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Feedback Stabilization by Means of a Horizontal Field Coil Surrounding the whole INTOR Structure

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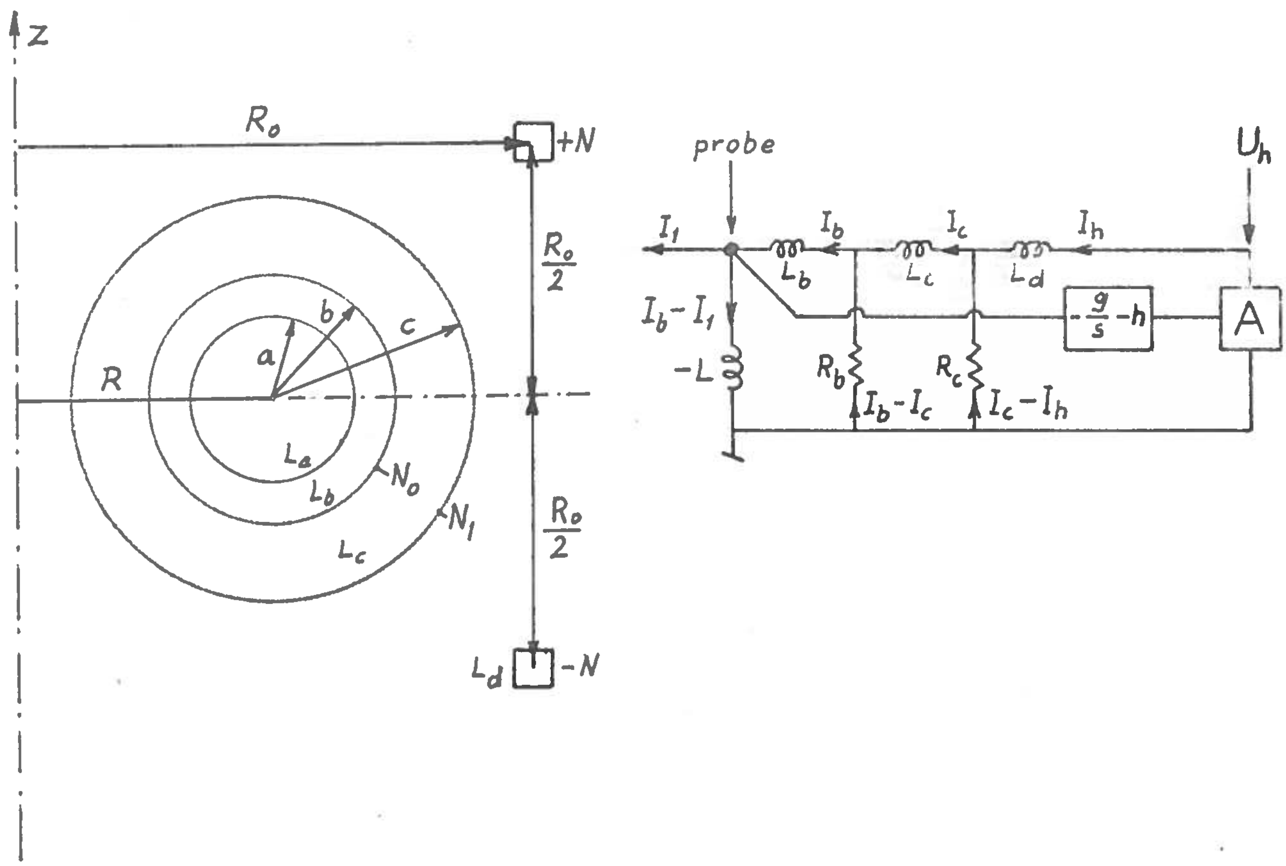
1. Introduction

The response of the vertical motion is calculated for a plasma suffering an external perturbation. The system can be represented by an electrical analog circuit constructed according to a method explained in the laboratory report LRP 166/80. The stability conditions and the power are related to the time constants of the Al blanket, the vacuum vessel and the support structure.

2. The Model



2.1 Transformer Ratio

The Al blanket is supposed to be sliced in N_o turns in toroidal direction, the winding density being proportional to $\sin\theta$. In a similar way, the vacuum vessel is sliced in N_1 turns. The field produced by a constant current I is equal to

$$B_h = \frac{\mu_o N_o I}{4b}$$

where b denotes the mean radius of the blanket. If the current flows in the vessel instead of the blanket, the field is given by $\mu_o N_1 I/4c$ where c denotes the mean radius of the vessel. The force on the plasma will be the same if the field is the same in each case. Equating the above expressions, we find the transformer ratio

$$\frac{N_1}{N_o} = \frac{c}{b}$$

Proceeding in the same manner, the number of turns N of the horizontal field coils is determined by equating the field at the center of the plasma with the above field. If R_o is the radius of the coils, the optimum spacing is also R_o. In this case the horizontal field is given by

$$B_h \cong 0{,}43 \frac{\mu_o N R I}{R_o^2}$$

This formula is valid if R is smaller than R_o, which is the case. Thus the transformer ratio coil-blanket becomes

$$\frac{N}{N_o} = \frac{R_o^2}{4 \cdot 0{,}43\, R b} \cong 8$$

N holds for the number of turns of the upper coil, the lower coil has the same number of turns, in opposite direction.

Instead of creating the horizontal field with separate coils, it would be possible to create the field by feeding the existing vertical field coils in a differential mode. But their position indicated on the drawing is very unfavorable. For simplicity, we assume $N_o = 1$ and $N = 8$. In the analog circuit all voltages as N_o and all currents scale as $1/N_o$.

2.2 The Inductances

The inductance of the blanket without plasma is

$$L_a = \frac{\pi^2}{4} \mu_0 R N_0^2 = 16{,}1 \cdot 10^{-6}$$

The inductance of the space between plasma and blanket is

$$L_b = \left(1 - \frac{a^2}{b^2}\right) L_a = 5{,}3 \cdot 10^{-6}$$

The inductance between blanket and vessel is proportional to the volume :

$$L_c = \left(\frac{c^2}{b^2} - 1\right) L_a = 27 \cdot 10^{-6}$$

The horizontal field coil has an inductance of about

$$L_d \cong 5000 \cdot 10^{-6}$$

L_d goes like $N^2 \sim R_o^4$. The feedback power is proportional to the 4th power of the coil radius.

2.3 The Resistances

The inductances L_b, L_c and L_d are connected in series, and the resistances are connected between the bifurcation points and ground. The blanket resistance is given by

$$R_b = \frac{L_a}{\tau_b} = 62 \cdot 10^{-6}$$

where τ_b = 0.26 sec denotes the blanket time constant. The vessel resistance is given by

$$R_c = \frac{L_a}{\tau_c} \left(\frac{c}{b}\right)^2 = 1000 \cdot 10^{-6}$$

The vessel time constant is 33 msec. It is correct to add the time constant of the support structure, because it is very short : 12 msec. The sum is

τ_c = 0.041 sec. {see P. Reynolds, INTOR Workshop, 2nd Draft, 1.10.80}.
We neglect the effect of the prompt penetration of a small partial field through the slits and the bellows.

2.4 The Dynamic Inductance of the Plasma

The dynamic inductance L_p depends on the coefficient of elasticity K_h of the plasma in vertical direction. From LRP 166/80 we find

$$L_p = \frac{\varkappa^2}{K_h} \quad with \qquad K_h = \frac{n \mu_0 \Gamma I_p^2}{2R}$$

The quantity $\varkappa$ denotes the ratio between force and coil current :

$$\varkappa = \frac{\pi \mu_0 R N_0 I_p}{2b} = 33{,}7$$

I_p is the plasma current and Γ is the Shafranov factor

$$\Gamma = \ln \frac{8R}{a} + \beta_p + \frac{l_i}{2} - \frac{3}{2} = 4{,}2 \quad for \ \beta_p = 2 \ \ and \ \ l_i = 1$$

n is the field index defined as follows

$$n = - \frac{R}{B_v} \cdot \frac{dB_v}{dR}$$

n is negative for an elongated cross section. Its value may be deduced from the radius of curvature r = 4,8 of the field lines at R = 5,2 {see Bobbio, Coccorese, Martone : SOFT, Oxford 1980, Fig. 7}.

$$n = - \frac{R}{r} = -1{,}1$$

Therefore L_p is negative, that is the reason why the plasma is vertically unstable. We write

$$L = -L_p = - \frac{\pi^2 \mu_0 R^3 N_0^2}{2 n \Gamma b^2} = +50 \cdot 10^{-6}$$

n is computed for a flat current profile. Its value is bigger for a peaked profile. A further study of this problem is required.

See for instance :

K. Lackner, A.B. MacMahon : Nucl. Fusion 14, 575 (1974)

G. Cima et al : 6th Int.Conf. Berchtesgaden 1976, Vol. I, p. 335.

J.P. Somon; Tokamaks with non-circular cross section :
G.I.R/TOR.FI/76.2/E, Frascati 1976

2.5 The Probe, the Feedback Gain

The probe produces a signal proportional to the vertical velocity of the plasma. This signal has to be computed on line like the ASDEX system, for instance. The signal is then integrated by a preamplifier having a transfer function of the form

$$-\frac{g}{s} - h$$

s is the Laplace variable and g is the gain of the integrator. A velocity feedback gain h is necessary for stability. Any other probe may be used, i.e. an optical system.

The quantity A represents the power amplifier with unit gain. I_1 is a current which represents an external force on the plasma. This current serves to impose the initial conditions.

The plasma motion ξ is related to the currents in the following way

$$\xi = \frac{L}{x}(I_b - I_1)$$

If the perturbation I_1 is constant, the coil current I_h tends to this value and all currents reach the same value, as a consequence of the feedback action. So $I_h \doteqdot I_c \doteqdot I_b \doteqdot I_1$ and the displacement ξ tends to zero.

The probe (see figure) measures a voltage $- L(\dot{I}_b - \dot{I}_1)$ proportional to the velocity $\dot{\xi}$.

Data Table in MKSA Units

$a = 1.3\sqrt{1.6} \cong 1.6$	$\Gamma = 4.2$	$\varkappa = 33.7$
$b = 1.95$	$N = 8$	$L_a = 16.1 \cdot 10^{-6}$
$c = 3,2$	$\tau_b = 0.26$	$L_b = 5.3 \cdot 10^{-6}$
$R = 5.2$	$\tau_c = 0.041$	$L_c = 27.0 \cdot 10^{-6}$
$R_o = 12$	$R_b = 62 \cdot 10^{-6}$	$L_d = 5000 \cdot 10^{-6}$
$I_p = 6.4 \cdot 10^6$	$R_c = 1000 \cdot 10^{-6}$	$L = 50.0 \cdot 10^{-6}$

3. The Determinant and the Stability

The equations for the 3 unknown I_b, I_c and I_h are

I_b	I_c	I_h	
$(R_b + sL_b - sL)$	$- R_b$	0	$- sL\, I_1$
$- R_b$	$(R_b + R_c + sL_c)$	$- R_c$	0
$(g + sh)L$	R_c	$- (R_c + sL_d)$	$(g + sh)L\, I_1$

The determinant of the system is

$$D = s^3 L L_c L_d + s^2 \left[R_b L_d (L - L_c) + R_c L (L_c + L_d)\right] + s R_b R_c \left[L(1+h) - L_c - L_d\right] + R_b R_c g L$$

We neglect L_b compared with L. The determinant has 3 poles. We consider first the case without vacuum vessel, $R_c \to \infty$. Only two poles remain. The stability condition is simply

$$L(1+h) - L_c - L_d = \alpha (L_c + L_d) \qquad \alpha > 0$$

The best choice of h is the case of critical damping. The double pole is then

$$s_1 = - \frac{\alpha R_b}{2L}$$

and the corresponding gains are

$$g = \frac{\alpha^2 R_b}{4L^2}(L_c + L_d) \qquad \text{and} \quad h+1 = (\alpha+1)\frac{L_c + L_d}{L}$$

For the choice of α we make the logical assumption that the characteristic time $-1/s_1$ of the system has to be similar to the blanket time constant

$$-\frac{1}{s_1} = \tau_b$$

It follows

$$\alpha = \frac{2L}{L_a} = 6,2 \qquad g = 1200\ sec^{-1} \qquad h = 723$$

The vessel has practically no effect. By introducing the actual R_c a 3d stable pole appears, and the 2 other poles undergo a very little displacement. A numerical test shows that the system remains stable and weakly ~~by~~ oscillating as long as

$$\tau_c < \tau_b$$

This condition is amply fulfilled.

4. The Response of the Circuit

By introducing a simple step function

$$I_1(s) = \frac{\hat{I}_1}{s}$$

as a perturbation, we obtain a typical behaviour of the plasma motion and of the current and the voltage. The Laplace transform of the motion is then

$$\xi(s) = \frac{\hat{I}_1}{x(s-s_1)^2}(sL_b - \alpha R_b)$$

The term sL_b may be neglected. The time function is

$$\xi(t) = \frac{2L\hat{I}_1}{x} s_1 t e^{s_1 t} \qquad \xi_{max} = \frac{-2L\hat{I}_1}{e x}$$

As ξ_{max} is prescribed, say - 0.05 m, the current is determined

$$\hat{I}_1 = -\frac{e x \xi_{max}}{2L} = \underline{+45800\,A \triangleq I_h}$$

This is also the maximum current of the horizontal field coils having 8 turns each.

The voltage is given by

$$U_h = (g+sh)x\xi = \frac{\hat{I}_1}{(s-s_1)^2}\left[s^2hL_b + sgL_b - s\alpha hR_b - \alpha gR_b\right]$$

The term sgL_b is negligeable. The first term s^2hL_b produces a delta function. This voltage spike is necessary to create the initial condition. It is supposed to be produced by an external source, but not by the amplifier itself. The amplifier voltage experiences an initial jump followed by a small overshot. For the voltage maximum we find

$$\underline{U_h(t=+0)} \cong -\alpha(\alpha+1)\frac{L_c+L_d}{L}R_b\hat{I}_1 = \underline{-12740\,V}$$

Finally, the power of the feedback amplifier amounts to

$$\underline{P = 584\,MVA}$$

5. Conclusions

The power depends very much on the amplitude and of the time constant of the plasma motion. If the expected amplitude is only 2.5 cm the power will be about 150 MVA. On the other hand, for an expected amplitude of 5 cm and a twice as fast a response, say $|s_1| = 2/\tau_b$, the power would exceed 2000 MVA. But a response which is faster than the blanket time constant can never be excited.

The power scales as $1/L\tau_b$, which means that a thinner Al blanket or a SS blanket requires more power. And the power is proportional to the field index. In conclusion, the blanket resistivity has to be as low as possible in respect to power consumption.

The high feedback gain is a consequence of a large horizontal field coil, far away from the plasma. A careful compensation of the probe against stray fields is necessary in order to prevent unstable coupling.

The expected time constants of the vessel and the support structure are far below the limiting value. The power is proportional to the 4th power of the coil radius.