $R \gg a$, R being the major radius and a the radius of curvature of the racetrack. If $2h_p$ is the distance between the plasma rings, and E the elongation, we have

$$h_p = (E-1)a \tag{1}$$

The plasma is surrounded by a rectangular vacuum vessel at a distance $\surd$. The half-height $h_v$ and the width w of the vessel is thus

$$h_v = Ea+\surd \quad \text{and} \quad w = 2(a+\surd) \tag{2}$$

The corners of the vessel are cut out in order to place the active coils as near as possible to the plasma. The wires are separated vertically by $h_v$ and horizontally by w. The 4 coils are connected in series, with the current $I_1$ flowing in the direction shown in Fig. 1. The same figure shows the assumed path of the image current flowing in the vessel wall. We suppose the current $I_c$ to be split in two paths separated horizontally by an appropriate distance. The two top wires are connected in parallel as well as the bottom wires. The current $I_c$ is forced to flow in opposite direction (a so called m=1 mode) by means of vertical connectors having a negligible inductance.

### 1.1 The plasma model

In order to write the equations of the system we have to describe first the elements of the circuit. The upper plasma ring is subject to a downward force caused by the current of the lower ring. It amounts to

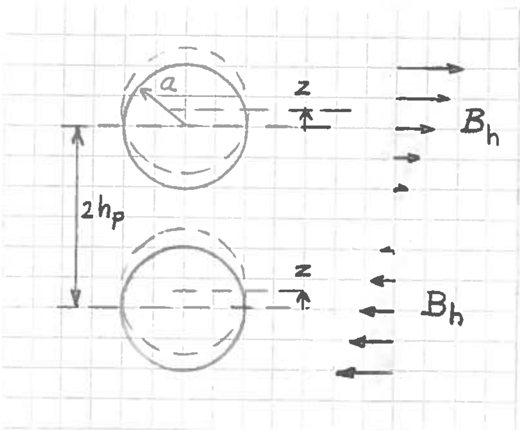
$$-\frac{\mu_0 I_p^2 R}{8h_p}$$

The ring has to be kept in equilibrium by the horizontal component of the field created by the shaping coils. We assume that the horizontal field $B_h$ varies linearly with the vertical coordinate z, going from 0 at $z = h_p - a$ to $2B_{ho}$ at $z = h_p + a$. The force due to this field must be

$$+\frac{\mu_0 I_p^2 R}{8h_p}\left(\frac{z}{a} + 1\right)$$

Now we assume that the distance of the two rings is kept constant, i.e. the plasma moves like a rigid body. The total "positive" vertical force on the whole plasma will be twice the sum of the above expressions:

$$F_V = \frac{\mu_0 I_p^2 R}{4h_p} \cdot \frac{z}{a} \qquad (3)$$




We are looking for the variation of this force, i.e. the coefficient of elasticity

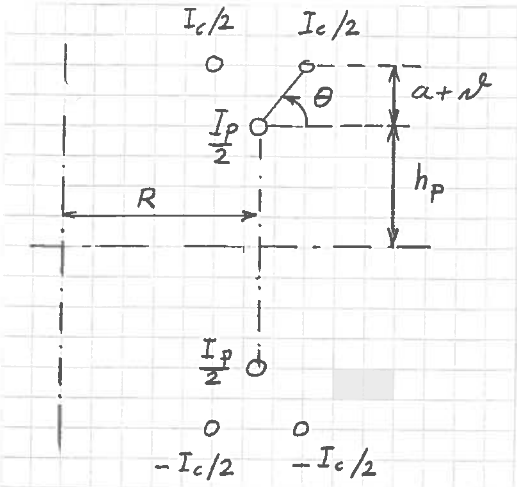
$$K = \frac{dF_V}{dz} = \frac{\mu_0 I_p^2 R}{4h_p a} \qquad (4)$$

The positive value of K gives rise to the vertical instability.

A correction of this formula has to be introduced in a heuristic way. K should not diverge when $h_p$ vanishes. As soon as the elongation gets smaller than 2:1 the lobes of the doublet start to interpenetrate and the force must go to zero when the elongation reaches 1:1. The following expression is more appropriate

$$K = \frac{\mu_0 I_p^2 R}{4a^2} \cdot \frac{1}{h_p/a + a/h_p} = \frac{\mu_0 I_p^2 R}{4a^2} \cdot \frac{(E-1)}{(E-1)^2+1} \qquad (5)$$

1.2 The plasma-vessel model




For the calculation of the inductance of the vessel we assume that the vessel is decomposed in just two plates, the top and the bottom plate. It is easy to show that a thin plate of width w has the same inductance as a round wire or radius w/4. Thus the bifilar conductor simulating the vessel has an inductance given by $L_c = 2\mu_0 R \ell n(8h_v/w)$ or

$$L_c = 2\mu_0 R \; \ell n \frac{4(E+\surd/a)}{1 + \surd/a} = 5.4 \cdot 10^{-6} \qquad (6)$$

R = 0.815  E = 4  a = 0.18  $\surd$ = 0.045

The force acting on the whole plasma, due to the vessel current, is given by

$$F_{cp} = I_p I_c N_{cp} \qquad (7)$$

where $N_{cp} = dM_{cp}/dz$ means the derivative of the mutual inductance between the vessel and the plasma. Instead of calculating $M_{cp}$ it is easier to calculate directly the forces resulting from the currents. As the current in the vessel is not concentrated in the middle of the plates, we assume the current to be spread out in two paths in the top plate, lying at an angle θ and 180 °-θ (see figure) with respect to the upper plasma filament. In the bottom plate the situation is symmetrical. So we find

$$N_{cp} = \frac{\mu_0 R}{a} \left[ \frac{(\sin\theta)^2}{1 + \surd/a} + \frac{1}{2E - 1 + \surd/a} \right] \qquad (8)$$

$N_{cp} = 3.7 \text{x} 10^{-6}$ for $\sin\theta = 0.8$.

$\sin\theta$ is a parameter suitable to match our model with the result of the numerical code (in the calculation of $L_c$ we ignored $\theta$ because the inductance is not very sensitive to the current distribution in the plate).

In order to express the circuit equations we finally need the equivalent resistance $R_c$ of the vessel. The resistance depends on the current distribution, but no attempt is made here to calculate $R_c$. The circuit equations are

$$L_c \frac{dI_c}{dt} + \frac{d}{dt}\left(M_{cp}I_p\right) + R_cI_c = 0 \qquad (9)$$

$$\frac{d}{dt}\left(M_{cp}I_c\right) + L_p \frac{dI_p}{dt} = 0 \qquad (10)$$

$$N_{cp}I_pI_c + Kz - M\frac{d^2z}{dt^2} = 0 \qquad (11)$$

The term $d/dt(M_{cp}I_p)$ equals $N_{cp}I_p dz/dt$ because $M_{cp} = 0$ at the equilibrium position of the plasma. The term $d/dt(M_{cp}I_c)$ equals $N_{cp}I_c dz/dt$ where $I_c$ is a first order term. So $I_c dz/dt$ is of second order and can be neglected. Consequently eq. (10) states that $I_p$ = const. Eq. (11) is nothing else than the Newton equation. Switching to the Laplace transform, by setting $s = d/dt$ the two remaining equations are

$$(R_c+sL_c)I_c + sN_{cp}I_pz = 0 \qquad (12)$$

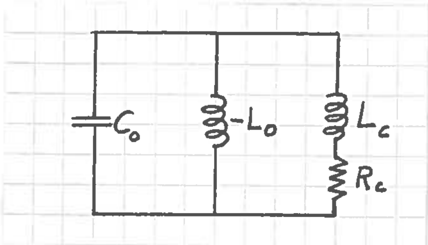
$$N_{cp}I_pI_c + (K-s^2M)z = 0 \qquad (13)$$

M stands for the total mass of the plasma. The meaning of $I_c$ and z are now the Laplace transform of the current and the displacement. For the true variables with time we adopt the notation $I_c(t)$ and $z(t)$. The solutions of the above equations are obtained by putting the determinant equal to zero. The eigenfunctions are $e^{s_it}$ and $s_i$ are the poles. The equation for the poles turns out to be

$$-\frac{M}{K}\left(s^3\frac{L_C}{R_C}+s^2\right)-s\frac{L_C}{R_C}\left[\frac{(N_{cp}I_p)^2}{L_CK}-1\right]+1=0 \qquad (14)$$

We observe that this equation is identical with the equation of the following electrical circuit

$$-L_0C_0\left(s^3\frac{L_C}{R_C}+s^2\right)-s\frac{L_C}{R_C}\left(\frac{L_0}{L_C}-1\right)+1=0 \qquad (15)$$




By indentification we find the meaning of $L_0$ and $C_0$:

$$L_0=\frac{(N_{cp}I_p)^2}{K} \qquad C_0=\frac{M}{(N_{cp}I_p)^2} \qquad (16)$$

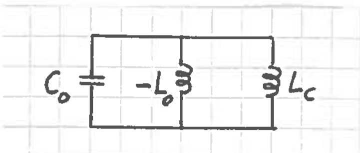
We call $-L_0$ the dynamic inductance and $C_0$ the dynamic capacity of the plasma. $C_0$ represents the inertial term. The negative value of $-L_0$ explains the instability of the system. As $R_C \ll \omega L_C$ for the ringing circuit the solution of the fast mode can be found by neglecting the $s^2$ term and the constant:

$$-L_0C_0s^2-\left(\frac{L_0}{L_C}-1\right)=0$$

thus

$$s_1{}^2=\frac{-(L_0/L_C-1)}{L_0C_0}\equiv-\omega^2 \qquad (17)$$

The frequency $\omega$ is in the range of the very high Alfvén frequency. Stability in that short time scale is achieved if $\omega$ is real:




$$\frac{L_0}{L_C}-1>0 \qquad (18)$$

It means that the resulting inductance of $L_0$ and $L_C$ connected in parallel must be positive.

On the other hand, for a time scale of the order of the vessel time constant

$$\tau_C = \frac{L_C}{R_C} \tag{19}$$

the second solution $s_2$ does not contribute to the terms $s^2$ and $s^3$ in eq. (15). The capacity $C_0$ can be ignored. It remains

$$-s_2\tau_C \left(\frac{L_0}{L_C} -1\right) + 1 \cong 0 \qquad \text{or} \qquad \gamma = \frac{1}{\tau_C(L_0/L_C-1)} \tag{20}$$

$\gamma = s_2$ stands for the growth rate of the slow motion. This mode is stable if $\gamma$ is negative (decay rate):


$$\frac{L_0}{L_C} - 1 < 0 \tag{21}$$

It means that the resulting inductance of $L_0$ and $L_C$ connected in series must be positive.

Conditions (21) and (18) are contradictory. The fast mode cannot be stabilised by technical means, condition (18) must be fulfilled in any case. Hence the system has a residual groth rate $\gamma$. By the way, the capacity, which represents the mass of the plasma, has no effect during this slow motion. The plasma follows a moving equilibrium position.

From now on we introduce a quantity called the stability margin $\Lambda_C-1$ where $\Lambda_C = L_0/L_C$ is given by (16), (8), (6) and (5)

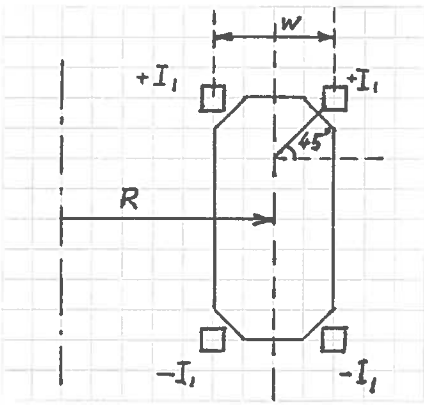
$$\Lambda_c = \frac{2\left[\frac{(\sin\theta)^2}{1+\vartheta/a} + \frac{1}{2E-1+\vartheta/a}\right]^2 \cdot \left[(E-1)^2+1\right]}{(E-1)\ln\frac{4(E+\vartheta/a)}{1+\vartheta/a}} \tag{22}$$

Fig. 2 shows $\Lambda_C$ vs the elongation E, for a plasma-wall distance of 4.5 cm and 6 cm. The stability margin found by numerical calculation is 4% at an elongation of 4:1. In order to fit the formula with the code we must admit sin $\theta$ = 0.8. It is satisfactory to find a very plausible value of $\theta$ with our crude calculation. If stability is obtained at 4.5 cm with a margin of 4%, the plasma is no more stable at 6 cm. The residual growth rate

$$\gamma = \frac{1}{\tau_C(\Lambda_C - 1)} \tag{23}$$

is dramatically increased, compared with the reciprocal vessel time constant.

1.3 Plasma-vessel-active coil system




In order to stabilise the residual instability a feedback action has to be provided by an amplifier powering an active coil system connected in a way shown in the figure. The 4 coils are connected in series but the currents in the bottom coils must be opposite to the currents in the top coils. We have to introduce the new parameters $L_1$, $R_1$, $M_{1C}$, $M_{1p}$. The inductance $L_1$ for 4 turns is given by the following approximate formula

$$L_1 \cong 8\mu_0 R \ln \frac{2R}{\sqrt{a_1 w}\sqrt{1+\frac{R^2}{h_v^2}}} = 18 \cdot 10^{-6} \tag{24}$$

With a probable cross section of 50 $cm^2$ the mean radius $a_1$ of the conductor is 4 cm, and we obtain $L_1 = 18 \cdot 10^{-6}$.

For the derivative of the mutual inuductance between coil and plasma we may use formula (8) but multiplied by 2 because there are 4 turns instead of 2. Here the coils are at $\theta = 45°$:

$$N_{1p} = \frac{2\mu_0 R}{a}\left[\frac{0.5}{1+\vartheta/a} + \frac{1}{2E-1+\vartheta/a}\right] = 6 \cdot 10^{-6} \tag{25}$$

$R_1$ and $M_{1c}$ will be considered later.

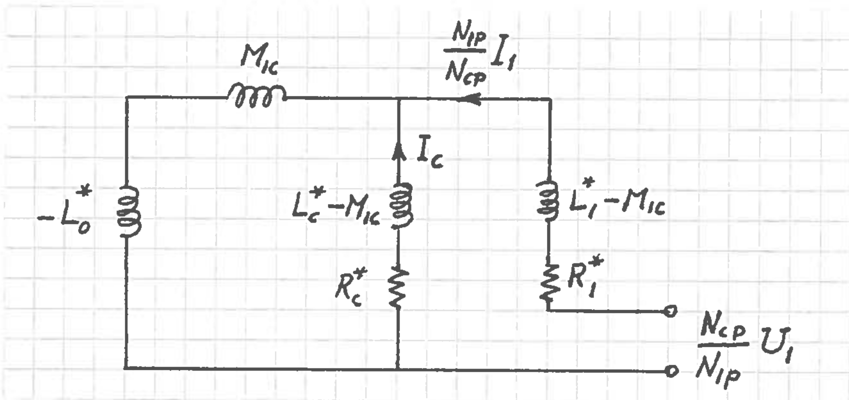
Following the same argument as before there are now 3 equations to be solved

$$(R_1+sL_1)I_1 + sM_{1c}I_c + sN_{1p}I_pz = U_1 \tag{26}$$

$$sM_{1c}I_1 + (R_c+sL_c)I_c + sN_{cp}I_pz = 0 \tag{27}$$

$$N_{1p}I_pI_1 + N_{cp}I_pI_c + Kz = 0 \tag{28}$$

We let the reader to establish the determinant of the system of equations. We again observe that these equations are identical with the circuit equations of the following circuit:




The new quantities are

$$L_1^* = \frac{N_{cp}}{N_{1p}} L_1 \qquad L_c^* = \frac{N_{1p}}{N_{cp}} L_c \qquad R_1^* = \frac{N_{cp}}{N_{1p}} R_1 \qquad R_c^* = \frac{N_{1p}}{N_{cp}} R_c$$

$$L_o^* = \frac{N_{1p}}{N_{cp}} L_o \qquad M_{1c}^* = M_{1c} \tag{29}$$

We shall not make use of the above quantities, but we have to keep in mind the transformer ratio $N_{1p}/N_{cp}$.

There exists actually a very good coupling from the vessel in direction to the active coil, i.e. nearly all the flux from the vessel current crosses the $L_1$ coil system. In other words $M_{1c} \approx L^*_c$ and the circuit as well as the equations are very much simplified. From now on we assume

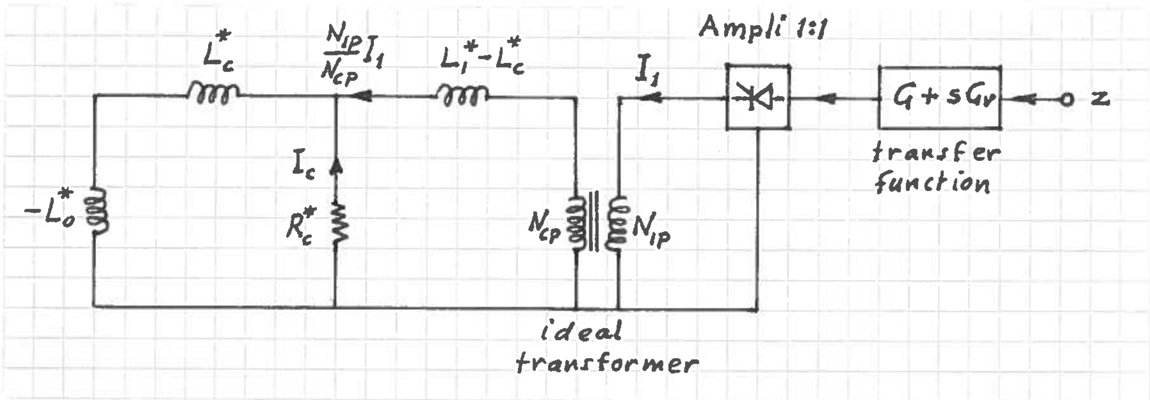
$$M_{1c}N_{cp} \equiv N_{1p}L_c \tag{30}$$

## 2. THE FEEDBACK SYSTEM

The applied voltage $U_1$ in eq. (26) is produced by an amplifier driven by the signal of a flux coil (a probe) inside the vacuum vessel, which detects the motion of the plasma. We assume a linear voltage amplifier, and the detected signal is formed by the following combination

$$U_1 = -Gz - G_V\dot{z} \qquad (31)$$

G and $G_V$ are the position and velocity feedback gains. (As the load is mainly inductive the output current of the amplifier resembles the integral of $Gz + G_V\dot{z}$.) The minus sign means negative feedback. The equivalent circuit is shown below




The ideal transformer is only an artefact of the model. The current in the physical passive coil is of course $I_1$. With (31) the equation of the determinant of (26)-(28) looks like

$$D = -\frac{R_c N_{IP}^2}{\Lambda_1}\left\{\tau_c(\Lambda_c - 1)\left(1 - \frac{\Lambda_1}{\Lambda_c}\right)s^2 + (g_v - 1 + \Lambda_1)s + g\right\} = 0 \qquad (32)$$

Normalised position and velocity feedback gains are introduced:

$$g = \frac{G\Lambda_1}{I_p N_{IP}} \qquad g_v = \frac{G_v \Lambda_1}{I_p N_{IP}} \qquad (33)$$

The new parameter $\Lambda_1$ is analogous to $\Lambda_c$

$$\Lambda_1 = \frac{(N_{IP} I_p)^2}{K L_1} \qquad \Lambda_c = \frac{(N_{cp} I_p)^2}{K L_c} \qquad (34)$$

The factor $1-\Lambda_1/\Lambda_c = (L_1^*-L^*_c)/L_1^*$ appearing in (32) expresses the stray flux between the active coil and the vessel caused by the current $I_1$. The resistance $R_1$ is ignored because the voltage drop on it can be compensated by adding an appropriate term to the transfer function.

The determinant immediately furnishes the stability condition, i.e.: $g_v > 1-\Lambda_1$. The response of the feedback system is governed by the two poles of eq. (32). The smoothest behaviour is obtained by chosing the gains in such a way as to create a double pole $s_0$. This provides the two following conditions for $g_v$ and $g$:

$$s_0 = -\frac{r(g_v-1+\Lambda_1)}{2\left(1-\frac{\Lambda_1}{\Lambda_c}\right)} \quad \text{and} \quad s_0^2 = \frac{r g}{1-\frac{\Lambda_1}{\Lambda_c}} \tag{35}$$

Let $T_0 = -1/s_0$ be the typical response time of the feedback system. Voltage, current and displacement will then have a dominant term $e^{-t/T_0}$.

## 2.1 The problem of the response time

Looking at (35) it is theoretically possible, in the frame of our model, to choose an arbitrary long response time $T_0$ by lowering the gain $g$ and by adjusting $g_v$. There is of course a limit because the feedback system will be very sensitive to small changes of $\Lambda_c$ due to a modification of the plasma current profile. A good compromise, in our opinion, is to set the response time at the geometrical mean between the residual growth time $1/\gamma$ and the vessel time constant:

$$T_0 = \tau_c \sqrt{\Lambda_c-1} \tag{36}$$

For instance with $\tau_c$ = 3 msec resp. 10 msec and $\Lambda_c-1$ = 4% we have $T_0$ = 0.6 msec or 2 msec respectively. This fixes $g$ and $g_v$:

$$g = \frac{1}{\tau_c}\left(1-\frac{\Lambda_1}{\Lambda_c}\right) \qquad g_v = 2\sqrt{\Lambda_c-1}\left(1-\frac{\Lambda_1}{\Lambda_c}\right)+1-\Lambda_1 \tag{37}$$

## 2.2 Waveform of the voltage, the current and the displacement

The system of equations (26)-(28) together with (31) is solved by the standard method of Laplace transform, with the following initial conditions: the displacement is hold at $z_0$ = 2 cm by a constant injected current $I_1$ and then the amplifier is switched on. For the plasma motion we find

$$z(t) = z_0 \cdot (1 + t/T_0)e^{-t/T_0} \tag{38}$$

The displacement z(t), the current $I_1(t)$ and the voltage $U_1(t)$ are plotted in Fig. 3. The vessel time constant is fixed at $\tau_c$ = 3 msec. The number of turns per coil is assumed to be n=20, i.e. the voltage $U_1(t)$ is meant for a total of 80 turns. A factor of 2 is included in the formula for the current in order to take into account the adverse coupling of the shaping coils (see next chapter). We obtain

$$-I_1(t) \cong 2\,\frac{I_p z_0 N_{1p}}{n L_1 \Lambda_1}\left[1 + \left(1 + \sqrt{\Lambda_c - 1}\right)\frac{t}{T_0}\right] e^{-\frac{t}{T_0}} \tag{39}$$

$$-U_1(t) \cong \frac{n I_p N_{1p} z_0}{\Lambda_1 \tau_c}\left[1 - \frac{\Lambda_1}{\Lambda_c} - \left(1 - \frac{\Lambda_1}{\Lambda_c} + \frac{1-\Lambda_1}{\sqrt{\Lambda_c - 1}}\right)\frac{t}{T_0}\right] e^{-\frac{t}{T_0}} \tag{40}$$

$(I_1)_{max}$ = 800 A   $(U_1)_{max}$ = 330 V   for $\tau_c$ = 3 msec
$\Lambda_c$ = 1.04   $\Lambda_1$ = 0.82   $I_p = 10^6$ A   $z_0$ = 2 cm   $T_0$ = 0.6 msec

# 3. THE COUPLING WITH THE SHAPING COILS

We are taking into account the two upper and the two lower shaping coils. A rough estimate gives (in the symmetrical case and for one-turn coils) for the inductance $L_2$ = 18 µH and for the mutual inductance $M_{12}$ = 13 µH between the 4 shaping coils and the 4 feedback coils. Three cases of coupling are considered.

## 3.1 Open circuit

If the power supply of the shaping coil presents a high impedance for the rapidly varying feedback current, the voltage appearing at the

4 coils in series is $U_2 = U_1M_{12}/L_1 = 0.72\ U_1$. In the present design the shaping coils have 60 turns, so the voltage according to (40) will be 710 V.

3.2 Short circuit

If the power supply presents a very low impedance, the ratio $I_2/I_1$ is equal to $M_{12}/L_2 = 0.72$. For the same current $I_1$ the new voltage $U_1$ applied on the feedback coils equals the previous voltage multiplied by $1-M^2_{12}/L_1L_2 = 0.48$. On the other hand, the adverse force created by $I_2$ on the plasma is given by $N_{2p}$, which is about 1.5 smaller than $N_{1p}$ (because the distance of the shaping coil is 1.5 times further away from the centre of curvature of the racetrack than the corresponding distance of the feedback coil). Hence, for an unchanged force the current $I_1$, has to be multiplied by $(1-0.72/1.5)^{-1} = 1.92$. This factor of about 2 is included in formula (39). Finally the new voltage $U_1$ will be greater by a factor of 0.48 x 1.92, that means nearly no change. The required power is thus about twice as much. We obtain 260 kVA. In addition, the ohmic loss with an estimated resistance of 0.13 Ω is about 80 kW. The total required power of the feedback amplifier is

$$W = 340\ \text{kVA}$$

If the vessel time constant is 10 msec instead of 3 msec the inductive power will be 100 kVA but the ohmic loss remains unchanged.

3.3 The shaping coils used as active feedback coils

The feedback amplifier is connected to the shaping coils $L_2$ and the slow power supply of the shaping coils is decoupled by some filtering. Formula (39) and (40) applied for this case produce (without the factor of 2 in $I_1$):

$$(I_2)_{max} = 203\ \text{A} \qquad (U_2)_{max} = 5200\ \text{V}$$

for $\tau_c = 3$ msec. $\Lambda_2$ is now 0.365 and we assumed four 60 turn coils. The inductive power amounts to 1050 kVA, and it is maybe twice as much if the energy loss in the decoupling system is included. The ohmic losses are small in this case. The required power is about

$$\underline{W = 2000 \text{ kVA}}$$

or 6 times more as compared with special separate active coils.

## 4. THE UNSYMMETICAL CASE

During the buildup phase the plasma edge at the top is hold at a fixed distance from the vessel by means of feedback, and the lower edge moves accordingly to the programmed elongation. Our calculation (not shown here) predicts that the vertical stabilisation is continuously working by using the same symmetrical active coils and by using only the probe which detects the position of the upper edge. The feedback gain can be hold constant provided the stability margin $\Lambda_C-1$ does not diminish during the course of elongation.

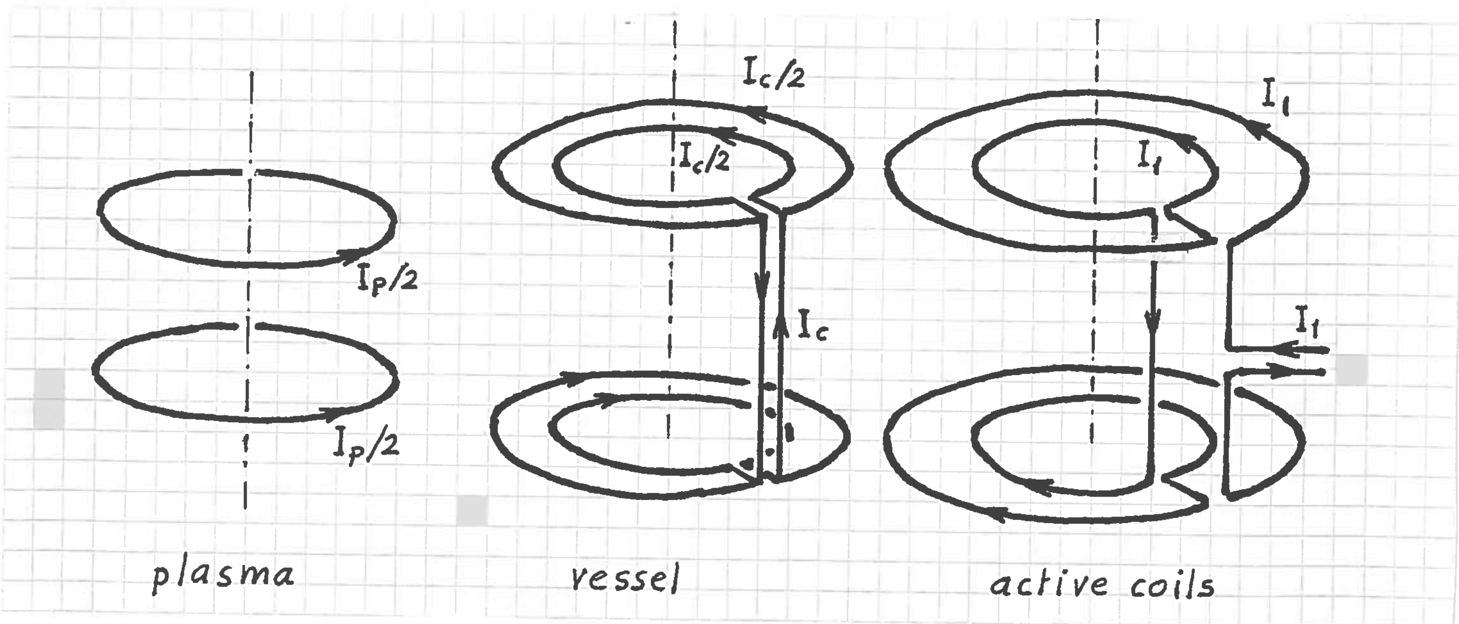



Fig. 1

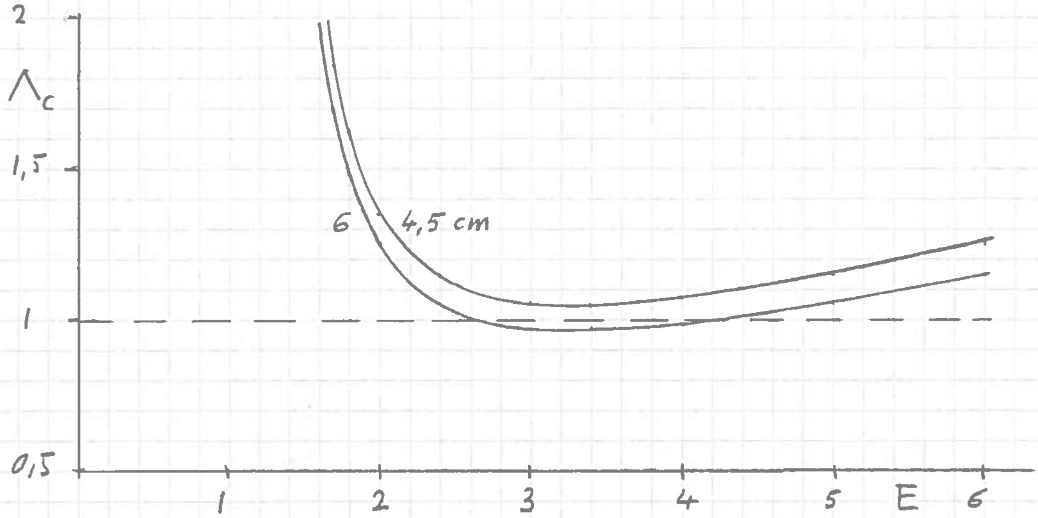



**Fig. 2** Stability margin $\Lambda_c - 1$

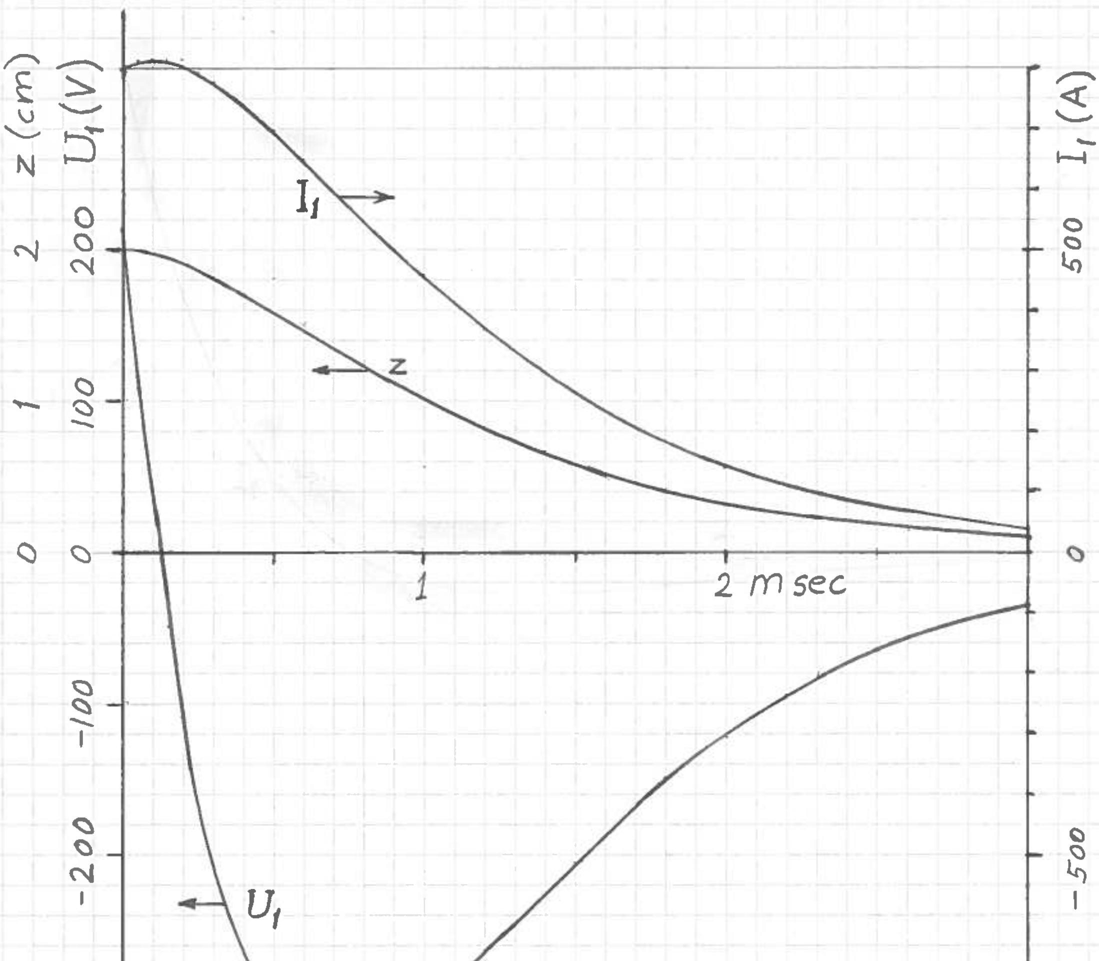



**Fig 3** Waveform