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FREE BOUNDARY TOKAMAK EQUILIBRIUM CODE (FBT)

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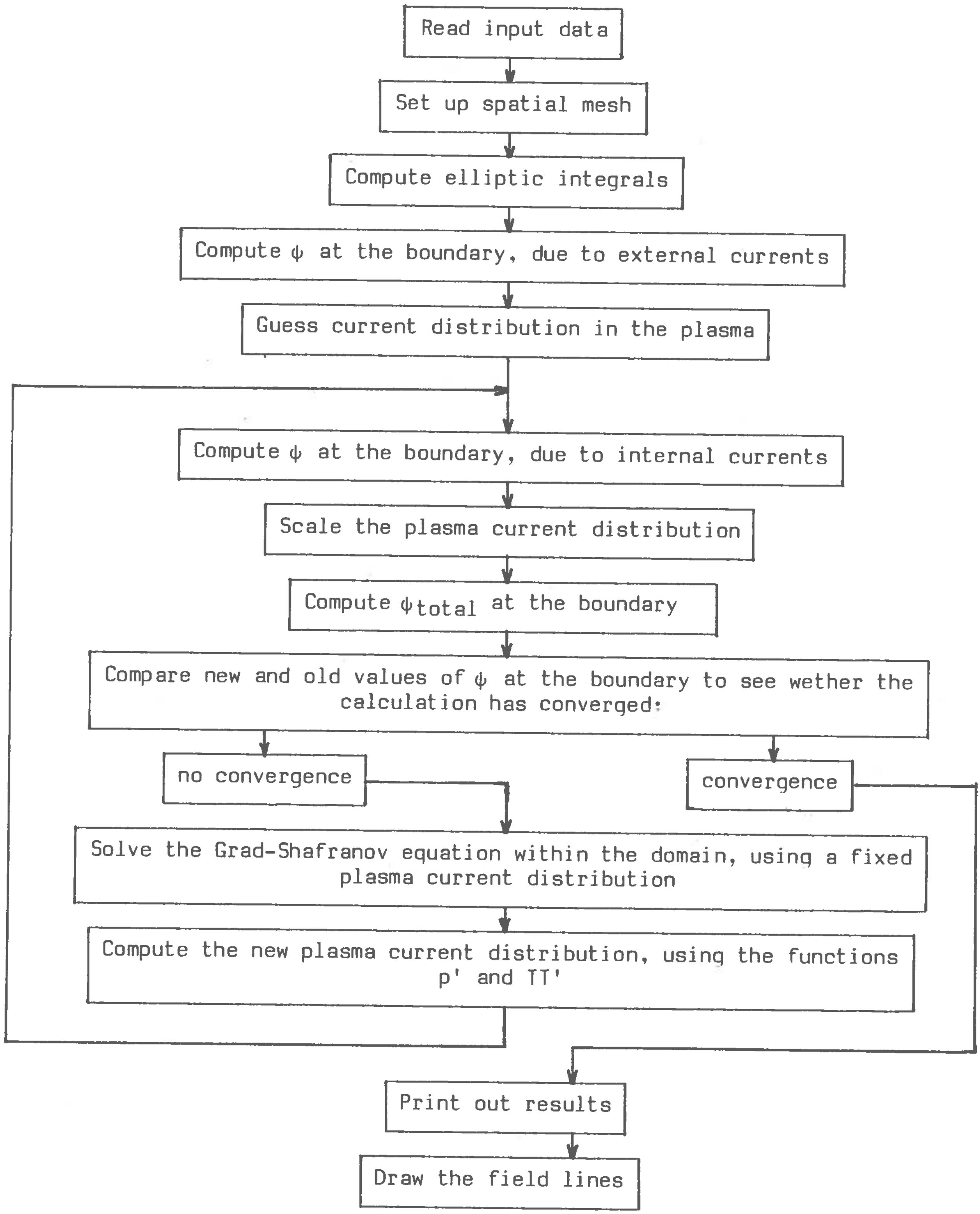
Abstract

A Computer Code (FBT) has been written to calculate axisymmetric tokamak equilibria with arbitrary plasma cross section. The code solves the Grad-Shafranov equation in a rectangular domain, in r-z coordinates. The accuracy of the calculation has been tested by varying the mesh size and by changing the number of terms used in evaluating elliptic integrals. The code has been applied to the computation of a number of TCA and TCD equilibria. Some of these equilibria have also been computed in Garching, using Lackner's code. The results of the two codes agree to within 0,25 %.

1. Description of the FBT Equilibrium Code

1.1 Strategy

The basic strategy of the FBT code is outlined in the flow-chart below:



1.2 Vacuum Fields

The toroidal component of the vector potential, A_θ, due to a current loop (I) located at $r = r_I$, $z = z_I$ is given by

$$A_\theta(r,z) = \frac{\mu_0}{2\pi} \lim_{N\to\infty} \sum_{i=1}^{N} \frac{I\, r_I\, \Delta\alpha \cos\alpha_i}{\left[r^2 + r_I^2 - 2 r r_I \cos\alpha_i + (z-z_I)^2\right]^{1/2}} \tag{1}$$

where $\Delta\alpha = \dfrac{\pi}{N}$, $\alpha_i = (i-\frac{1}{2})\Delta\alpha$

The corresponding poloidal flux function, $\psi = rA_\theta$, can be written as

$$\psi(r,z) = \frac{\mu_0}{2\pi} I k \left[\frac{r\, r_I}{2}\right]^{1/2} F(k^2) \tag{2}$$

where

$$k^2 = \frac{2 r r_I}{r^2 + r_I^2 + (z-z_I)^2} ,$$

$$F(k^2) = \lim_{N\to\infty} \sum_{i=1}^{N} \frac{\Delta\alpha \cos\alpha_i}{\left[1 - k^2 \cos\alpha_i\right]^{1/2}}$$

The elliptic integral $F(k^2)$ is computed for M different values of k^2 in the internal $0 < k^2 < 1$. Values of F at intermediate points are obtained by linear interpolation.

1.3 Grad-Shafranov equation

The poloidal flux function ψ describing an axisymmetric toroidal equilibrium obeys the equation

$$\frac{\partial^2 \psi}{\partial z^2} + r \frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial \psi}{\partial r}\right] = -\mu_0 r j_\theta \qquad (3)$$

where j_θ is the toroidal current density. In finite difference form, the equation is written as

$$\psi_{m,n+1} + \psi_{m,n-1} + E_m \psi_{m+1,n} + W_m \psi_{m-1,n} + C_m \psi_{m,n} = D_{m,n} \qquad (4)$$

where

$$E_m = \frac{(\Delta z)^2}{(\Delta r)^2} \frac{r_m}{r_{m+\frac{1}{2}}}$$

$$W_m = \frac{(\Delta z)^2}{(\Delta r)^2} \frac{r_m}{r_{m-\frac{1}{2}}}$$

$$C_m = -2 - E_m - W_m$$

$$D_{m,n} = -\mu_0 r_m (j_\theta)_{m,n} (\Delta z)^2$$

In the present version of the code, equation (4) is solved by an iterative SOR method. In future versions, it is planned to use an optimized Gauss elimination technique in order to save computation time.

1.4 Current Distribution

A. Configurations without separatrix

The toroidal current density is given by

$$
\begin{aligned}
j_\theta &= r p' + \frac{1}{r \mu_0} T T' \quad , \psi > \psi_{lim} \\
j_\theta &= 0 \quad , \psi < \psi_{lim}
\end{aligned}
\qquad (5)
$$

where $$p' = \frac{\partial p}{\partial \psi} \quad , \quad T' = \frac{\partial T}{\partial \psi} \quad , \quad T = r B_\theta$$

and ψ_{lim} is the value of ψ at the limiter surface. The quantities p' and TT' are arbitrary functions of ψ. If TT' is proportional to p'. an approximate value of β_p is given by

$$\beta_p \cong \left[1 + \frac{T T'}{R_c^2 \mu_0 p'} \right]^{-1} \qquad (6)$$

R_0 is the mean plasma radius, $R_0 = (R_{LI} + R_{LO})/2$ where R_{LI} and R_{LO} are the inner and outer limiter radii. respectively.

B. Configurations with separatrix

When ψ has a saddle point. ψ_{lim} in Eq (5) must be replaced by ψ_{sep}, the value of ψ at the separatrix. In addition. Eq. (5) must be supplemented by a condition stating that $j_\theta = 0$ in any domain where $\psi > \psi_{sep}$, if that domain is in contact with the boundary.

2. Results

2.1 Effect of varying the mesh size

A typical TCA equilibrium was computed. using the following input parameters:

$$R_{LI} = 0.4349 \text{ m}$$

$$R_{LO} = 0.8099 \text{ m}$$

$$\beta_p = 1.53382$$

$$I_{VF} = 10 \text{ kA}$$

$$p' = \text{const.} \times \left[1 - (1-\varphi)^2\right]$$

$$\varphi = \frac{\psi - \psi_{lim}}{\psi_{axis} - \psi_{lim}}$$

$$TT' = \text{const.} \times p'$$

Five runs were performed with different numbers of mesh points. Fig. 1 shows the plasma current as a function of the parameter $(NR \times NZ)^{-1}$. NR and NZ are the numbers of radial and axial mesh points, respectively. Note, that when NR x NZ is changed by a factor of 9, the plasma current changes by 0.2 % only.

2.2 Number of Terms in Elliptic Integrals

N is the number of terms used in the calculation of the elliptic integrals (Eq. 1). A series of runs were performed, using four different values of N. Input parameters were identical with the ones listed in section 2.1. Fig. 2 shows the plasma current as a function of N. By extrapolating to $N = \infty$ we find that $n = 200$ produces an error which is less than 0.1 %.

2.3 Interpolation Errors

The precision of the interpolation procedure which is used to compute $F(k^2)$ depends on the number (M) of values of k^2 at which F is actually computed. Fig. 3 shows the variation of the plasma current as a function of M. Again we find, by extrapolating to $M = \infty$, that $M = 2000$ is sufficient for obtaining a precision of 0.1 %.

2.4 Comparison with Lackner's Code

In September 1982 a number of TCA and TCD equilibria were computed at Garching, using Lackner's equilibrium code (TCD = proposed upgrade of TCA with poloidal divertor). Some of these equilibria were then recomputed at Lausanne, using the FBT-code described in this report. Identical pressure and toroidal field profiles (INTOR profiles) were used in both calculations. The results of the two codes are compared in Table I and in Figs. 4 through 11. Note, that the corresponding plasma currents differ by less than 0.25 %. The flux surface plots are practically identical.

TABLE 1

EQUILIBRIUM	LOW-β TCA		HIGH-β TCA		LOW-β TCD		HIGH-β TCD	
Limiter radii inner, R_{LI} [m]	0.4319		0.4349		0.4603		0.4706	
outer, R_{LO} [m]	0.7849		0.8099		0.7746		0.8115	
β_p	0.29884		1.53328		0.30528		1.49260	
Vertical field current, I_{VF} [k A]	10		10		10		10	
divertor current I_D [kA]	0		0		56.0403		43.5866	
CODE	LACKNER	FBT	LACKNER	FBT	LACKNER	FBT	LACKNER	FBT
Plasma current I_p [kA]	87.32	87.22	60.25	60.15	80.06	79.90	62.27	62.12
ψ-plot, Fig.	4	5	6	7	8	9	10	11
Size:								
r_{min} [m]	0.370	0.370	0.370	0.370	0.370	0.370	0.370	0.370
r_{max} [m]	0.850	0.850	0.850	0.850	0.850	0.850	0.850	0.850
z_{min} [m]	-0.360	-0.257	-0.360	-0.257	-0.360	-0.330	-0.360	-0.330
z_{max} [m]	0.360	0.257	0.360	0.257	0.360	0.330	0.360	0.330