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OPTIMIZATION OF THE OHMIC HEATING SYSTEM FOR T.C.A.

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INTRODUCTION

The heating of the TCA Tokamak will be provided in the first instance by an Ohmic heating transformer. Due to the small aspect ratio (R/a = 3.3) and hence the compact nature of the machine, the amount of space required for an iron core is not available. Thus, the OH system will operate with an air core and the stray fields produced in the plasma region by the primary OH coil, must be compensated by the strategic placement of supplementary coils around the machine.

In this report we present an outline of the theory and results of the calculation of the sizes and positions of the compensation coils that will ensure minimization of the stray fields in the plasma volume. Finally, some comments are made on the problem of magnetic fields outside the Tokamak, that could have a deleterious effect on some diagnostic instruments.

THEORY

The problem may be stated as follows: as shown in Fig. IA the main OH coil (A) produces a magnetic field inside the plasma area that would normally be eliminated by the use of an iron core. This field is undesirable since it will interact with the plasma current producing unwanted forces on the plasma column. To direct the flux around the plasma, supplementary coils are placed about the plasma as shown in Fig. IB. Ideally the plasma would be completely surrounded by coils to provide zero field; this is, however, impractical and in reality a few coils are placed about the plasma to provide a minimum field with maximum access to the plasma. Thus, the positions of the suplèmentary coils and the currents they carry must be determined such that, when combined with the OH coil, they will produce as low a field as possible in the plasma volume. The method employed was to calculate analytically the field in the plasma due to a certain coil configuration and to minimise this field, with respect to the supplementary currents, by the method of least squares.

Beginning with the law of Biot and Savart:

$$\underline{dB}^{ij} = \frac{I\,\underline{dl} \times \hat{\underline{X}}^{ij}}{|X^{ij}|^2} \tag{1}$$

where $I\,\underline{dl}$ is a current element and $\underline{X}^{ij}$ is the vector connecting the current element to the point $(r_i\,,\ z_j)$ at which the field, $\underline{dB}^{ij}$, is to be calculated. Transposing into right cylindrical coordinates and assuming that all currents are purely azimuthal and that $\underline{dB}$ is to be calculated in the $\theta=0$ plane, yields:

$$\begin{aligned} dB_r^{ij} &= I\,dl_\theta \cos\theta \; X_z^{ij} / |X^{ij}|^3 \\ dB_z^{ij} &= -I\,dl_\theta \; X_r^{ij} / |X^{ij}|^3 \\ dB_\theta^{ij} &= 0 \end{aligned} \tag{2}$$

Integrating over the current path (θ=0 to 2π) gives the field at $(r_i,\ z_j)$ due to the mth current loop, the coordinates of which are $(\rho^m,\ z^m)$.

$$\begin{aligned} B_r^{mij} &= 2I_m \int_0^{\pi} \frac{\rho^m \cos\theta \; X_z^{ijm}}{|X^{ijm}|^3}\, d\theta \\ B_z^{mij} &= -2I_m \int_0^{\pi} \frac{\rho^m \; X_r^{ijm}}{|X^{ijm}|^3}\, d\theta \end{aligned} \tag{3}$$

where $\underline{X}^{ijm}$ is defined by (see Fig. 2)

$$|x^{ijm}| = \left\{ (z_j - z^m)^2 + r_i^2 + \rho^{m\,2} - 2 r_i \rho^m \cos\theta \right\}^{1/2}$$

$$x_r^{ijm} = r_i \cos\theta - \rho^m \qquad (4)$$

$$x_z^{ijm} = z_j - z^m$$

Summing Eq. 3 for m=1,M yields the magnetic field at (r^i, z^j) due to all the current loops:

$$B_r^{ij} = 2 \sum_m I_m \int_0^\pi \frac{\rho^m \cos\theta \, x_z^{ijm}}{|x^{ijm}|^3} \, d\theta \, d\theta + C_r \qquad (5)$$

$$B_z^{ij} = -2 \sum_m I_m \int_0^\pi \frac{\rho^m \, x_r^{ijm}}{|x^{ijm}|^3} \, d\theta + C_z$$

Where the terms C_r and C_z are identical to the preceding terms except that the sign of z^m is reversed. This takes into account the currents below the z=0 plane, which are assumed to be the mirror image of those above. For convenience we can write 5 as:

$$B_r^{ij} = 2 \sum_m I_m X_{mij}$$

$$B_z^{ij} = 2 \sum_m I_m Y_{mij} \qquad (6)$$

Where the definitions of Xmij and Ymij are apparent from 5. Using the method of least squares, the total field in the plasma region must be minimised with respect to the currents.

The total field in the plasma is given by:

$$B^2 = B_r^2 + B_z^2 = \sum_{ij}\left\{ B_r^{ij\,2} + B_z^{ij\,2} \right\}$$

Using 6 this becomes:

$$B^2 = 4\sum_{ij}\sum_{mm'}\left\{ I_m I_{m'}\left(X_{mij}X_{m'ij} + Y_{mij}Y_{m'ij}\right)\right\} \tag{7}$$

Of the M currents under consideration the first k of these represent the currents in the primary OH coil. These currents are fixed and known and are therefore not included in the minimisation procedure. To minimise 7 we differentiate with respect to the supplementary currents, (M-k) of them, and set to zero. ie:

$$\frac{d}{dI_\mu}\left(B^2\right) = 8\sum_{ij}\sum_{m} I_m\left\{ X_{\mu ij}X_{mij} + Y_{\mu ij}Y_{mij}\right\} \tag{8}$$

where $\mu = k+1,\ k+2,\ \ldots\ldots\ldots\ M$

Thus we have a system of (M-k) equations, one for each supplementary current. Equation 8 can be simplified by representing the first k terms of the sum as:

$$\beta_\mu = \sum_{m=1}^{k} I_m \sum_{ij}\left\{ X_{\mu ij}X_{mij} + Y_{\mu ij}Y_{mij}\right\}$$

and by defining,

$$\alpha_{\mu\nu} = \sum_{ij}\left\{ X_{\mu ij}X_{\nu ij} + Y_{\mu ij}Y_{\nu ij}\right\}$$

where υ = k+1, k+2, ... M. Thereby reducing Eq. 8 to the form:

$$\sum_{\upsilon = k+1}^{M} \{ I_{\upsilon} \alpha_{\mu\upsilon} \} + \beta_{\mu} = 0 \qquad (9)$$

where μ = k+1, ... M, which can be solved numerically using the method of Gaussian elimination[1], ie triangulation of the augmented matrix $[\alpha_{\mu\upsilon} \beta_{\mu}]$. There are, however, some analytical points worth considering, namely: the possibility of a maximum or minimum solution, uniqueness of the solution and degeneracy.

(1) The solution so obtained must represent a minimum field in the plasma area. If equation 8 is again differentiated with respect to current, this would produce a second derivative that is independent of I and positive (sum of squares), thus indicating a solution that is a minimum. Physically, magnetic field varies linearly with current and therefore, in principle, has no maximum.

(2) The solution will be unique because equation 7 is quadratic in all variables and thus has only one extremum. Also the set of equations are linearly independent and therefore, since there are as many unknowns as equations, the solution is unique.

(3) The set of equations become degenerate if two or more of the unknown currents have the same coordinates. This degeneracy manifests itself by giving infinite solutions. What is perhaps more important is that if two currents are put very close together then one is approaching the degenerate condition and unrealistic solutions will result, eg: the two coils will carry large currents of opposite sign. If the two coils are replaced by a single coil the solution would indicate a current of approximately the difference between the two former solutions.

The programme CCOIL was developed to solve equation 9. The sizes and positions of the known currents as well as the positions of the unknown currents are read in and the programme solves equation 9 over a specified grid. The solution is printed out along with a display of the resulting field components B_r, B_z and $B = (B_r^2 + B_z^2)^{\frac{1}{2}}$.

CALCULATIONS

The programme was written to calculate the required currents given the coil positions. The positions of the coils may be adjusted to obtain the most satisfactory minimum of the stray fields. However, there are some constraints on the positioning of the coils:

(1) The entire OH system must be designed to be compatible with TCA and TCV. ie the coils must be able to cope with the larger OH current as well as provide minimum stray fields over the larger TCV plasma volume[2].

(2) The coils must be demountable to allow the removal of the vacuum vessel without having to totally dismantle the machine.

(3) Large current densities should be avoided for reasons of heat dissipation.

When these constraints are applied to the positioning of the coils, the choice of coil positions is already limited. The proceedure for optimisation went as follows. The current in the primary OH coil was fixed at 1.5 MA and was represented by 30 currents distributed over the area occupied by the coil in the upper half plane. Four supplementary coils were allowed per half plane and these were represented by point currents in order to avoid degenerate solutions. However, these single currents do not give a good representation of the coil size, thus, when a reasonable solution has been found by varying the coil positions, one of the currents is fixed and distributed over the area to be occupied by the coil. After fixing one coil the optimisation proceedure is repeated for the remaining coils and then another coil is fixed etc, etc. From a practical point of view the currents are not fixed exactly to the computed value, but to a value that gives a reasonable number of turns for a current of, say, 10 kA/turn. Using this optimisation proceedure the stray fields were minimised, the final results being shown in Table I.

COIL	CURRENT (kA)	X-sec AREA (cm^2)	CURRENT DENSITY (kA/cm^2)	TURNS et 10 kA/T
A (fixed)	1500	406	3.7	150
B	160	60	2.7	16
C	280	98	2.9	28
D	100	36	2.8	10
E	10	4	2.5	1

Table I

Fig. 3 shows the configuration of the coils around the torus and Fig. 4 shows the resulting stray fields in the plasma area. It can be seen that all the constraints have been satisfied and that, except for the corners, most of the area has a field of less than 10 gauss, which is to be compared to a proposed vertical field of 200 to 300 gauss. It was found that the coils B and C (Fig. 3) were the most critical to position. Ideally coil B should have a smaller radius, this would improve the stray fields at the expense of demountability.

Finally, Figs 5 and 6 display the stray fields generated outside the plasma. The fields shown are representative of peak current conditions and will decay to about a third of the value during the course of a shot. The fields immediately above the machine will be very high, requiring the removal of any magnetic materials from this area. The fields in the plane of the machine are still high enough such that field sensitive instruments would be affected. Such instruments (eg. photomultipliers) would have to be magnetically shielded in, for example, mu-metal containers. It appears that the data aquisition system and the computing equipment will be far enough removed from the machine so as to be unaffected by the stray fields.

REFERENCES

1. C.E. Fröberg - Introduction to numerical Analysis, Addison - Wesley 1970, page 82

2. R. Gruber et al. - The TCV Tokamak Project, CRPP - May 1977

FIGURE CAPTIONS

Figure 1 : (a) A schematic representation of the stray fields of the OH coil (A) of the Tokamak.
(b) as for (a) but with compensating coils.

Figure 2 : (a) Coordinate system based on right cylindrical coordinates, the symbols are explained in the text.
(b) projection of (a) onto the x-z plane.
(c) projection of (a) onto the x-y plane.

Figure 3 : Showing the positioning of the supplementary coils relative to the plasma and the toroidal field coils.

Figure 4 : Map of the stray fields in the plasma cross-section using the coil configuration of Table I and Fig. 3. The lines are equipotential surfaces labeled in gauss.

Figure 5 : Map of stray fields outside the machine, ie in the laboratory area.

Figure 6 : Detailed map of stray fields in the immediate vicinity of the machine.

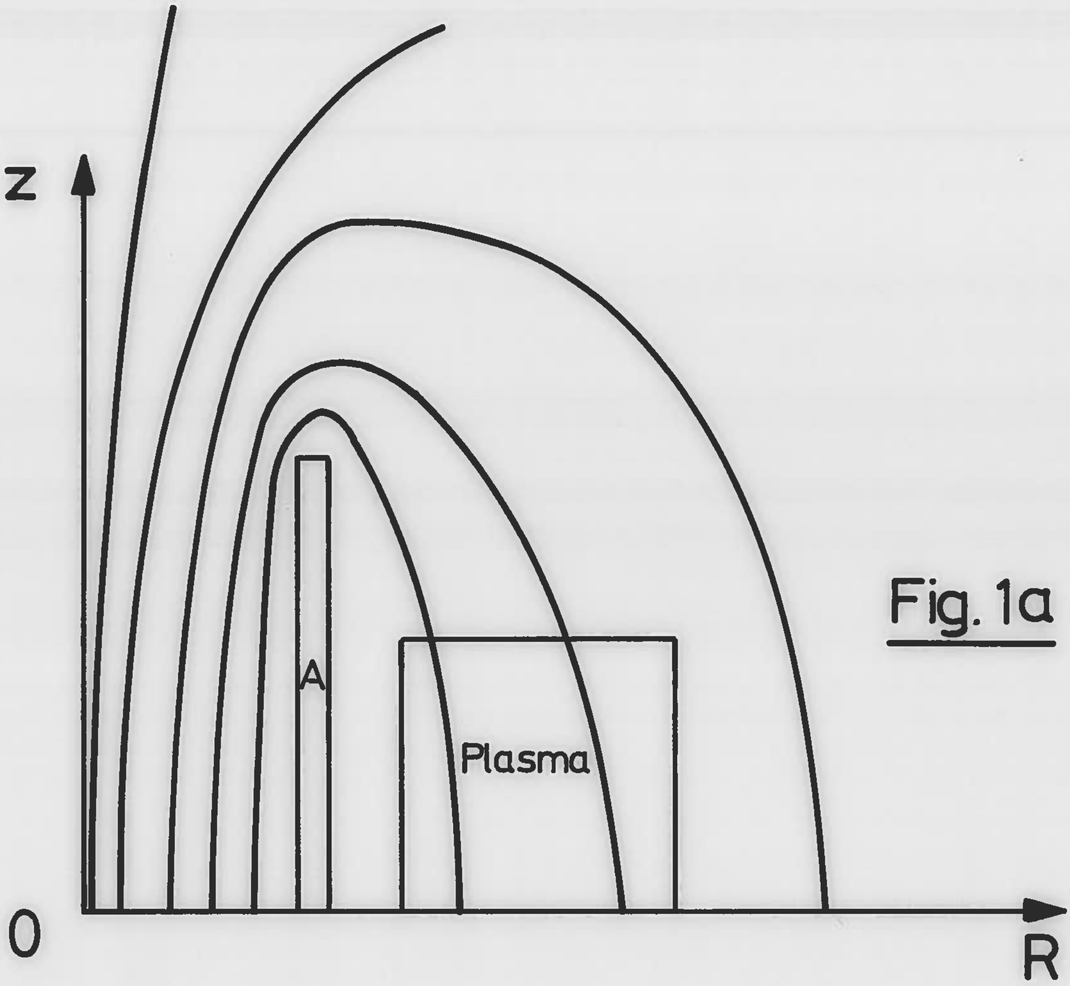


Fig. 1a

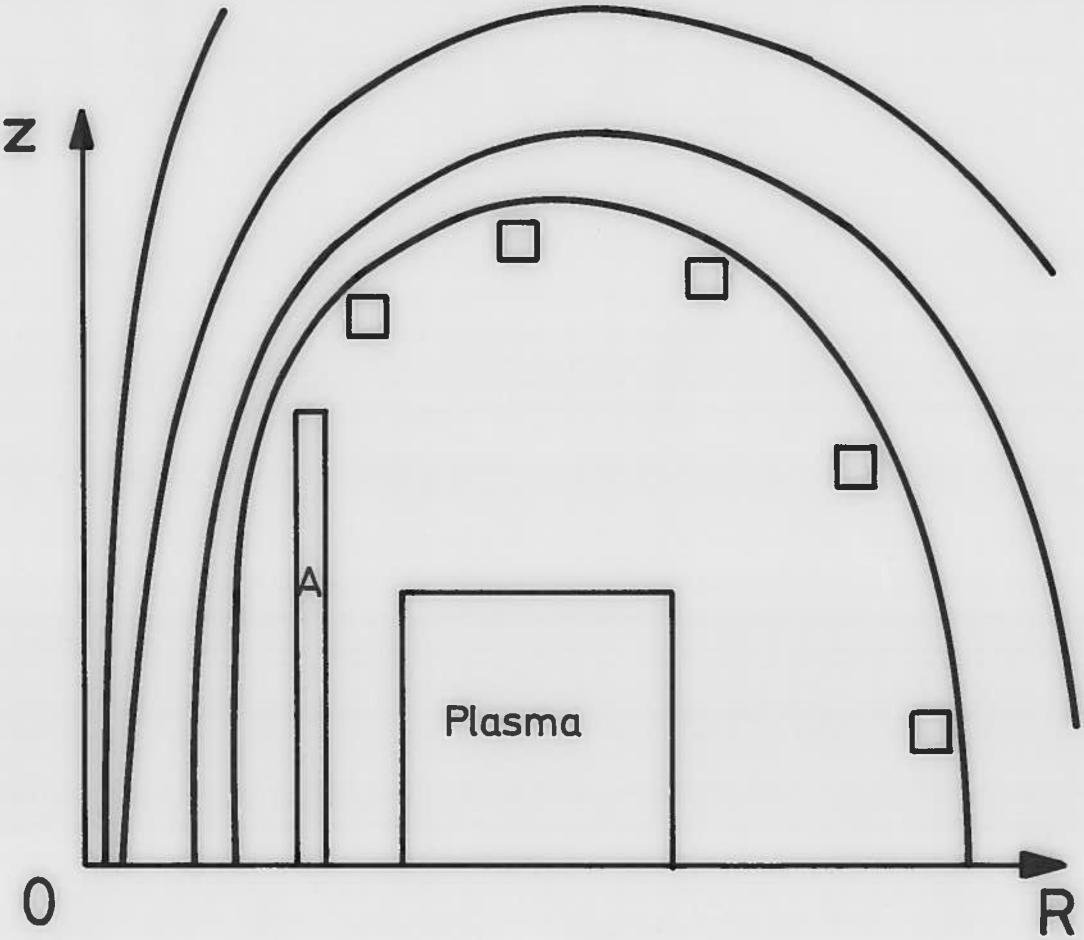


Fig. 1b

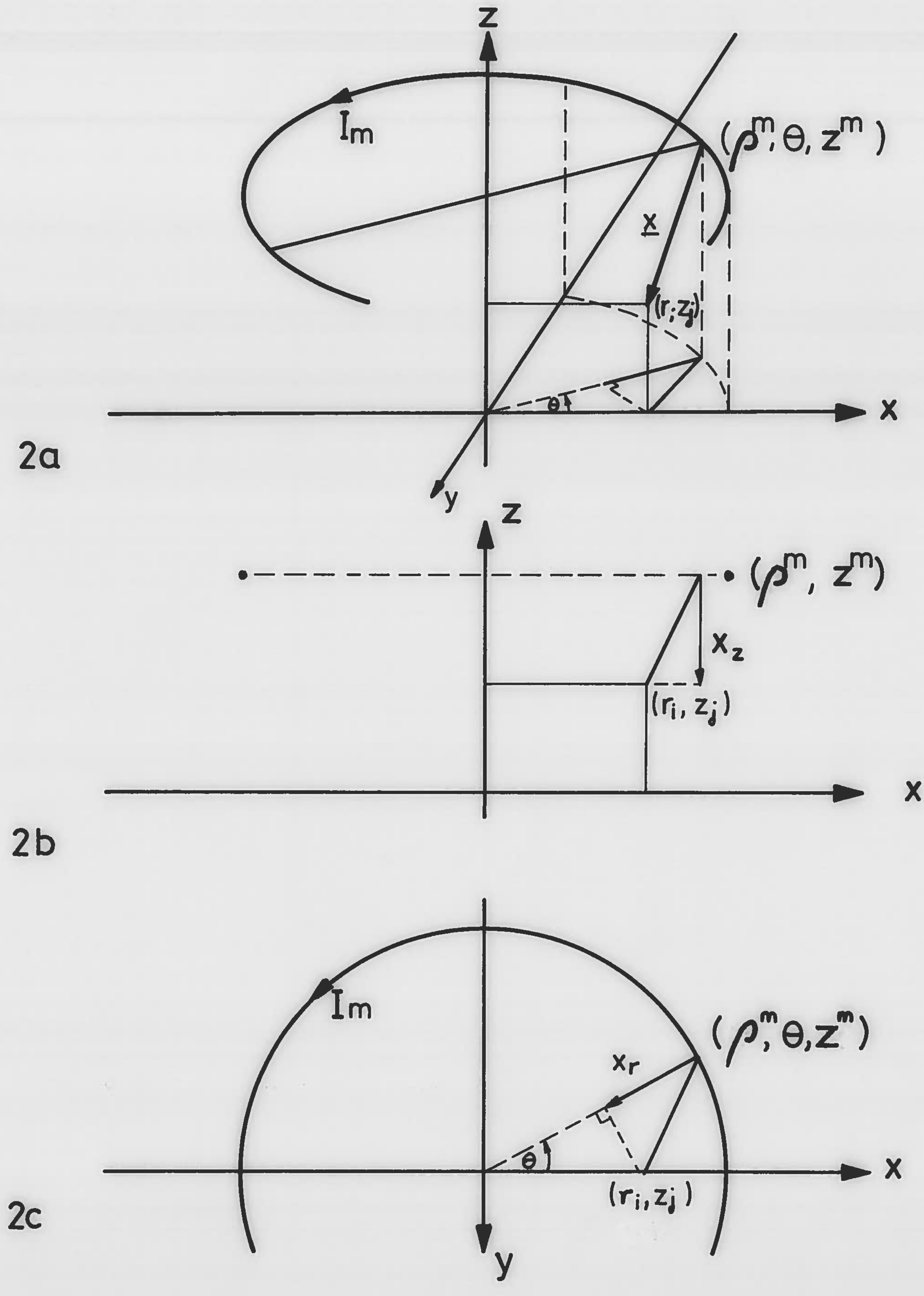

z
I_m
(ρ^m, θ, z^m)
x
(r_i, z_j)
θ
x
2a
y
z
(ρ^m, z^m)
x_z
(r_i, z_j)
x
2b
I_m
(ρ^m, θ, z^m)
x_r
θ
x
(r_i, z_j)
2c
y

Fig. 2

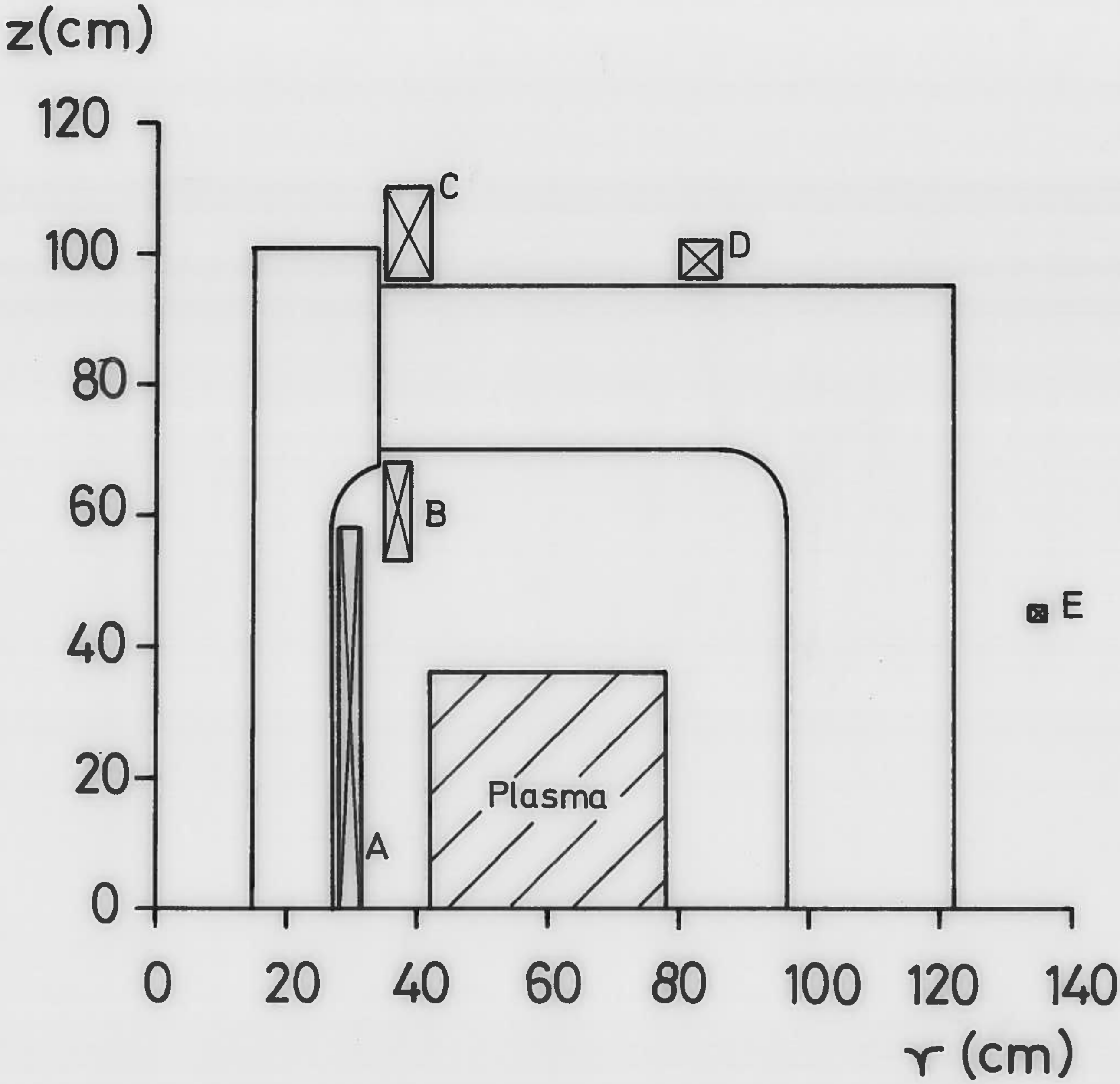

z(cm)
120
100
80
60
40
20
0
C
D
B
E
A
Plasma
0
20
40
60
80
100
120
140
r (cm)

Fig. 3

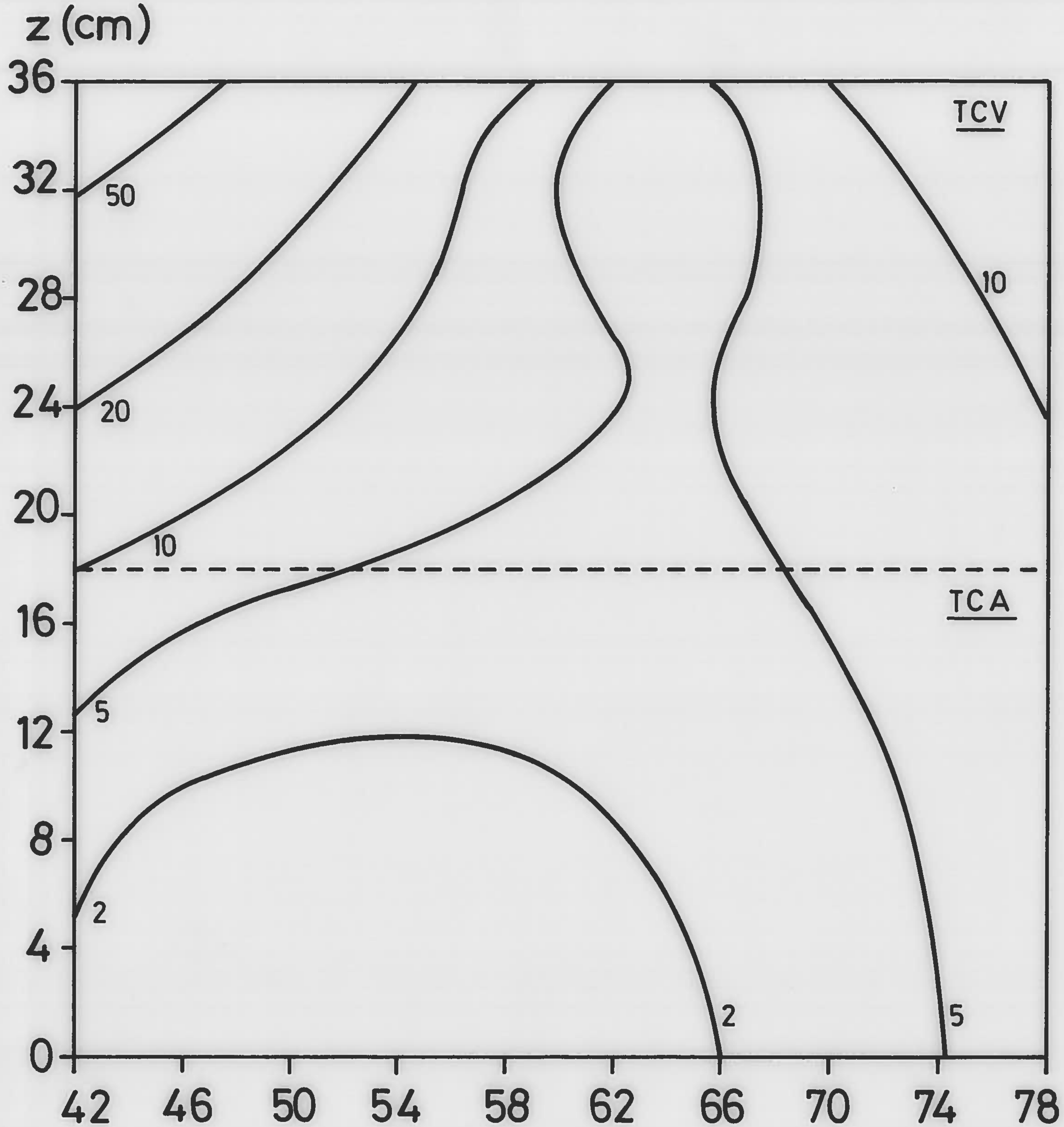

z (cm)
36
32
28
24
20
16
12
8
4
0
50
20
10
5
2
2
5
10
TCV
TCA
42
46
50
54
58
62
66
70
74
78
R (cm)

Fig. 4

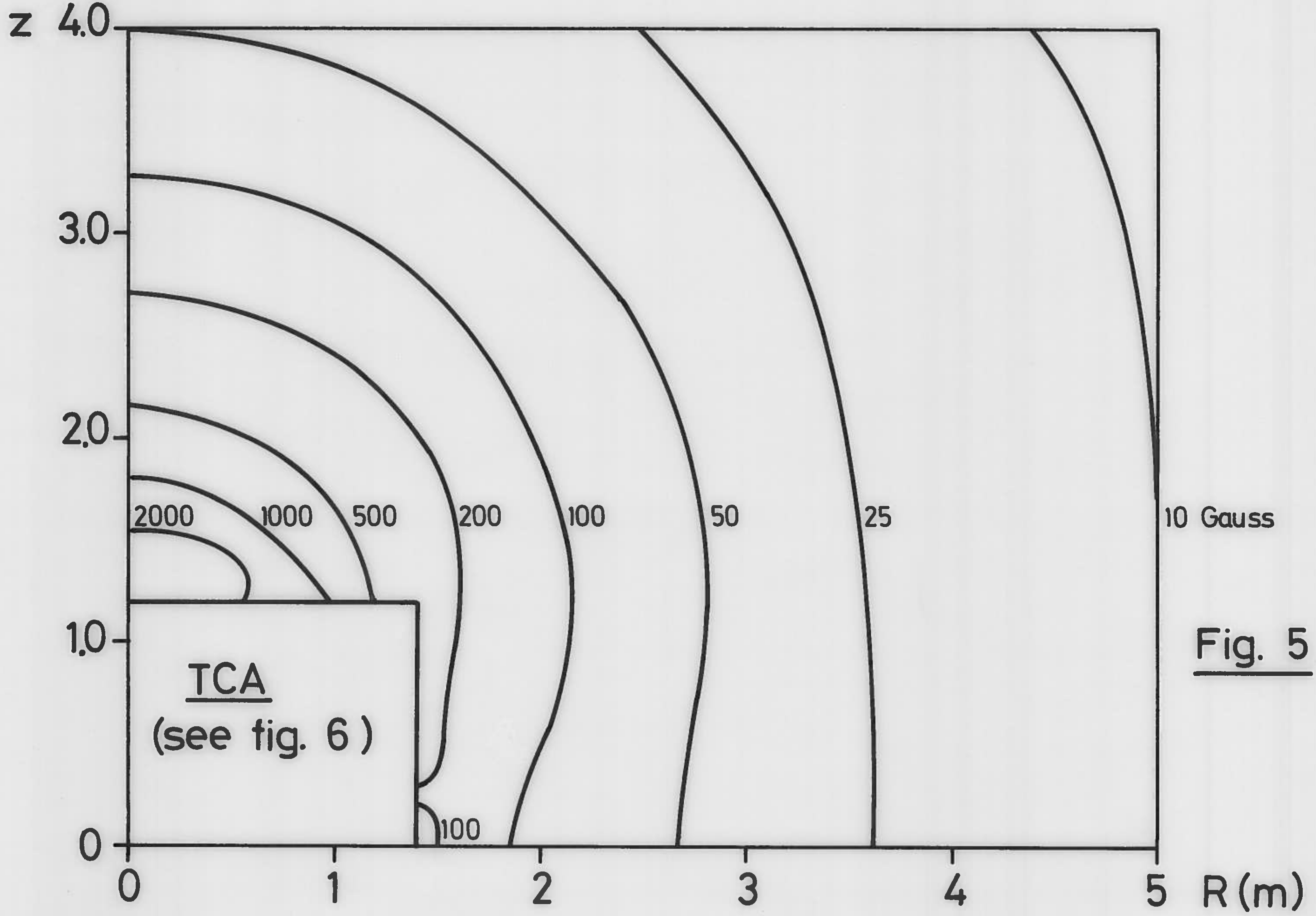
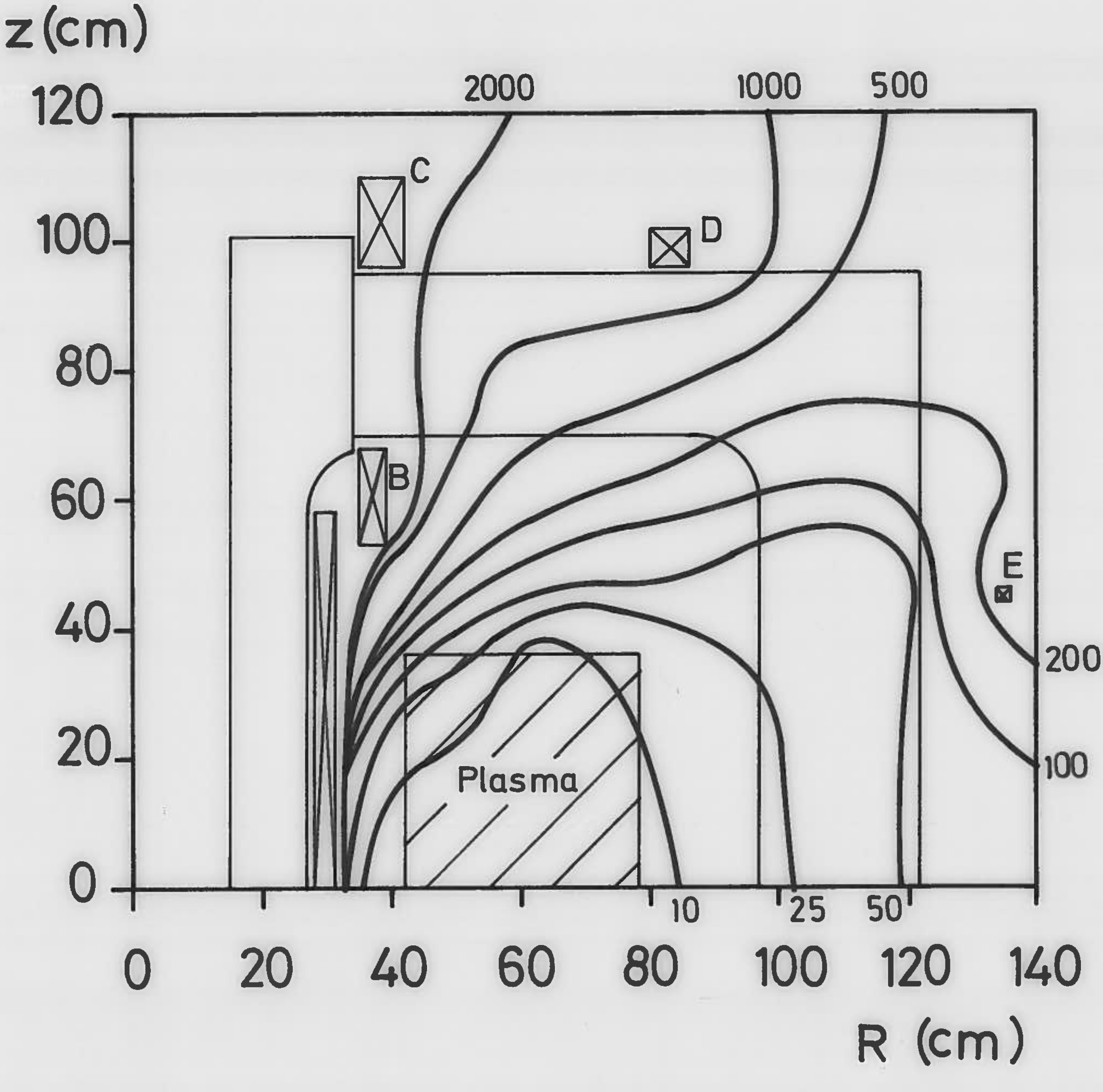

z
4.0
3.0
2.0
1.0
0
2000
1000
500
200
100
50
25
10 Gauss
100
TCA
(see fig. 6)
0
1
2
3
4
5
R (m)

Fig. 5



z (cm)
120
100
80
60
40
20
0
0
20
40
60
80
100
120
140
R (cm)
2000
1000
500
C
D
B
E
200
100
Plasma
10
25
50

Fig. 6