May 1979 INT 94/79

# THE POTENTIAL OF ALFVEN WAVE HEATING

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Submitted to the Advisory Group on R.F. Heating,
Lausanne, 28 - 30 May 1979

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## 1. The nature of the wave

The classification of the waves resulting from ideal MHD are considerably simplified in the limit $\omega \ll \omega_{ci}$. In plane geometry the solution of the equations leads to the following determinant:

$$D = \left[-\rho\omega^2 + (k\cdot B_0)^2/\mu_0\right]\cdot\left[\rho^2\omega^4 - \rho\omega^2 k^2\left(B_0^2/\mu_0 + \gamma p_0\right) + k^2\gamma p_0 (k\cdot B_0)^2/\mu_0\right] \tag{1}$$

$B_o$ and $p_o$ are the equilibrium values of the magnetic field and the plasma pressure. The condition D = o produces the dispersion relation. It splits in two factors, the first one is

$$-\rho\omega^2 + (k\cdot B_0)^2/\mu_0 = 0 \tag{2}$$

The phase velocity of this propagation mode is

$$\frac{\omega}{k} = v_A \cos\varphi \tag{3}$$

where $\phi$ is the angle between the wave vector k and the field $B_o$. The Alfvén speed is

$$v_A = \frac{B_0}{\sqrt{\mu_0 \rho}} \tag{4}$$

This propagation mode is the shear Alfvén wave (AW) which is the subject of this rapport.

The second factor contains two solutions for the phase velocity, the fast and the slow magnetoacoustic wave. If the orders of magnitude of $v_A$ and $v_S$ are different, the dispersion relation of the fast wave approches the form

$$\left(\frac{\omega}{k}\right)^2 = v_A^2 + v_S^2 \tag{5}$$

and for the slow wave it is

$$\left(\frac{\omega}{k}\right)^{-2} = \left(v_A^{-2} + v_S^{-2}\right)(\cos\varphi)^{-2} \tag{6}$$

where $v_S$ stands for the sound speed

$$v_S = \sqrt{\frac{\gamma P_0}{\rho}} \tag{7}$$

The equations in cylindrical geometry were discussed by Hain and Lüst {1}. The AW takes now the following dispersion relation

$$-\rho\omega^2 + k^2 B_z^2 (1 - m\nu)^2 / \mu_0 = 0 \tag{8}$$

The meaning of $\nu$ is

$$\nu = -\frac{B_\theta}{r k B_z} \tag{9}$$

The propagation vector k is parallel to the axis z. The wave function $(\xi_r, \xi_\theta, \xi_z)$ related to the AW behaves in a singular way. For instance, in the case of a flat current and density profile, the wave function can adopt an arbitrary shape, which at the same time does not change during propagation. This happens because the local phase velocity $\omega/k$ is independent of the radius r. In other words, there exists no coupling between regions of different radii , and consequently, no excitation of AW's can be originated by an external force. For real current and density profiles the phase velocity is a variable function of the radius. If there exists a radius at which this velocity coincides with the phase velocity of a perturbation applied from outside (antenna), a singular surface is formed at this radius. The normal mode analysis shows that $\xi_r$ presents a logarithmic singularity and $\xi_\theta$ a hyperbolic singularity. The plasma energy goes to infinity. By analytic continuation of $\xi_r$ through the singular surface {2,3} it is possible to join the internal to the external domain, and to calculate the coupling with the excitation force. The computation shows an inflowing energy which accumulates at the singular surface at a continuous rate. This result has a general meaning; it occurs when a conservative system has a continuous spectrum, the AW continuum in our case. The spectral range is, according to (8)

$$\omega_A^2 = \left(kB_z + \frac{m}{r_1}B_\theta\right)^2 \Big/ \mu_0\rho \qquad (10)$$

where $r_1$ is the radius of the singular surface. As soon as the driving frequency $\omega$ is turned between the limits $\omega_A$ (o)<$\omega$ < $\omega_A$ (a) there is absorption (if the function $\omega_A$ (r) is supposed to be monotonically increasing).

The differential equation for the radial component $\xi_r$ has the following form, in the long wave length approximation {4,5} :

$$\frac{d}{dr}\left[\rho(\omega^2-\omega_A^2)r^3\frac{d\xi_r}{dr}\right] + \left[\omega^2 r^2\frac{d\rho}{dr} - r(m^2-1)\rho(\omega^2-\omega_A^2)\right]\xi_r = 0 \qquad (11)$$

The origin of the continuous spectrum of the eigenfrequency comes from the existence of a zero in the coefficient of the second order term.

A time-dependent solution of this equation has been found by reintroducing the time variable $\omega = i\delta/\delta t$. The energy builds up mostly in the azimuthal motion, because $\xi_\theta \gg \xi_r$ knowing that div $\xi \overset{\sim}{=}$ o. The energy grows proportionally in time, it concentrates in a singular layer whose half width decreases like 1/t. The wave magnetic field and the current grow until other phenomena appear (after about 100 periods) like ohmic and viscous losses, and the creation of a new type of wave. The singular layer stops decreasing and the phenomenon reaches a stationary state.

In Tokamak configurations the best energy coupling is achieved with the m = 1 mode. Antennae with the corresponding topology excite a kink mode. In equation (11) one sees that the density gradient $d\rho/dr$ acts like a local source term which creates the AW. Thus the antennae transfer the energy at first to the global kink motion, upon which an energy flow sets in to the AW.

## 2. Thermalisation

One peculiarity of the AW heating is that the total energy flux entering the plasma does not depend on the various mechanisms which occur in the singular layer {6}. The singular layer extends over a range of approximately 10 ion Larmor radii {7}. But the region where heating occurs is much larger owing to another phenomenon: the linear mode conversion of the MHD wave to a so called kinetic wave {8,9,10} whose dispersion ralation is given by

$$\omega^2 = k_{\parallel}^2 v_A^2 (1 + k_{\perp}^2 r_s^2) \tag{12}$$

where $r_s$ is the ion Larmor radius, $k_{\parallel}$ and $k_{\perp}$ are parallel and perpendicular wave numbers. The plasma is heated by the damping of this kinetic AW. This wave is found to propagate towards the higher density side, which is favorable for core heating.

In the region near the singular layer, where the mode conversion takes place, the magnetic field of the wave is enhanced by a factor 10 to 30 (the Airy factor $(k_{\perp} r_s)^{-2/3}$). A 25 MW heating power for JET necessitates an externally applied field of only 50 Gauss. Thus the field of the kinetic AW is of the order of 1000 Gauss. Linear or non-linear heating occurs whether or not the field is lower or higher than about 500 Gauss. The linear heating occurs due to the Landau damping of the kinetic AW; the Landau damping rate of electrons dominates if $\beta < 0.1$, hence in a low $\beta$ plasma electrons are predominantly heated in the linear heating regime. The non-linear heating occurs by the resonant decay of the kinetic AW to the ion acoustic wave (if $T_e \gtrsim 3\ T_i$) or by the non-linear Landau damping (if $T_e \lesssim 3\ T_i$). In either case both species are heated approximately at an equal rate. The range of minor radius over which the heating takes place is about 1 m, corresponding to the absorption length of the kinetic AW. Consequently the whole area inside the singular layer is heated.

## 3. Theoretical feasibility of AW heating in JET

Our calculations are applied to toroidal geometry. The torus is treated like a cylinder of length $2\pi R$ and radius a. The approximation is crude in the case of a small aspect ratio and for an elongated cross section like JET.

According to calculations for the Lausanne TCA, the antenna topology giving the best coupling to the plasma should have the mode numbers $m = -1$, $n = 2$. But such an antenna would have a helical shape, which is technically not feasible. It can be shown that the poloidal component of the current produces a stronger force on the plasma than the toroidal component. Therefore we are proposing antennae oriented in the poloidal direction. The 8 antennae form current loops which are placed between the plasma and the top or the bottom of the vacuum vessel. Each of the 4 upper and the 4 lower antennae covers a sector of 20° (see Fig. 1) and they are 90° apart in toroidal direction. The return current takes place via the vessel wall.

The 4 major ports are used for the connections to the antennae. They can be easily arranged without interfering with the neutral beams. In each part both antennae are in phase, so they could be connected together. The material is the same as for the vacuum vessel, so the antennae can be considered as part of the vacuum vessel. New first wall problems should not arise. Heating power up to 25 MW can be supported without substantial temperature increase, even in the CW regime. Wall losses are negligible. It may well be necessary to provide some kind of electrostatic shielding in order to avoid discharges in the toroidal direction between two antennae.

By virtue of the antenna geometry, two global kink modes of opposite circular polarization are excited: $m = +1$, $n = +2$ and $m = -1$, $n = +2$. This superposition creates a vertical polarized wave. The eigenfrequency depends on the spacing between plasma and wall. It depends also on the way the antennae are connected. For instance short circuited antennae bring the wall closer to the plasma and, consequently, change the eigenfrequency. By analogy with coupled ringing circuits it is found that the antennae introduce a coupling of both modes $m = \pm 1$. An equivalent circuit, Fig. 2, may be drawn. We assume the following JET parameters:

$B_o = 3,5$ Tesla $\quad\quad \bar{n}_e = 3,5.10^{19}\ m^{-3}$ deuterium

$J_p = 4.10^6$ Amp $\quad\quad q_a = 3$

The applied frequency for this case lies in the range of 1,4 MHz ± 20 %. For a total heating power of 25 MW the peak current per antenna is 7,5 kA and the peak voltage is 15 kV at the antenna itself. At the feedthrough the voltage can be maintained below 25 kV by proper design of the conductor. The values of the equivalent circuit are the following, in nH, nF and Ω:

| $L_+$ | $L_-$ | $\Lambda_+$ | $\Lambda_-$ | $L$ | $C_+$ | $C_-$ | $R_+$ | $R_-$ |
|---|---|---|---|---|---|---|---|---|
| 80 | 45 | 540 | 300 | 100 | 70 | 70 | 1.5 | 2 |

Note that the bulk motion of the plasma is less than 1 mm.

One drawback of the JET geometry for AW heating is the small clearance of the plasma in the vertical direction. In order to decrease the kink frequency and to bring it closer to the center of the continuous spectrum we decreased the elongation ratio in this proposal to $b/a = 1.5$ (instead of 1.68). The Q factor of the antenna is 18. The area inside the singular layer is 85 % of the plasma cross section for the $m = + 1$ mode, and 60 % for the $m = -1$ mode. The full area is heated by the absorption of the kinetic AW. Both modes consume about the same power. With a larger spacing between antenna and wall the necessary current and the Q factor would decrease and the singular layer could be shifted deeper into the plasma.

In this respect the situation of ASDEX is more favorable. The relative plasma-wall spacing is much greater. The presence of the divertor coils forces the antennae to be centered at a poloidal angle of $\theta = 0^o$ instead of $\pm 90^o$. As the inside is not accessible no antenna can be placed at $\theta = 180^o$. Only 4 antennae have to be installed (see Fig. 3) which shake the plasma in a horizontal direction. We assume the following ASDEX parameters:

$B_o = 2.8$ Tesla $\qquad \bar{n}_e = 5.10^{19}$ $m^{-3}$ deuterium

$J_p = 450$ kA $\qquad q_a = 3$

The applied frequency for this case lies in the range of 1.9 MHz ± 20 %. For a total heating power of 4 MW the peak current per antenna is 2.2 kA and the peak voltage is 2.5 kV between both ends of the antenna facing the plasma. At the feedtrough the peak voltage will be 6 kV between the two leads. The Q factor of the antenna is 2.5. The area inside the singular layer is 80 % of the plasma cross section for the $m = + 1$ mode, and 45 % for the $m = - 1$ mode. The values of the equivalent circuit of one antenna are the following:

| $L_+$ | $L_-$ | $\Lambda_+$ | $\Lambda_-$ | $L$ | $C_+$ | $C_-$ | $R_+$ | $R_-$ |
|---|---|---|---|---|---|---|---|---|
| 50 | 30 | 200 | 125 | 150 | 180 | 180 | 0.8 | 1.6 |

## 4. Heating experiments

Several experiments have been performed attempting to heat plasmas confined in linear and stellarator type devices by means of AW. {11}. While significant heating has been observed in all of these experiments, they have also been characterized by loss of plasma during the heating.

An explanation of the enhanced transport in Proto-Cleo at Wisconsin is given in ref. {10}. It is based on the formation of magnetic islands due to the interaction of the magnetic field produced by the heating coil with the confining field. But this effect is probably absent in higher $\beta$ plasmas like Tokamaks {12}, because the penetration of the magnetic islands is less pronounced. On the other hand the wave field may reduce the number of trapped electrons which will eventually reduce the diffusion rate {8}.

The Uragan II at Kharkov {13} is an $\ell = 2$ race-track stellarator. Densities were up to $5.10^{12}$ $cm^{-3}$. Temperature, density and RF field were measured as functions of toroidal magnetic field. The power was in the 30 kW range. Peaks in these quantities occured, which correlated with the region of excitation of longitudinal AW modes. On can observe heating, but the density falls.

The Heliotron - D at Kyoto {14,15} is an $\ell = 2$ ohmically heated (2 ~ 15 kA) torsatron with magnetic limiters. The toroidal field was set at 3 kG. Up to 1 MW of RF at 400 kHz was supplied. Plasma densities were as high as $2.10^{13}$ $cm^{-3}$. An $m = \pm 2$, $n = 2$ AW was launched. It can be seen that after an initial increase in temperature, there appears to be a decrease in the electron temperature. Both electrons and ions are heated significantly, i.e., from 95 eV to 100 eV for the electrons, and from 25 to 40 eV for the ions at an input power level of 300 kW. And, although no pump-out is observed, there appears to be some increased ionization due to the RF which may mask enhanced transport.

In a subsequent work where new improved antennae were installed, the electron temperature does not decrease so quickly, and the RF heating seems not to cause a significant increase of the plasma loss. This is an important statement which shows that the figure of merit of a heating scheme can be confused by spurious effects. In the same experiment it is found that the plasma core is mainly heated by the absorption of the kinetic AW which is propagating inwards. This happens at the higher temperature regime (100 eV). But at low temperature (10 eV) the kinetic AW cannot be excited, thus the heating is concentrated at two radii, indicating the location of the singular layers m = ± 2. Only $T_e$ measurements are reported {16}.

The R - 02 stellarator at Sukuhmi {17} is an ohmically heated $\ell = 2$ stellarator of toroidal field 15 kG and ohmic heating currents up to 2 kA. Plasma densities are of the order of $5.10^{13}$ $cm^{-3}$ and electron temperautres between 10 and 20 eV. The diamagnetic signal shows peaks corresponding to Alfvén resonances. The dependence of the heating on the RF field strength is found to be non-linear, and it experiences a threshold. The ion heating is probably due to parametric decay in an ion acoustic and an Alfvén branch.

In linear geometries experimental evidence of AW absorption was seen in the ISAR-I 5 m theta pinch at Garching {18}, and in a 3.5 m theta pinch at Culham {19}. A theta-screw pinch experiment at Lausanne has been used to examine AW heating. The m = 1 launching coil was driven by an LC ringing circuit. The peak power was of the order of 10 MW, and the frequency was in the 1 MHz range. The observed resonance curve of the heating power agrees quantitatively with the predicted behaviour {20}.

## 5. Conclusion

In examining the present status of the AW heating scheme the first remark to be put forward is the lack of experimental results on Tokamaks. Nevertheless the positive results on other machines show evidence of AW heating.

But still more theoretical effort should be made, especially the study of toroidal effects and the application to non-cicular cross sections. For instance, toroidicity will introduce mode coupling of different m values, and there exists no information yet on how energy will divide among the various resonances {5}. Similar effects arise for elliptic cross sections {21}. The possible heating of electrons in phase with the wave is not yet studied. For a reactor plasma this heating method has the basic merit of using a low frequency field of the order of 1 MHz for which a high power source is currently available. Only the density but not the kind of particle plays a role. - The Q factor of the antennae is low. - No mode tracking is necessary. - Profile shaping is achievable by proper choice of the frequency. - The massive antennae can be considered as part of the vacuum vessel. - Finally, the additional heating by Alfvén Waves could be one of the cheapest and most efficient methods.

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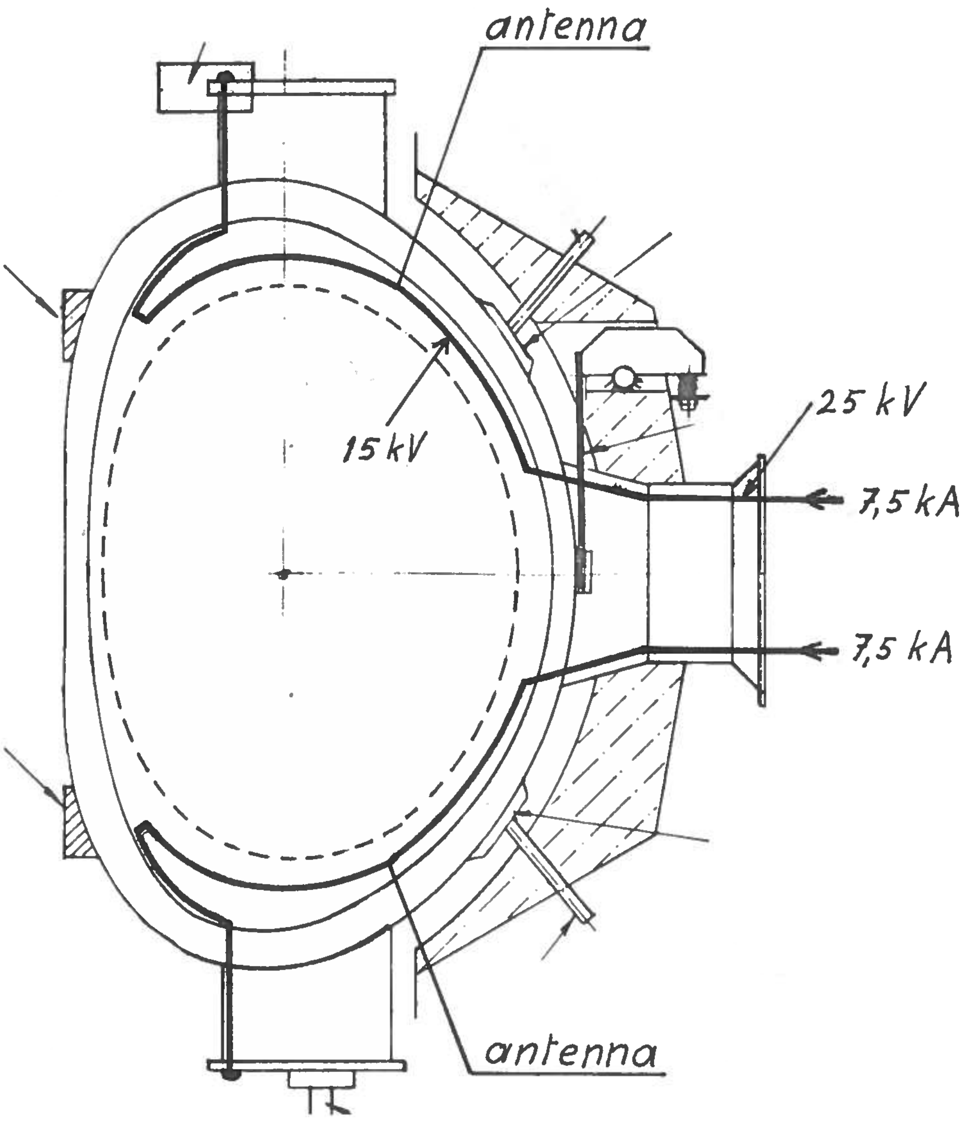

Fig. 1. JET 25 MW

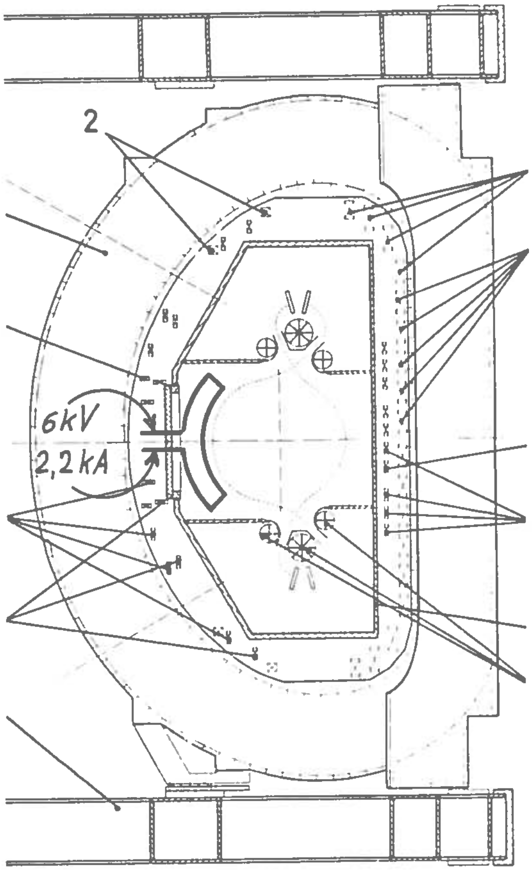

Fig. 3. ASDEX 4MW

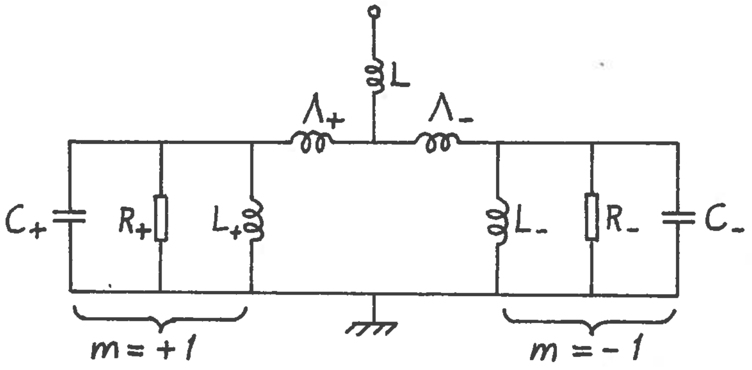

Fig. 2.