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TCA Toroidal Field Coil Design and Analysis

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1. Introduction

TCA is a small tokamak device, to be built at CRPP in order to study Alfvèn wave heating. Its main parameters are as follows :

Major radius	60 cm
Minor radius	18 cm
Toroidal magnetic field	10 kG
Plasma current	75 kA
Electron temperature	$\sim$ 500 eV
Ion temperature	$\sim$ 250 eV
Electron density	$\sim 2 \times 10^{13} cm^{-3}$
Energy confinement time	$\sim$ 2 ms

The purpose of this report is to present the design philosophy for the TCA toroidal field coil and to summarize the stress calculations and heat transfer analysis.

2. Design Goals

(1) The toroidal field coil should be demountable, such that the inner components of the machine (vacuum vessel, OH coil and vertical field coils) can be removed vertically {1}.

(2) The coil should be made from copper plate having sufficient structural strength to carry all induced loads {1}.

(3) The joints should be located close to the points where the bending moment is zero {1}.

(4) The net centering force on each turn of the coil should be reacted by a central fiberglass column.

(5) Out-of-plane forces (which arise from the interaction between the vertical field and the currents in the horizontal sections of the toroidal field coil {2}) should be reacted by a stainless-steel machine frame.

(6) The various sections of the coil should be cut from standard 1 x 2m copper plates, with as little waste as possible.

(7) The coil should be large enough for accommodating an elongated, TCV-like vacuum vessel, with R_o = 60 cm, a = 18 cm and b = 36cm.

(8) The cooling system should be designed in such a way that continuous operation at a rate of one shot every 5 minutes is possible.

3. Coil Dimensions

Coil dimensions are shown in Fig. 1. They were determined through an optimization procedure, taking into account the various design goals mentioned above. The coil is made from 15 mm thick copper plate. It has 72 turns, grouped in 18 bundles of 4 turns each. The maximum current is 44 kA, which produces a toroidal magnetic field of 10.5 kG at R = 60 cm.

4. Magnetic Forces

In this section, radial and axial magnetic forces, acting on one turn of the toroidal field coil, are computed. Azimuthal forces have been discussed in {2}.

The force per unit length acting on a straight current-carrying conductor, immersed in a magnetic field, is given by

$$f = \int_A (\vec{j} \times \vec{B})\, dA$$

where A is the cross-sectional area at the conductor.

<u>Beam 1 (inner vertical beam)</u>

If I is the current in one turn, I(R) is the total current flowing inside a circle of radius R and n is the number of turns, then

$$I_1(R) = n I \frac{R - \rho_1}{\rho_2 - \rho_1}$$

$$B_1(R) = \frac{\mu_0}{2\pi R} I_1(R)$$

$$j_1 = \frac{I}{A_1}$$

$$dA_1 = \frac{A_1}{\rho_2 - \rho_1} dR$$

$$f_1 = \int_{\rho_1}^{\rho_2} \left(\frac{I}{\rho_2 - \rho_1}\right)^2 \frac{\mu_0}{2\pi R} n (R - \rho_1)\, dR$$

$$= \frac{n\mu_0}{2\pi} \left(\frac{I}{\rho_2 - \rho_1}\right)^2 \left\{ \rho_2 - \rho_1 - \rho_1 \ln \frac{\rho_2}{\rho_1} \right\}$$

$$= 61627 \frac{N}{m}$$

Beam 2 (horizontal beam)

$$B_2(R,z) = B_o \frac{R_o}{R} \left(\frac{z_2 - z}{z_2 - z_1} \right)$$

where

$$B_o R_o = \frac{n \mu_o}{2\pi} I$$

$$j_2 = \frac{I}{A_2}$$

$$dA_2 = \frac{A_2}{z_2 - z_1} dz$$

$$f_2 = \int_{z_1}^{z_2} \left(\frac{I}{z_2 - z_1} \right)^2 \frac{n \mu_o}{2\pi R} (z_2 - z)\, dz = \frac{n \mu_o I^2}{4\pi R} = \frac{13939}{R} \frac{N}{m}$$

Beam 3 (outer vertical beam)

$$I_3(R) = n I \frac{\rho_4 - R}{\rho_4 - \rho_3}$$

$$B_3(R) = \frac{\mu_o}{2\pi R} I_3(R)$$

$$j_3 = \frac{I}{A_3}$$

$$dA_3 = \frac{A_3}{\rho_4 - \rho_3} dR$$

$$f_3 = \int_{\rho_3}^{\rho_4} \left(\frac{I}{\rho_4 - \rho_3} \right)^2 \frac{n \mu_o}{2\pi R} (\rho_4 - R)\, dR$$

$$= \frac{n \mu_o}{2\pi} \left(\frac{I}{\rho_4 - \rho_3} \right)^2 \left\{ \rho_4 \ln \frac{\rho_4}{\rho_3} - \rho_4 + \rho_3 \right\} = 13293 \frac{N}{m}$$

Fig. 2 shows the distribution of magnetic forces as computed above.

5. Beam Model

The basic assumptions of the beam model are as follows :

(1) The coil is symmetric with respect to the midplane ($z = 0$).

(2) The beams are connected to each other rigidly at the corners.

(3) The net radial force is reacted by a fiberglass column which exerts a force per unit length, $f_c(z)$, on beam 1.

(4) There are no external forces acting on the system, except the magnetic forces (f_1, f_2, f_3) and the reaction force (f_c).

(5) The system is free to move in all directions. There are no external constraints for the deformation, except the central column.

(6) Lengthening of the beams as a result of longitudinal forces is assumed to be negligible (the validity of this assumption will be checked in section 7.5).

The deformed system is shown schematically in Fig. 3. The lengths and radii are given in terms of the beam dimensions (see Fig. 1) as

$$R_1 = \tfrac{1}{2}(\rho_1 + \rho_2) = 0.2100 \text{ m}$$

$$R_2 = \tfrac{1}{2}(\rho_3 + \rho_4) = 1.0925 \text{ m}$$

$$\ell_1 = \ell_3 = \tfrac{1}{2}(z_1+z_2) = 0.8225 \text{ m}$$

$$\ell_2 = (R_2 - R_1) = 0.8825 \text{ m}$$

The forces F_1 and F_3 can be computed immediately by applying the radial force balance condition :

$$F_1 = F_3 = f_3 \ell_3 = 10933 \text{ N} \tag{1}$$

Similarly, the axial force balance yields

$$F_0 = F_2 \tag{2}$$

$$F_5 = F_4 \tag{3}$$

Note that the forces F_0 end F_5 must be parallel to the z-axis because of the symmetry with respect to the midplane (z=0).

The boundary conditions for the displacements are :

$$\left[\frac{d\xi_1}{dz}\right]_{z=0} = 0 \tag{4}$$

$$\left[\frac{d\xi_1}{dz}\right]_{z=\ell_1} = \left[\frac{d\xi_2}{dR}\right]_{R=R_1} = \delta_1 \tag{5}$$

$$\xi_2(R_1) = \xi_2(R_2) \tag{6}$$

$$\left[\frac{d\xi_2}{dR}\right]_{R=R_2} = \left[\frac{d\xi_3}{dz}\right]_{z=\ell_3} = \delta_2 \tag{7}$$

$$\left[\frac{d\xi_3}{dz}\right]_{z=0} = 0 \tag{8}$$

The beams which are considered here have all rectangular cross-section, hence their moments of inertia are given by {3}

$$J = \frac{ab^3}{12}$$

where a and b are the width and height of the cross-section . Inserting numerical values, we have

$$J_1 = 2.16 \times 10^{-6}\ m^4$$
$$J_2 = 18.383 \times 10^{-6}\ m^4$$

The beams are made of copper having a modulus of elasticity $E = 1.2 \times 10^{11} Nm^{-2}$.

6. Central Column

Let us assume that the central column consists of an elastic material, such that the force which it exerts on beam 1 is proportional to the displacement ξ_1,

$$f_c(z) = \alpha\, \xi_1(z) \tag{9}$$

Let us further assume that the column is a solid cylinder without any hole in the center. The stresses and strains will then be independent of radius.

The proportionality constant, α, can be computed as follows : Since the length of the column is much larger than its radius, the problem is essentially a two-dimensional one, which implies that the axial strain must be zero, $\varepsilon_z = 0$. In addition, azimuthal symmetry requires that $\varepsilon_x = \varepsilon_y = \varepsilon_R$ and $\sigma_x = \sigma_y = \sigma_R$. Under these conditions, the standard stress-strain relation {3},

$$\begin{aligned} E\varepsilon_x &= \sigma_x - \nu(\sigma_y + \sigma_z) \\ E\varepsilon_y &= \sigma_y - \nu(\sigma_z + \sigma_x) \\ E\varepsilon_z &= \sigma_z - \nu(\sigma_x + \sigma_y) \end{aligned} \tag{10}$$

where E is the modulus of elasticity and ν is Poisson's ratio, reduce to the single equation

$$E\varepsilon_R = \sigma_R(1 - \nu - 2\nu^2) \tag{11}$$

Since $\rho_i \varepsilon_R = \xi_1$ and $2\pi\rho_1 \sigma_R = nf_c$, we finally have

$$\alpha = \frac{2\pi E}{n(1-\nu-2\nu^2)} \tag{12}$$

where n is the number of turns (n = 72). It should be noted that, although eq. (12) has been derived for a solid column, it will also be approximately valid for a column with a central hole if the diameter of the hole is small compared with the outer diameter of the column.

Using typical values for epoxy, $E_e = 3.12 \times 10^{10} Nm^{-2}$ and $\nu = 0.3$ we obtain

$$\alpha = 5.24 * 10^9 \ Nm^{-2}$$

7. Bending Moments and Displacements

The bending moment M(x), the displacement $\xi(x)$ and the force distribution f(x) are related through the equations {3}

$$EJ \frac{d^4 \xi}{dx^4} = f(x) \tag{13}$$

$$EJ \frac{d^2 \xi}{dx^2} = M(x) \tag{14}$$

where E is the modulus of elasticity and J is the moment of inertia of the beam cross section, with respect to the axis of symmetry which is perpendicular to the force f(x).

7.1 Beam 1 Analysis

When eq. (13) is applied to beam 1, we obtain

$$EJ_1 \frac{d^4 \xi_1}{dz^4} = f_1(z) - \alpha \xi_1(z) \tag{15}$$

where use has been made of eq. (9). The solution of (15) which is symmetric with respect to z = 0 is given by

$$\xi_1 = \frac{f_1}{\alpha} + c_1 \cos Dz \cosh Dz + c_2 \sin Dz \sinh Dz \tag{16}$$

where

$$D \equiv \left\{ \frac{\alpha}{4EJ_1} \right\}^{1/4}$$

According to eq. (14), the bending moment is written as

$$M(z) = EJ_1 \frac{d^2 \xi_1}{dz^2}$$

$$= 2EJ_1 D^2 \left\{ -c_1 \sin Dz \sinh Dz + c_2 \cos Dz \cosh Dz \right\} \tag{17}$$

At $z = \ell_1$, we have

$$M_1 = 2EJ_1 D^2 \left\{ -c_1 \sin D\ell_1 \sinh D\ell_1 + c_2 \cos D\ell_1 \cosh D\ell_1 \right\} \tag{18}$$

and

$$\delta_1 = \left[\frac{d\xi_1}{dz} \right]_{z=\ell_1} = D \left[c_1 \left\{ \cos D\ell_1 \sinh D\ell_1 - \sin D\ell_1 \cosh D\ell_1 \right\} \right.$$

$$\left. + c_2 \left\{ \sin D\ell_1 \cosh D\ell_1 + \cos D\ell_1 \sinh D\ell_1 \right\} \right] \tag{19}$$

The radial force balance is expressed as

$$\int_0^{\ell_1} (f_1 - \alpha \xi_1)\, dz = F_1$$

which leads to

$$(c_1 - c_2) \cos D\ell_1 \sinh D\ell_1 + (c_1 + c_2) \sin D\ell_1 \cosh D\ell_1 = - \frac{2DF_1}{\alpha} \tag{20}$$

The constants c_1 and c_2 can be eliminated from eqs. (18), (19) and (20). The result is

$$EJ_1 \delta_1 = V M_1 + W \tag{21}$$

where

$$V = \frac{C_s^2 + S_c^2}{D\left\{C_s(C_c - S_s) + S_c(C_c + S_s)\right\}}$$

$$W = \frac{-F_1\left\{C_c(C_s - S_c) + S_s(C_s + S_c)\right\}}{2D^2\left\{C_s(C_c - S_c) + S_c(C_c + S_s)\right\}}$$

and

$$S_s = \sin D\ell_1 \sinh D\ell_1$$

$$S_c = \sin D\ell_1 \cosh D\ell_1$$

$$C_s = \cos D\ell_1 \sinh D\ell_1$$

$$C_c = \cos D\ell_1 \cosh D\ell_1$$

Note that in eq. (21) all quantities are known except δ_1 and M_1.

7.2 Beam 2 Analysis

Here, eq. (15) takes the form

$$EJ_2 \frac{d^4 \xi_2}{dR^4} = f_2(R) = \frac{K}{R} \tag{22}$$

where K is a constant. Integration yields

$$EJ_2 \frac{d^2 \xi_2}{dR^2} = M(R) = K\,(R \ln R - R) + c_3 R + c_4 \tag{23}$$

The constants c_3 and c_4 are determined by the boundary conditions, $M(R_1) = M_1$ and $M(R_2) = M_3$:

$$c_3 = \frac{1}{R_2 - R_1} \left\{ M_3 - M_1 - K\left(R_2 \ln R_2 - R_1 \ln R_1 - R_2 + R_1\right) \right\} \tag{24}$$

$$c_4 = \frac{1}{R_2 - R_1} \left\{ M_1 R_2 - M_3 R_1 + K R_1 R_2 \ln \frac{R_2}{R_1} \right\} \tag{25}$$

Further integration leads to

$$EJ_2 \frac{d\xi_2}{dR} = \frac{K}{2}\left(R^2 \ln R - \frac{3}{2} R^2\right) + \frac{c_3}{2} R^2 + c_4 R + c_5 \tag{26}$$

with the boundary conditions

$$EJ_2 \left[\frac{d\xi_2}{dR}\right]_{R=R_1} = EJ_2 \delta_1 = \frac{K}{2}\left(R_1^2 \ln R_1 - \frac{3}{2} R_1^2\right) + \frac{c_3}{2} R_1^2 + c_4 R_1 + c_5 \tag{27}$$

$$EJ_2 \left[\frac{d\xi_2}{dR}\right]_{R=R_2} = EJ_2 \delta_2 = \frac{K}{2}\left(R_2^2 \ln R_2 - \frac{3}{2} R_2^2\right) + \frac{c_3}{2} R_2^2 + c_4 R_2 + c_5 \tag{28}$$

Integration of (26) finally gives the displacement ξ_2

$$EJ_2\xi_2 = \frac{K}{6}\left(R^3 \ln R - \frac{11}{6}R^3\right) + \frac{c_3}{6}R^3 + \frac{c_4}{2}R^2 + c_5 R + c_6 \qquad (29)$$

where the boundary condition (6) must be applied :

$$\frac{K}{6}\left(R_1^3 \ln R_1 - \frac{11}{6}R_1^3\right) + \frac{c_3}{6}R_1^3 + \frac{c_4}{2}R_1^2 + c_5 R_1 =$$
$$\frac{K}{6}\left(R_2^3 \ln R_2 - \frac{11}{6}R_2^3\right) + \frac{c_3}{6}R_2^3 + \frac{c_4}{2}R_2^2 + c_5 R_2 \qquad (30)$$

The constant c_5 may be eliminated from eqs. (27), (28) and (30), and we obtain

$$2EJ_2(\delta_2 - \delta_1) = KR_1R_2 \ln\frac{R_2}{R_1} - \frac{K}{2}(R_2^2 - R_1^2) + (R_2 - R_1)(M_1 + M_3) \qquad (31)$$

and

$$6EJ_2(\delta_2 R_2 - \delta_1 R_1) = KR_1R_2(R_1 + R_2)\ln\frac{R_2}{R_1} - \frac{2K}{3}(R_2^3 - R_1^3)$$
$$+ \left\{M_1(R_2 + 2R_1) + M_3(2R_2 + R_1)\right\}(R_2 - R_1) \qquad (32)$$

where we have used eqs. (24) and (25) to express the constants c_3 and c_4. Note that eqs. (31) and (32) contain four unknowns δ_1, M_1, δ_2 and M_3.

7.3 Beam 3 Analysis

Again using eq. (13), we have

$$EJ_3 \frac{d^4 \xi_3}{dz^4} = -f_3 \tag{33}$$

The minus sign mus be introduced here because ξ_3 and f_3 are pointing in opposite directions (Figs. 2 and 3). Integrating twice with respect to z yields

$$M(z) = EJ_3 \frac{d^2 \xi_3}{dz^2} = -f_3 \frac{z^2}{2} + c_7 z + c_8 \tag{34}$$

Since the solution must be symmetric with respect to z = 0, we must take $c_7 = 0$. The constant c_8 may be expressed in terms of the bending moment at $z = \ell_3$:

$$-M_3 = M(\ell_3) = -f_3 \frac{\ell_3^2}{2} + c_8$$

and we obtain

$$M(z) = -f_3 \frac{1}{2} (z^2 - \ell_3^2) - M_3 \tag{35}$$

Integrating once more yields

$$EJ_3 \frac{d\xi_3}{dz} = -\frac{1}{2} f_3 \left\{ \frac{z^3}{3} - \ell_3^2 z \right\} - M_3 z + c_9 \tag{36}$$

and the boundary condition (8) requires that $c_9 = 0$.

At $z = \ell_3$, we have

$$EJ_3 \left[\frac{d\xi_3}{dz}\right]_{z=\ell_3} = EJ_3 \delta_2 = \frac{1}{3} f_3 \ell_3^3 - M_3 \ell_3 \qquad (37)$$

The displacement ξ_3 is obtained by integrating (36) :

$$EJ_3 \xi_3 = -\frac{1}{2} f_3 \left\{ \frac{z^4}{12} - \ell_3^2 \frac{z^2}{2} \right\} - M_3 \frac{z^2}{2} + c_{10} \qquad (38)$$

7.4 Solution

The four equations (21), (31), (32) and (37) can now be solved for the four unknown δ_1, M_1, δ_2 and M_3. The result is :

$$M_1 = \frac{Q_3 Q_5 - Q_6 Q_2}{Q_1 Q_5 - Q_4 Q_2} \qquad (39)$$

$$M_3 = \frac{Q_3 Q_4 - Q_6 Q_1}{Q_2 Q_4 - Q_5 Q_1} \qquad (40)$$

where

$$Q_1 = \frac{2 J_2 V}{J_1} + (R_2 - R_1)$$

$$Q_2 = \frac{2 J_2 \ell_3}{J_3} + (R_2 - R_1)$$

$$Q_3 = 2 J_2 \left\{ \frac{f_3 \ell_3^3}{3 J_3} - \frac{W}{J_1} \right\} - K R_1 R_2 \ln \frac{R_2}{R_1} + \frac{K}{2} (R_2^2 - R_1^2)$$

$$Q_4 = \frac{6J_2R_1V}{J_1} + (R_2-R_1)(R_2+2R_1)$$

$$Q_5 = \frac{6J_2R_2\ell_3}{J_3} + (R_2-R_1)(2R_2+R_1)$$

$$Q_6 = 6J_2\left\{\frac{R_2f_3\ell_3^3}{3J_3} - \frac{R_1W}{J_1}\right\} - KR_1R_2(R_1+R_2)\ln\frac{R_2}{R_1} + \frac{2K}{3}(R_2^3-R_1^3)$$

and δ_1 and δ_2 may be obtained from equations (21) and (37).

Inserting numerical values, we have

$$M_1 = 750.4 \text{ Nm}$$

$$M_3 = 2690.6 \text{ Nm}$$

$$\delta_1 = 4.67 \times 10^{-5}$$

$$\delta_2 = 11.45 \times 10^{-5}$$

The bending moment can now be computed at any position, using eqs. (17), (23) and (35). The result is shown in Fig. 4.

The displacements ξ_1, ξ_2 and ξ_3 are given by eqs. (16), (29) and (38). They are shown in Fig. 5.

7.5 Longitudinal Forces

In this section, the forces F_2 and F_4 will be computed in order to verify whether the stretching of the beams is really negligible as was assumed in section 5.

The axial force balance for beam 2 may be expressed as

$$F_2 + F_4 = \int_{R_1}^{R_2} f_2(R)\, dR = K \ln \frac{R_2}{R_1} \qquad (41)$$

and the moment balance for the point $R = R_1$ is given by

$$M_1 - M_3 + F_4 \ell_2 = \int_{R_1}^{R_2} f_2(R)\,(R - R_1)\, dR = K \left\{ R_2 - R_1 - R_1 \ln \frac{R_2}{R_1} \right\} \qquad (42)$$

Using the values of M_1 and M_3 which were computed in section 7.4, we obtain

$$F_2 = 12320 \quad N$$

$$F_4 = 10668 \quad N$$

As a result of these forces, the lengths of beams 1 and 3 will be increased by $\Delta \ell_1$, and $\Delta \ell_3$, respectively. The increments are given by

$$\Delta \ell_1 = \frac{\ell_1 F_2}{E A_1} = 4.691 * 10^{-5} \ m$$

$$\Delta \ell_3 = \frac{\ell_3 F_4}{E A_3} = 1.990 * 10^{-5} \ m$$

where A_1 and A_2 are beam cross-sections. Although these deformations are very samll, they still give rise to bending moments since the entire system is extremely rigid. The bending moments that are produced by such deformations are computed as follows : Let us consider the L-shaped system consisting of beams 2 and 3 and calculate the force F_Δ that would have to be applied at $R = R_1$, in order to produce a displacement $\Delta = \Delta\ell_1 - \Delta\ell_3$. The problem is analyzed using the equations of sections 7.2 and 7.3 and assuming $f_1 = f_3 = 0$, $M_1 = 0$, $J_2 = J_3$, $\xi_2(R_1) = \Delta$ and $\xi_2(R_2) = 0$. The result is

$$M_3' = \frac{E J_2 \Delta}{\ell_2 (\ell_3 + \frac{1}{3}\ell_2)} \tag{43}$$

and

$$F_\Delta = \frac{M_3'}{\ell_2} \tag{44}$$

Inserting the numerical values gives

$$M_3' = 60.46 \quad \text{N m}$$

$$F_\Delta = 68.51 \quad \text{N}$$

These quantities are small compared to M_3 and F_2 and, hence, the stretching of the beams can be neglected.

8. Maximum absolute stresses

Inspection of Fig. 4 shows that the critical points are the corners, at $R = R_1$ and $R = R_2$, as well as the locations of the joints where the beam cross-section is locally reduced to 30% of its normal value. The joints are located at R = 0.295 m (beam 2) and z = 0.656 m (beam 3). Let us calculate the maximum stresses at these critical points :

Corner $R = R_1$ (beam 1, because $J_1 \ll J_2$)

$M = M_1 = 750.4$ Nm
$F = F_2 = 12320$ N

$$\sigma_{max} = \frac{|M_1| b_1}{2 J_1} + \frac{F_2}{a_1 b_1} = 276.8 * 10^5 \text{ N m}^{-2}$$

Corner $R = R_2$ (beam 2, because $F_3 > F_4$)

$M = M_3 = 2690.6$ Nm
$F = F_3 = 10933$ N {eq. (1)}

$$\sigma_{max} = \frac{|M_3| b_3}{2 J_3} + \frac{F_3}{a_3 b_3} = 209.0 * 10^5 \text{ N m}^{-2}$$

Junction at z = 0.656 m (beam 3)

$M = - 1054.4$ Nm
$F = F_4 = 10668$ N
$a_4 = 0.3\ a_3 = 0.0045$ m
$J_4 = 0.3\ J_3 = 5.515 \times 10^{-6}\ m^4$

$$\sigma_{max} = \frac{|M|\, b_3}{2 J_4} + \frac{F_3}{a_4 b_4} = 331.0 \times 10^5 \frac{N}{m^2}$$

The junction at R = 0.295 m is not critical because the bending moment is much smaller than the one computed above, and $J_2 = J_3$.

The yield strength of cold rolled copper is

$$\sigma_y \cong 1800 * 10^5 \; N\,m^{-2}$$

According to BBC experts {4}, a safety factor of at least 3 must be assumed, so that

$$\sigma_{max} = 600 * 10^5 \; N\,m^{-2}$$

We conclude that the stresses in the copper (for B_o = 10.5 kG) are about a factor of 2 below the maximum allowable limit. Consequently, the coil should be able to operate at a higher magnetic field, i.e. up to B_o = 15 kG.

9. Heat transfer

9.1 Necessity of water cooling

Assuming one shot every 5 minutes, the average power dissipation in the toroidal field coil is :

$$P = 60\,kW \tag{45}$$

The coil consists of 9.96 tons of copper having a total heat capacity, Q_c = 3.84 MJ/^{o}C. If the coil is not cooled, its temperature will increase at a rate of

$$\frac{dT_c}{dt} = \frac{P}{Q_c} = 56\ ^\circ C/hour$$

This cannot be tolerated since the maximum operating temperature of the fiberglass insulation is 150°C. If the coil is cooled by the ambient air, the air temperature will increase at a rate of

$$\frac{dT_A}{dt} = \frac{P}{Q_A}$$

where Q_A is the heat capacity of the air in the room, and T_c = const. has been assumed. The room measures 14.4 x 28.8 x 7.5 m; it contains 4.02 tons of air with a total heat capacity, Q_A = 4.06 MJ/$^\circ$C. The rate of increase of the air temperature is therefore

$$\frac{dT_A}{dt} = 53\ ^\circ C/hour$$

which, of course, cannot be tolerated either. We conclude that the coil must be water-cooled. For this purpose, grooves are cut into the edges of the copper plates and copper tubes are fitted into the grooves. The inside diameter of the copper tubes is D = 6 mm and the length for one turn of the coil is L_T = 4 m. The total surface area available for cooling is therefore given by

$$A = \pi D L_T n = 5.43\ m^2 \qquad (46)$$

where n is the number of turns (n = 72).

9.2 Water Inlet Temperature

Condensation of atmospheric moisture onto the coil must be avoided under any circumstances, since this might produce a short circuit. The water inlet temperature must therefore always be above room temperature. On the other hand, the coil should be kept as cool as possible, since its electrical resistance increases with temperature. We assume

$$T_{in} = 30\,^{\circ}C \tag{47}$$

9.3 Reynolds Number

The flow in a tube can be laminar or turbulent, depending on the value of the Reynolds number, defined as

$$Re = \frac{V\,D}{\nu} \tag{48}$$

where D is the diameter of the tube, V is the velocity and ν the kinematic viscosity of the fluid. If Re $\lesssim$ 2000, the flow is laminar and if Re $\gtrsim$ 12000, it is turbulent. The heat transfer between the tube wall and the fluid may be characterized by the Nusselt number {5},

$$Nu = \frac{P\,D}{A\,\Delta T\,k} \tag{49}$$

where P/A is the thermal power per unit area, ΔT is the difference between wall and fluid temperatures, and k is the thermal conductivity of the fluid. For laminar flow {5} and constant P/A,

$$Nu = 4.36 \tag{50}$$

For fully developed turbulent flow {5},

$$Nu = 0.023 \; Re^{0.8} \; Pr^{1/3} \tag{51}$$

where Pr is the Prandtl number. Pr, k and ν depend on temperature; values for water {6} are given in the table, below :

Temperature {°C}	Pr	k {W/m °C}	ν {m^2/sec}
0	13.67	0.5642	1.79×10^{-6}
20	7.01	0.5981	1.01×10^{-6}
40	4.35	0.6274	0.656×10^{-6}
60	3.00	0.6516	0.469×10^{-6}

Using the values of P, A and D as given in section 8.1, and assuming $T_{water} = 40^{o}C$ and $Re_{turb} = 12000$, we find

$$\Delta T_{lam} = 24.2 \; {}^{o}C$$

and

$$\Delta T_{turb} = 1.5 \; {}^{o}C$$

The advantage of the turbulent flow is evident and we assume

$$Re = 12000 \tag{52}$$

9.4 Flow Rate, Pumping Power and Copper Temperature

For a given Reynolds number, the fluid velocity, V, is obtained from (48). The flow through one channel is given by

$$\Phi_1 = \frac{\pi}{4} D^2 V \tag{53}$$

Assuming that N turns are connected in series, the length of one channel is given by

$$L = N L_T \tag{54}$$

where L_T is the length of tubing in one turn (L_T = 4m). The total flow of water is then

$$\Phi = \Phi_1 \frac{n}{N} \tag{55}$$

where n is the nuber of turns (n=72). The temperature rise, (T_{out} - T_{in}), is related to the total flow by

$$T_{out} - T_{in} = \frac{P}{\mu \rho \Phi} \tag{56}$$

where μ and ρ are the specific heat and the density of water, respectively. The pressure drop, Δp, between inlet and outlet is given {5} in terms of the friction factor λ :

$$\Delta p = \frac{2 \lambda L \rho V^2}{D} \tag{57}$$

λ depends on the surface roughness and the Reynolds number. For our conditions, $\lambda \approx 0.011$. Finally, the power which is necessary to pump the cooling water through the coil, is given by

$$P_p = \Phi \, \Delta p \tag{58}$$

Using equations (53) - (58) and assuming various N's, we obtain the results shown in the table below :

N	L {m}	Φ {lit/sec}	$T_{out}-T_{in}$ {^{o}C}	Δ_p {atm}	P_p {Watts}	$\frac{T_{out}+T_{in}}{2}$ {^{o}C}^{2}
1	4	2.67	5.4	0.25	67	32.7
2	8	1.34	10.7	0.50	67	35.3
4	16	0.67	21.5	1.01	67	40.7
8	32	0.33	43.0	2.02	67	51.5

The last column shows the average water temperature, assuming $T_{in} = 30^{o}$C (see section 8.2). The average copper temperature is then obtained by adding the various ΔT's :

$$\bar{T}_{Cu} = \bar{T}_{H_2O} + \Delta T_{turb} + \Delta T_1 + \Delta T_2 \tag{59}$$

ΔT_{turb} has been computed in section 8.3. ΔT_1 is the temperature difference between the inner wall of the tube and the edge of the copper plate, and ΔT_2 is the difference between the temperature at the edge of the plate and the average copper temperature. ΔT_1 and ΔT_2 were measured on a prototype section of the coil; they are 5.0°C and 6.0°C, respectively. It follows that

$$\overline{T}_{Cu} = \overline{T}_{H_2O} + 12.5\ °C \tag{60}$$

It is clear that all the options given in the table are feasible. However, the best compromise, as far as flow rate and copper temperature is concerned, seems to be the case N = 2. We assume therefore

N	=	2
Φ	=	1.34 liters/sec
T_{in}	=	30°C
T_{out}	=	40.7°C
Δp	=	0.5 atm.
P_p	=	67 Watts
$\overline{T}_{Cu}$	=	47.8°C

10. References

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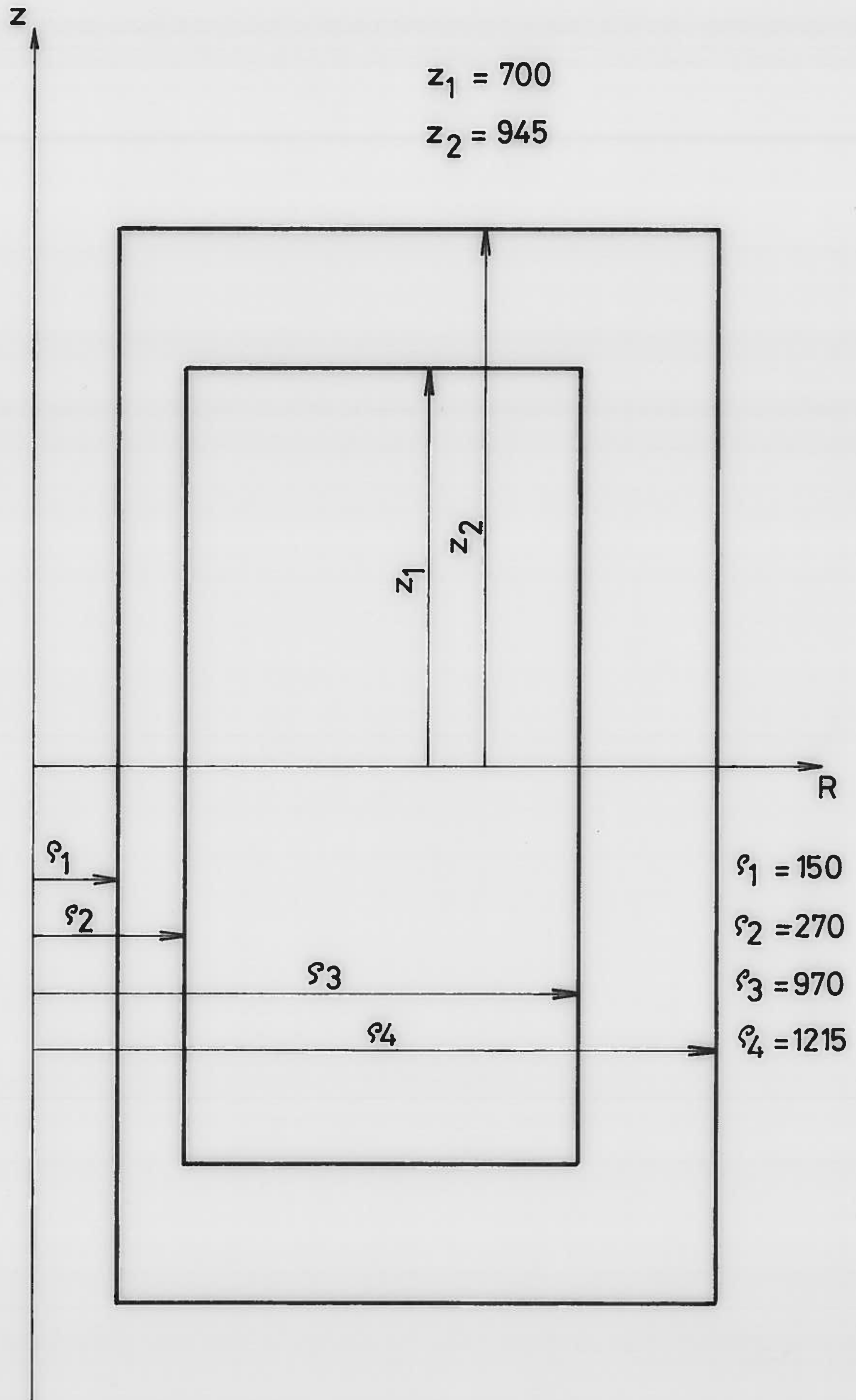

FIG.1 : COIL DIMENSIONS (in mm)

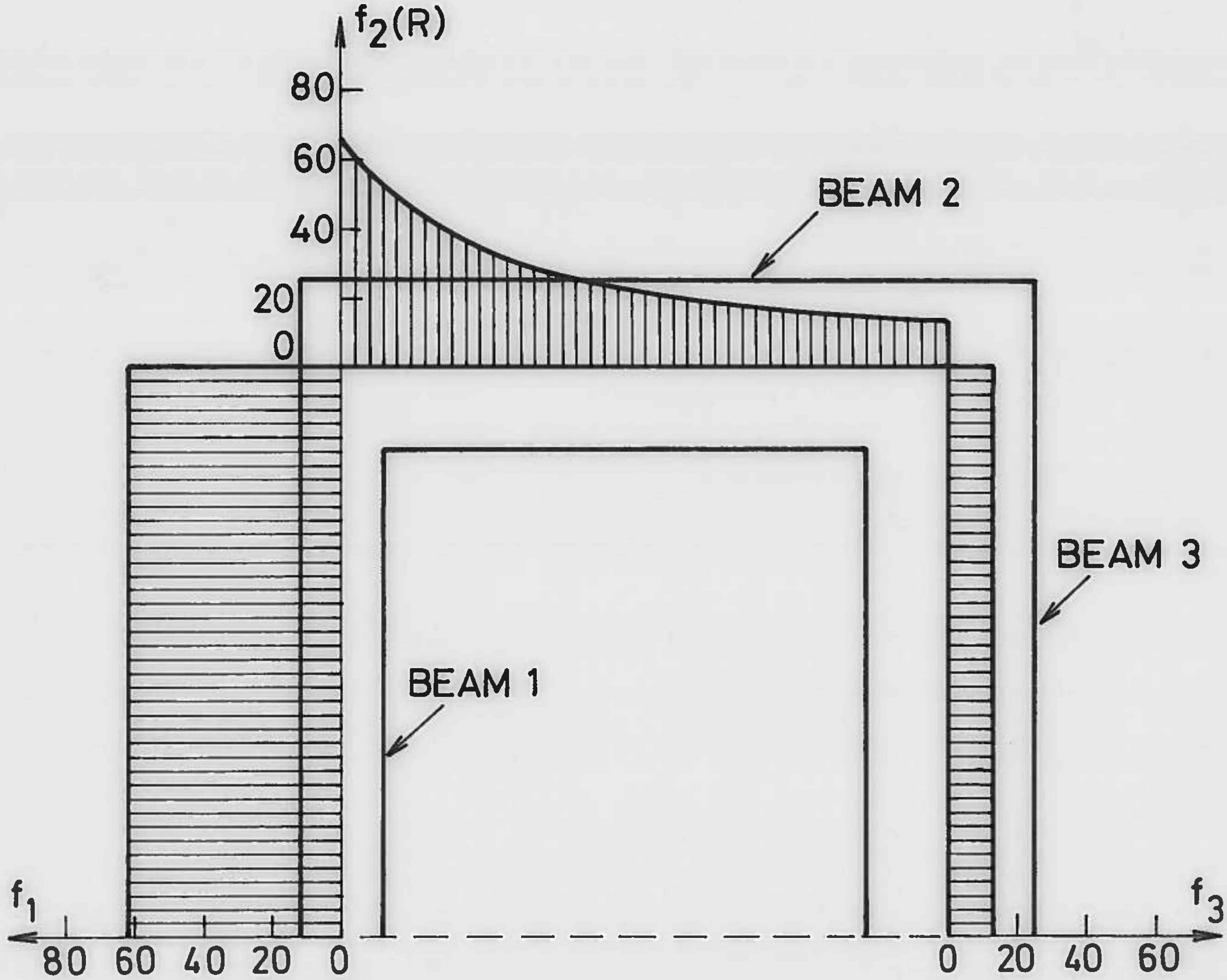

FIG.2: MAGNETIC FORCES (in kN/m)

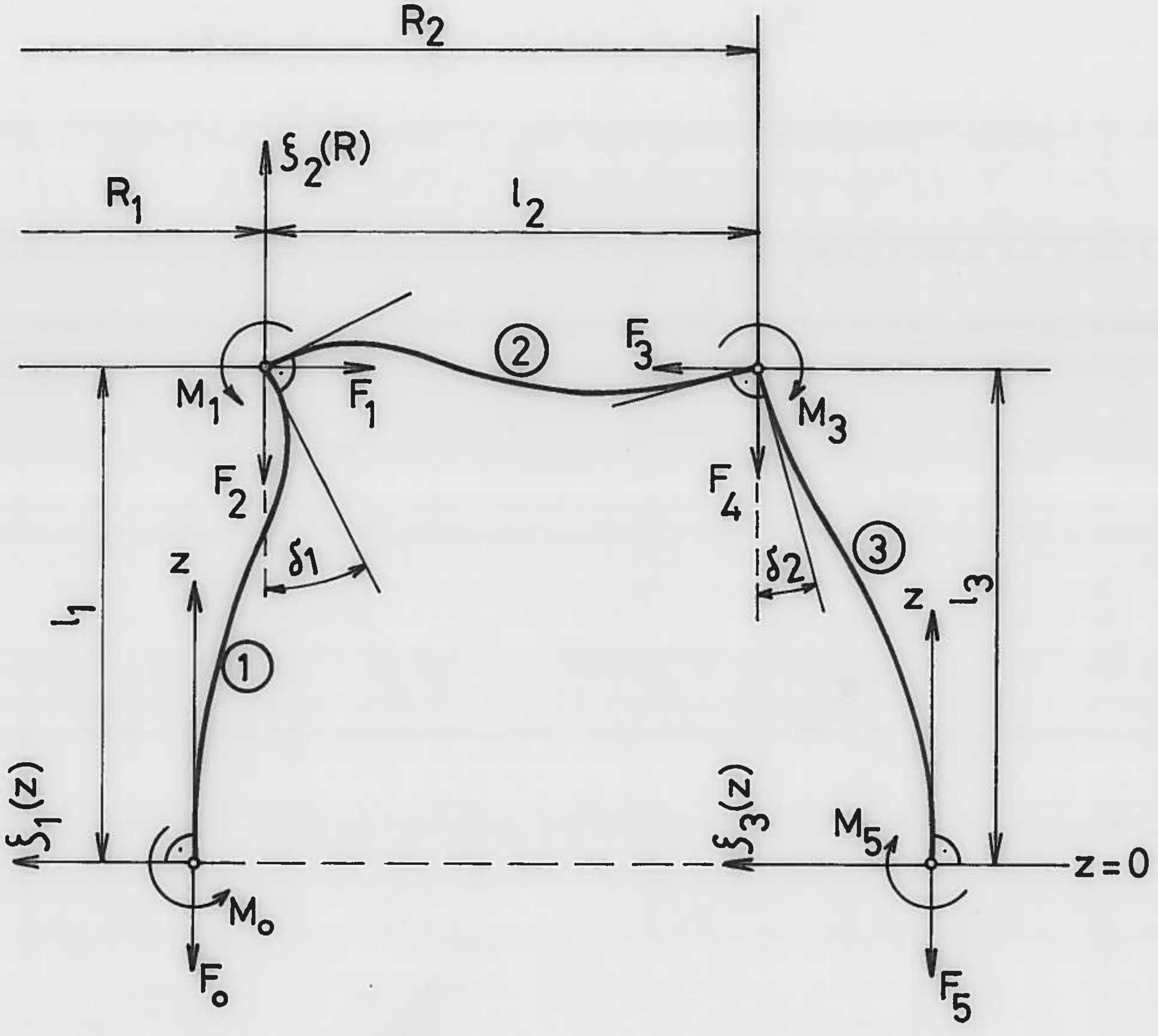

FIG.3: BEAM MODEL

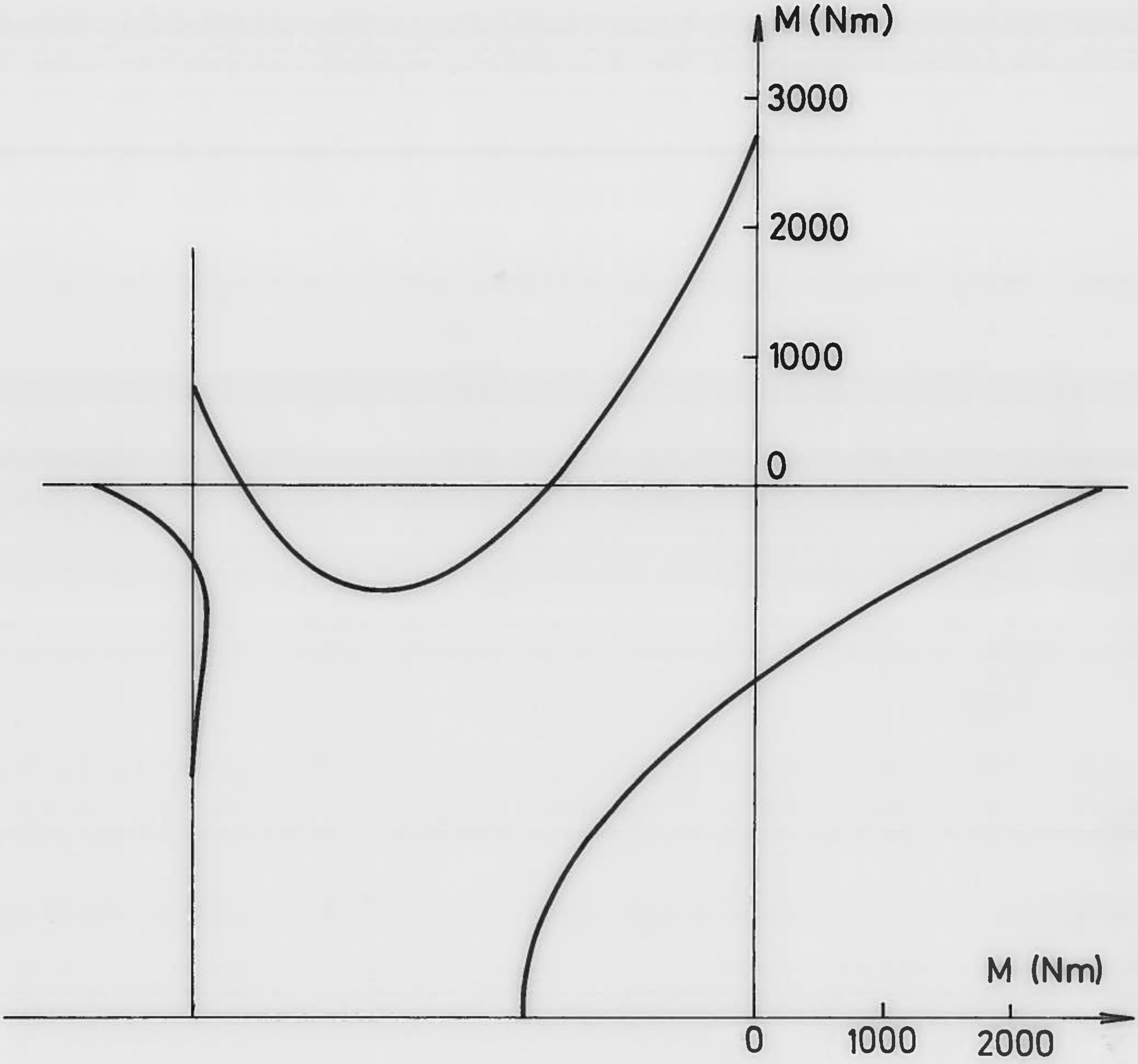

FIG.4:

IN-PLANE BENDING MOMENT

ONE TURN, I=44kA, B =10.5kG

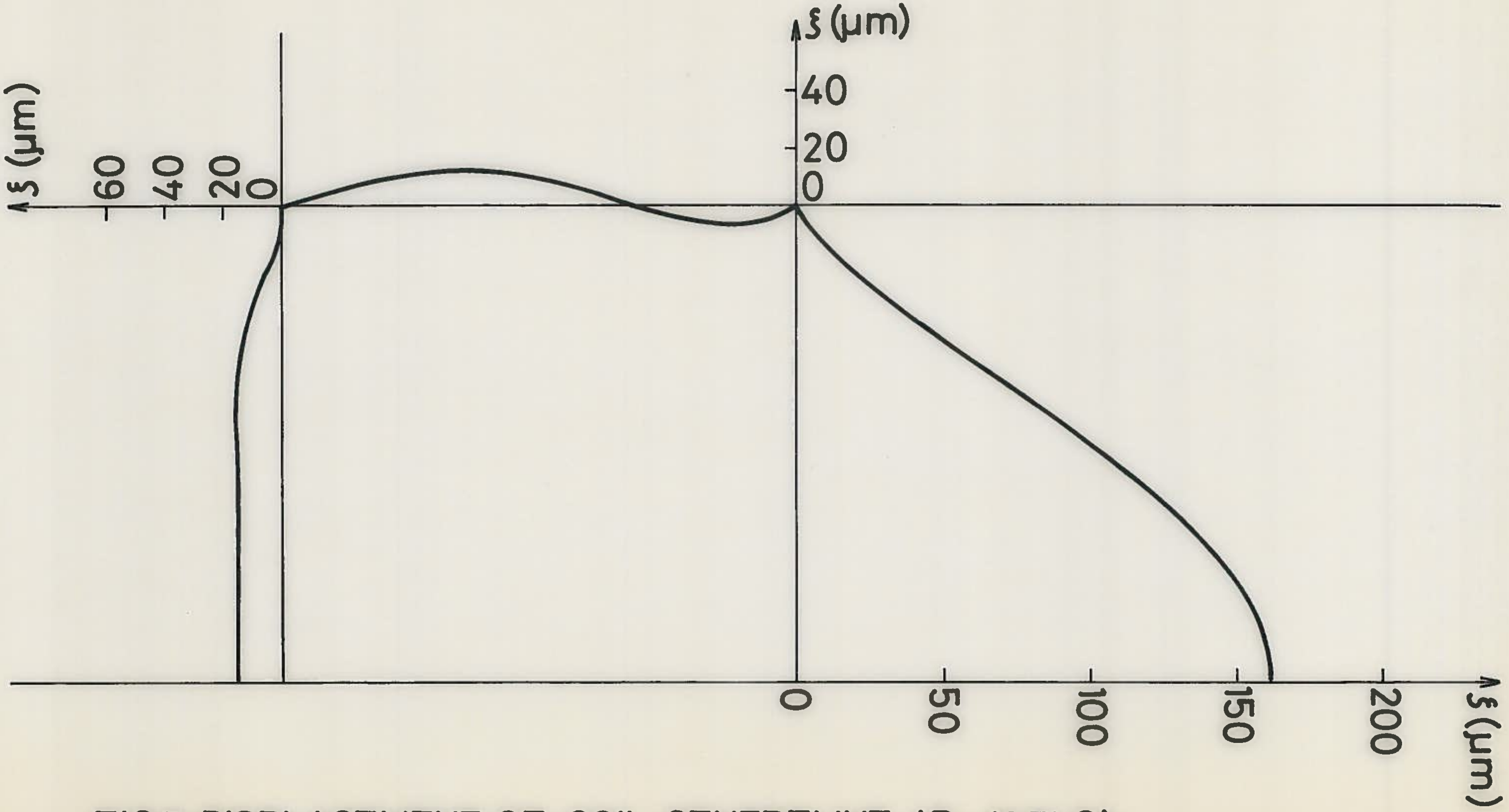

FIG.5: DISPLACEMENT OF COIL CENTRELINE (B_o=10.5kG)