Robust Tracking Commitment with Application to Demand Response

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Abstract

Many engineering problems that involve hierarchical control applications, such as demand side ancillary service provision to the power grid, can be posed as an *optimal tracking commitment problem*. In this setting, the lower-level controller commits a set of possible reference trajectories over a finite horizon to an external entity, which requires guaranteed tracking of any reference trajectory that can be sampled from the committed set, with an allowed deviation, in exchange for a payment corresponding to the size of the reference set. This paper presents a method to solve the optimal tracking commitment problem for constrained linear systems subject to uncertain disturbance and reference signals. The proposed method allows tractable computations via convex optimization for conic representable reference sets and lends itself to distributed solution methods. We demonstrate the proposed method in a simulation based case study with a commercial building that offers a frequency regulation service to the power grid.

Index Terms

tracking, hierarchical control, distributed control, robust control, service commitment, ancillary service, demand response

I. Introduction

REGULATION of large and complex systems that include many agents is usually handled with several control layers that interact in a hierarchical fashion in order to break down the complex control task into simpler sub-tasks. Frequency regulation of the power grid is a good example of such a setting, where an upper level controller, run by the grid operator, sends reference signals to the lower level subsystems, reserve providers, which are expected to track the reference within an acceptable error bound. For the upper level controller, it is crucial to know the tracking capability of the subsystems. In the power grid, this information is provided to the grid operator in terms of a reserve commitment, which represents the set of robustly trackable trajectories by the lower level subsystem for a specific time window [1].

In today's market the reserves are usually described by simple limits on the maximum power and ramp rates which are available for power plants [2]. However, the integration of demand response and storage elements in frequency regulation impose several challenges on the commitment problem: First, more complicated reserve descriptions that incorporate energy limitations and time variations are required in order to capture fundamental characteristics of these new reserve providers [3]. Second, the assessment of the optimal reserve capacity, *i.e.* the uncertainty in the power consumption that the service provider can accommodate, is far from trivial to determine for a complex subsystem such as a commercial office building that is not designed for demand response and restrained by its primary objectives and external disturbances such as weather conditions and occupancy levels.

In this paper, motivated by the reserve commitment problem for demand response, we pose the robust tracking commitment problem, which corresponds to the assessment of a subsystem's tracking capability for a finite horizon and optimal commitment of this capability to an upper control layer, considering the subsystem's overall cost of operation and feasibility. The tracking capability is expressed in terms of a set of possible reference trajectories the system can track robustly, that is, not violating internal constraints and always staying close to the reference trajectory within an allowed error set, while being subjected to external disturbances. For a fixed reference set, considering that both the reference and disturbance are

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external uncertainty sources, the ability of the system to track any reference from this set can be assessed by means of a general robust control problem [4], [5]. The peculiarity of the commitment problem comes from the freedom in the choice of uncertainty sets that are admissible for robust tracking, and which therefore need to be optimized as well. In this paper, we propose a methodology to solve the robust tracking commitment problem in a tractable manner, by means of convex optimization. Our method is based on linear decision rules [6] together with modifier functions over sets and can be used for optimal sizing of reference sets while simultaneously guaranteeing robust tracking.

Finite horizon robust control for linear systems is well established in the model predictive control literature [7], however, the related work is mainly concerned with obtaining a control law that guarantees robust feasibility and stability under a given uncertainty set. Available methods mainly rely on choosing a nominal trajectory and a control policy that will keep the system around the nominal trajectory under the effect of the uncertainty [8], [9], [10]. The use of a closed-loop control policy reduces the conservatism considerably compared to open-loop robust policies. The control policies are often selected to be affine [10] as they lead to tractable computations that rely on convex optimization [6]. The work of [11] generalizes these linear policies by considering linear combinations of basis functions, a possibility also shown by [6].

An illustrative example of robust model predictive control for the operation of the power grid can be found in [12], where the authors allocate reserves while considering temporal correlation of the demand-generation forecast and assuming the forecast error to belong to a polytopic set defined over a finite prediction horizon. However, the uncertainty set is fixed prior to allocation, therefore the authors do not address the problem of assessing disturbance rejection capabilities of a given reserve fleet.

The commitment problem, on the other hand, requires searching over uncertainty sets that the system can accommodate rather than guaranteeing robustness against a fixed uncertainty set. The interest in this problem has peaked in recent years due to the prospect of Demand Response applications [13], where the reference set to be tracked that represents the reserve capacity of the system is not given, but should be computed by the service provider. In [5] the problem was posed in the robust model predictive control context, and simple up-down flexibility of a single actuator was optimized. [14] and [15] consider aggregation of several subsystems to track a reference signal and optimize maximum up-down limits on the reference, however the robust formulation is again limited either to single dedicated actuators or predetermined schemes that distributes the required change in the total power consumption among actuators. [16] considers reference sets that are norm balls and optimizes over linear mappings to modify the uncertain reference set utilizing dual norm formulations. However, in all aforementioned works, the authors do not consider the time correlation that will be present in the uncertain reference signal. This problem is tackled in our previous work [4], where we consider polytopic uncertainty sets defined over the whole prediction horizon and formulate a method for modification of the uncertainty set via linear maps. A similar work is [17], where the authors propose optimizing over a linear map to be applied to a polytopic reference set that represents energy constraints in frequency regulation signals.

Another related problem is the so called *output regulation*, which deals with the capability of the system to track a reference trajectory that is generated by an external dynamical system [18]. In the finite horizon framework, the external system serves as a generator for the reference trajectory set. Most of the work in output regulation deals with asymptotic tracking guarantees [18], [19], [20]. However, for the finite horizon tracking commitment context, it is necessary to guarantee tracking during the whole commitment period, which includes transients. The authors of [21] utilize robust invariant sets to guarantee tracking with specified error bounds during and after the finite prediction horizon, however the guarantees are sought for a given reference generator under the assumption that there exists a feasible solution to the problem.

The contribution of this paper is threefold: First, we formulate a general robust tracking commitment problem for linear systems that allows computation of optimal reference sets to be committed for guaranteed tracking with error bounds under additive state disturbance. Second, we propose a method that allows implicit modulation of conic representable uncertainty sets via modifier functions, which leads to tractable formulations for the problem of searching over admissible uncertainty sets and corresponding control policies simultaneously. Finally, we establish sufficient conditions on the modifier functions that ensure causality of the obtained control policies according to the available information on the uncertainty.

The paper is organized as follows. Section II lays out the problem formulation for constrained tracking under uncertainty and introduces information structures. Section III defines the robust tracking commitment problem, proposes a method for solving the problem in a computationally tractable manner with implicit modulation of uncertainty sets and closes with discussions of properties of the tractable problem formulation. Finally, Section IV illustrates our results with a demand response application.

Notation: $\mathbb{R}^n$ denotes the Euclidean space of dimension n, and $\mathbb{Z}$ denotes the set of integers. For two integers $i \in \mathbb{Z}$ and $j \in \mathbb{Z}$ such that $i < j$, let $\mathbb{Z}_{[i,j]} := \{i, i+1, \dots, j\}$. I_n denotes the identity matrix of dimension n and $\otimes$ denotes the Kronecker product. For a matrix $\mathrm{M} \in \mathbb{R}^{n \times m}$, an integer $i \in \mathbb{Z}_{[1,n]}$ and a set $\mathcal{J} \subseteq \mathbb{Z}_{[1,m]}$, $\mathrm{M}(i, \mathcal{J})$ indicates the set of components that belong to the ith row and columns for which the indices belong to $\mathcal{J}$. For a set $\mathcal{Q} \subseteq \mathbb{R}^n \times \mathbb{R}^m$, the orthogonal projection operator is defined as $\mathrm{Proj}_x(\mathcal{Q}) := \{x \in \mathbb{R}^n | \ \exists y \in \mathbb{R}^m, (x, y) \in \mathcal{Q}\}$. Given two functions $f : \mathbb{R}^n \to \mathbb{R}^m$ and $g : \mathbb{R}^m \to \mathbb{R}^l$, $f \circ g : \mathbb{R}^n \to \mathbb{R}^l$ denotes the composition of f and g , such that $f \circ g(x) = f(g(x))$.

II. Problem formulation

A. *Constrained tracking under uncertainty*

Consider the linear uncertain system

$$\begin{aligned} x_{k+1} &= Ax_k + Bu_k + w_i \\ y_k &= Cx_k + Du_k \end{aligned} \tag{1}$$

with constrained state and inputs $(x, u) \in X \times U \subseteq \mathbb{R}^{n_x} \times \mathbb{R}^{n_u}$, disturbance $w \in \mathbb{R}^{n_x}$ and output $y \in \mathbb{R}^{n_y}$. The sets X and U are assumed to be polytopic and to contain the origin.

Given that the system is in state x at time 0, the input sequence $\boldsymbol{u} = (u_0, \dots, u_{N-1})$ is applied, and the disturbance sequence $\boldsymbol{w} = (w_0, \dots, w_{N-1})$ is observed, the state at time i is denoted as $\phi_i(x, \boldsymbol{u}, \boldsymbol{w})$, and the resulting sequence of states $(\phi_1(x, \boldsymbol{u}, \boldsymbol{w}), \dots, \phi_N(x, \boldsymbol{u}, \boldsymbol{w}))$ as $\boldsymbol{\phi}(x, \boldsymbol{u}, \boldsymbol{w})$. Note that $\boldsymbol{\phi}$ is a linear function of its arguments. The output of the system over the horizon is also a linear function, and is denoted $\boldsymbol{y}(x, \boldsymbol{u}, \boldsymbol{w}) := (Cx_0 + Du_0, C\phi_1 + Du_1, \dots, C\phi_{N-1} + Du_{N-1})$, where the arguments have been removed for brevity.

The goal is to determine if an admissible control law exists such that the output of system (1) can be made to robustly track any signal within a given reference set over a finite horizon. To this end, we define a *reference set* $\mathcal{R} \subset \mathbb{R}^{Nn_y}$ and a *disturbance set* $\mathcal{W} \subset \mathbb{R}^{Nn_x}$ which represent possible reference and disturbance signals over the entire horizon of length N and assumed to be compact. Note that the reference and disturbance sets allow for coupling along the horizon which provides a critical flexibility that will be exploited in the application studied in Section IV.

Given a reference signal $r \in \mathbb{R}^{n_y}$ at time k, the tracking error is denoted as $e_k = y_k - r_k$ and the error over the horizon as

$$\boldsymbol{e} := \boldsymbol{r} - \boldsymbol{y}(x, \boldsymbol{u}, \boldsymbol{w}) \tag{2}$$

where $\boldsymbol{r} := (r_0, \dots, r_{N-1})$ is the reference trajectory. We further define the *tracking error set* $\mathcal{E} \subset \mathbb{R}^{Nn_y}$. The objective of the control action is to maintain the difference between the output of the system and the reference signal, that is $\boldsymbol{e}$, within the set $\mathcal{E}$.

We can now define the set of input, reference and disturbance sequences that satisfy the system and tracking constraints over an $N-$step horizon

$$\mathcal{Q}(x) := \{(\boldsymbol{u}, \boldsymbol{r}, \boldsymbol{w}) \mid \boldsymbol{\phi}(x, \boldsymbol{u}, \boldsymbol{w}) \in \mathcal{X}, \ \boldsymbol{u} \in \mathcal{U}, \ \boldsymbol{e} \in \mathcal{E}\} \tag{3}$$

where $\mathcal{X} := X^N$ and $\mathcal{U} := U^N$. As seen from the definition, the feasibility set $\mathcal{Q}$ is parameterized by the initial condition x of the system. In the following, we will drop the argument of $\mathcal{Q}$ for notational simplicity.

Remark 1. *The reference is only defined over the finite horizon and we do not discuss tracking guarantees for the infinite horizon case. Note that any terminal condition that would ensure robust invariance under a persistent disturbance after the finite horizon can easily be added to the set $\mathcal{Q}(x)$ without changing its structure. We refer the reader to [22] for computation of positively robust invariant sets under time invariant disturbance sets, to be used as the terminal constraint.*

Note that both the reference $\boldsymbol{r}$ and disturbance $\boldsymbol{w}$ are exogenous uncertain signals for the system. From the point of view of the controller, the main difference between these two is the time they are observed by the controller. This causality condition can be easily incorporated in the control policy to be applied over the horizon as we will show in the following sections. Therefore, we define $\boldsymbol{\xi} = (\boldsymbol{r}, \boldsymbol{w}) \in \mathbb{R}^{n_\xi}$ as the unified uncertain signal.

Let the map $\pi_k : \mathbb{R}^{Nn_\xi} \to \mathbb{R}^{n_u}$ be the control policy to be used at time step k. The control policy sequence for the finite horizon can be defined as

$$\boldsymbol{\pi}(\boldsymbol{\xi}) = (\pi_0(\boldsymbol{\xi}), \ldots, \pi_{N-1}(\boldsymbol{\xi})) \tag{4}$$

We further define the general uncertainty set as

$$\Xi = \mathcal{R} \times \mathcal{W} \ .$$

We can now define the set of all admissible finite-horizon control policies mapping from disturbance and reference sequences to input sequences

$$\Delta(\Xi) := \{\boldsymbol{\pi} : \Xi \to \mathbb{U} \mid \forall \boldsymbol{\xi} \in \Xi,\ (\boldsymbol{\pi}(\boldsymbol{\xi}), \boldsymbol{\xi}) \in \mathcal{Q}\} \tag{5}$$

Given the feasibility and tracking conditions and the uncertainty set, a controller using a policy that belongs to the set $\Delta(\Xi)$, starting from the state x, can keep the tracking error within the set $\mathcal{E}$ throughout the finite horizon for any realization of the disturbance $\boldsymbol{w}$ and the reference $\boldsymbol{r}$. The existence of such control policies is not guaranteed: if the system constraints and the tracking set is too restrictive or the uncertainty sets are too large, then it may not be possible for any controller to satisfy system feasibility and tracking requirements simultaneously.

Let us now characterize the uncertainty sets that allow existence of admissible control policies for tracking. This characterization will be instrumental in the following sections, when we optimize over reference sets that the system can track robustly.

Definition 1. *The set $\Xi \subset \mathbb{R}^{Nn_\xi}$ is admissible for tracking by system* (1) *in state x if*

$$\Delta(\Xi) \neq \emptyset \ . \tag{6}$$

The following lemma provides more insight into the geometry of the admissibility condition:

Lemma 1. *The set $\Xi \subset \mathbb{R}^{Nn_\xi}$ is admissible for tracking by system* (1) *in state x if and only if:*

$$\Xi \subseteq \mathit{Proj}_{\boldsymbol{\xi}}(\mathcal{Q}) \tag{7}$$

where $\mathit{Proj}_{\boldsymbol{\xi}}(\mathcal{Q})$ denotes the projection of the set $\mathcal{Q}$ onto the $\boldsymbol{\xi}$ subspace.

Proof. : The proof directly follows from the definition of the projection operator and the definition of $\Delta(\Xi)$. Suppose that $\Xi \subseteq \mathrm{Proj}_{\boldsymbol{\xi}}(\mathcal{Q})$, we have that $\forall \boldsymbol{\xi} \in \Xi,\ \exists \boldsymbol{u} : (\boldsymbol{u}, \boldsymbol{\xi}) \in \mathcal{Q}$. This indicates the existence of a function $\boldsymbol{\pi}$, which maps every $\boldsymbol{\xi} \in \Xi$ to a feasible $\boldsymbol{u} = \boldsymbol{\pi}(\boldsymbol{\xi})$, such that $(\boldsymbol{\pi}(\boldsymbol{\xi}), \boldsymbol{\xi}) \in \mathcal{Q}$, and hence $\Delta(\Xi) \neq \varnothing$. Conversely, suppose that $\Delta(\Xi) \neq \varnothing$ and let $\boldsymbol{\pi} \in \Delta(\Xi)$. By definition of $\Delta(\Xi)$, $\forall \boldsymbol{\xi} \in \Xi$, $\exists \boldsymbol{u}$ such that $\boldsymbol{u} = \boldsymbol{\pi}(\boldsymbol{\xi})$ and $(\boldsymbol{u}, \boldsymbol{\xi}) \in \mathcal{Q}$, and hence $\Xi \subseteq \mathrm{Proj}_{\boldsymbol{\xi}}(\mathcal{Q})$.

□

Lemma 1 also illuminates a method of testing the admissibility of a given uncertainty set for robust tracking, by means of verifying set inclusion.

B. Information structure of control policies

The control policy $\boldsymbol{\pi}$ should account for the fact that the uncertain exogenous signals are revealed partially to the controller as time progresses. Generally speaking, any decision variable u_k might depend on a subset of the uncertainty vector $\boldsymbol{\xi}$ and only on this subset. To make this claim more precise, the concept of the information structure of a function f is introduced. The presentation follows concepts from Section 14.2 of [6] but adopts a different presentation.

Definition 2. *Let $\mathcal{I}$ be a subset of $\{1, 2, \ldots, n\}$, and*

$$\mathcal{F}(\mathcal{I}) = \{f : \mathbb{R}^n \to \mathbb{R} \mid x_{\mathcal{I}} = \hat{x}_{\mathcal{I}} \Rightarrow f(x) = f(\hat{x})\} \tag{8}$$

where $x_{\mathcal{I}}$ denotes the entries of x defined by the indices of $\mathcal{I}$.
Let $\boldsymbol{\mathcal{I}} = (\mathcal{I}_k)_{k\in\mathbb{Z}_{[1,m]}}$ be a collection of index subsets and:

$$\mathcal{F}(\boldsymbol{\mathcal{I}}) = \{f : \mathbb{R}^n \to \mathbb{R}^m,\ f_k \in \mathcal{F}(\mathcal{I}_k)\ \forall k \in \mathbb{Z}_{[1,m]}\} \tag{9}$$

If $f \in \mathcal{F}(\boldsymbol{\mathcal{I}})$, then we refer to $\boldsymbol{\mathcal{I}}$ as the information structure of f.

Loosely speaking, $\mathcal{F}(\mathcal{I})$ denotes the set of real-valued functions that depend only on the input indexed in $\mathcal{I}$. For functions with multiple outputs, the information structure is defined output-wise. $\boldsymbol{\mathcal{I}}$ summarizes the information structure of the function f: the k^{th} component of f depends only on inputs indexed in $\mathcal{I}_k$. For example, in the robust multi-stage control setting considered here, a typical requirement of the control policy will be causality (also called non-anticipativity) which states that the current control action can depend on observations made in the past only: this translates in our notation to the fact that for each stage, every control action can depend on past measurements, so that $\pi_k \in \mathcal{F}(\mathcal{I}_k)$ with $\mathcal{I}_k = \{1, \ldots, k-1\}$. Notice here a small abuse of notation in the sense that π_k is a function with values in $\mathbb{R}^{n_u}$, and by $\pi_k \in \mathcal{F}(\mathcal{I}_k)$. We mean that every component of π_k is in $\mathcal{F}(\mathcal{I}_k)$.

Definition 3. *The set $\Xi \subset \mathbb{R}^{Nn_\xi}$ is causally admissible for tracking by system* (1) *in state x with respect to the information structure $\boldsymbol{\mathcal{I}}$ if*

$$\mathcal{F}(\boldsymbol{\mathcal{I}}) \cap \Delta(\Xi) \neq \emptyset\,. \tag{10}$$

In contrast with Definition 1, we now require that the control policy satisfies a particular information structure. For example, the reference trajectory to track will usually be known at the current time step but not the disturbance. It is also possible that the reference is known either partially or totally in advance. In the following section, we will see that our examples may display more complex information structures.

III. Robust Tracking Commitment

Consider the problem of finding a reference set, such that the combined uncertainty set $\Xi = \mathcal{R} \times \mathcal{W}$, composed of the disturbance and reference is admissible for robust tracking for system (1). The admissible reference set $\mathcal{R}$ is to be committed to an external agent together with the guarantee of robust tracking for a finite horizon. We call this problem, a *robust tracking commitment* problem, the application of which can be found in ancillary service provision to the power grid, as we will demonstrate in Section IV, or similar hierarchical control settings.

Essentially, tracking commitment requires evaluation of the admissibility of uncertainty sets, in order to be able to find a suitable reference set to be committed. Based on Definition 3, we can write the family of causally admissible uncertainty sets for tracking with respect to a given information structure

$$\Omega = \left\{\Xi \subset \mathbb{R}^{Nn_\xi} \mid \exists \boldsymbol{\pi} \in \mathcal{F}(\boldsymbol{\mathcal{I}}) \cap \Delta(\Xi)\right\} \tag{11}$$

First we tackle the problem of simply finding a causally admissible reference set, without attaching any cost function to the problem. The tracking commitment problem can be written as

$$\text{find } \mathcal{R} : \mathcal{R} \times \mathcal{W} \in \Omega \tag{12}$$

For a fixed uncertainty set, admissibility can be verified by searching over control policies. However it is not obvious how to search over possible admissible sets and corresponding control policies simultaneously. In order to treat the problem with a unified methodology, we will characterize admissible sets as images of a modifier function applied to an initial uncertainty set $\hat{\Xi}$. The advantage of this approach will be evident in the following section III-B, when we formulate tractable methods for evaluating the admissibility of uncertainty sets for tracking, utilizing parameterized function based techniques available in the robust optimization literature [6].

A. Implicit modification of uncertainty sets

Let us formalize characterization of uncertainty sets by modifier functions. We first define the *uncertainty modifier* function $\boldsymbol{\nu} : \mathbb{R}^{Nn_\xi} \to \mathbb{R}^{Nn_\xi}$, which is assumed to be bijective and used for reshaping a given uncertainty set.

$$\boldsymbol{\nu}(\hat{\Xi}) = \{\boldsymbol{\nu}(\boldsymbol{\xi}) : \boldsymbol{\xi} \in \hat{\Xi}\} \tag{13}$$

Remark 2. *Note that we do not distinguish between the reference and the disturbance as they are just parts of the combined uncertainty signal, and all presented results apply to general robust programs. However, in the context of tracking commitment, only the reference set $\mathcal{R}$ can be modulated, whereas the disturbance set $\mathcal{W}$ is fixed. Also in general, the information structure of the reference and the disturbance are different. Nevertheless, without loss of generality, these particularities of the uncertain signal can be easily incorporated in the definition of the information structure and the mapping $\boldsymbol{\nu}$ by letting*

$$\boldsymbol{\nu}(\boldsymbol{\xi}) = \boldsymbol{\nu}\left(\boldsymbol{r},\ \boldsymbol{w}\right) := \left(\boldsymbol{\nu}_r(\boldsymbol{r}),\ \boldsymbol{w}\right). \tag{14}$$

In the following, we will show that we can evaluate the admissibility of the set $\Xi = \boldsymbol{\nu}(\hat{\Xi})$ via conditions on the composite function $\hat{\boldsymbol{\pi}} = \boldsymbol{\pi} \circ \boldsymbol{\nu}$ that is applied to the initial set $\hat{\Xi}$, as depicted in Figure 1. This allows us to fix an initial uncertainty set $\hat{\Xi}$, embed the modifier function into the control policy and implicitly modulate uncertainty sets and control policies simultaneously. To this end we introduce the following lemma,

Lemma 2. *Let $\boldsymbol{\nu} : \mathbb{R}^{Nn_\xi} \to \mathbb{R}^{Nn_\xi}$, be a bijection and $\hat{\Xi}$ be a compact set with non-empty interior . The set $\Xi := \boldsymbol{\nu}(\hat{\Xi})$ is causally admissible for tracking by system* (1) *in state x with respect to the information structure $\boldsymbol{\mathcal{I}}$ if and only if*

$$\exists \hat{\boldsymbol{\pi}} \in \Delta_{\boldsymbol{\nu}}(\hat{\Xi}) : \hat{\boldsymbol{\pi}} \circ \boldsymbol{\nu}^{-1} \in \mathcal{F}(\boldsymbol{\mathcal{I}}) \tag{15}$$

where $\Delta_{\boldsymbol{\nu}}$ is defined as

$$\Delta_{\boldsymbol{\nu}}(\Xi) := \{\boldsymbol{\pi} : \forall \boldsymbol{\xi} \in \Xi,\ (\boldsymbol{\pi}(\boldsymbol{\xi}), \boldsymbol{\nu}(\boldsymbol{\xi})) \in \mathcal{Q}\} \tag{16}$$

Proof. : Suppose $\hat{\boldsymbol{\pi}} \in \Delta_{\boldsymbol{\nu}}(\hat{\Xi})$ and $\hat{\boldsymbol{\pi}} \circ \boldsymbol{\nu}^{-1} \in \mathcal{F}(\boldsymbol{\mathcal{I}})$. Then we have

$$\forall \hat{\boldsymbol{\xi}} \in \hat{\Xi},\ (\hat{\boldsymbol{\pi}}(\hat{\boldsymbol{\xi}}), \boldsymbol{\nu}(\hat{\boldsymbol{\xi}})) \in \mathcal{Q} \tag{17}$$

Let $\boldsymbol{\xi} := \boldsymbol{\nu}(\hat{\boldsymbol{\xi}})$. Since $\boldsymbol{\nu}$ is bijective, we have $\hat{\boldsymbol{\xi}} = \boldsymbol{\nu}^{-1}(\boldsymbol{\xi})$. Therefore (17) is equivalent to

$$\begin{aligned}
&\forall \boldsymbol{\nu}^{-1}(\boldsymbol{\xi}) \in \hat{\Xi},\ (\hat{\boldsymbol{\pi}} \circ \boldsymbol{\nu}^{-1}(\boldsymbol{\xi}), \boldsymbol{\nu} \circ \boldsymbol{\nu}^{-1}(\boldsymbol{\xi})) \in \mathcal{Q} \\
\Leftrightarrow &\forall \boldsymbol{\xi} \in \boldsymbol{\nu}(\hat{\Xi}),\ (\hat{\boldsymbol{\pi}} \circ \boldsymbol{\nu}^{-1}(\boldsymbol{\xi}), \boldsymbol{\xi}) \in \mathcal{Q} \\
\Leftrightarrow &\forall \boldsymbol{\xi} \in \Xi,\ (\boldsymbol{\pi}(\boldsymbol{\xi}), \boldsymbol{\xi}) \in \mathcal{Q} \text{ with } \boldsymbol{\pi} = \hat{\boldsymbol{\pi}} \circ \boldsymbol{\nu}^{-1} \\
\Leftrightarrow &\boldsymbol{\pi} \in \Delta(\Xi)
\end{aligned} \tag{18}$$

Moreover, we have that $\boldsymbol{\pi} = \hat{\boldsymbol{\pi}} \circ \boldsymbol{\nu}^{-1} \in \mathcal{F}(\boldsymbol{\mathcal{I}})$. This concludes that Ξ is causally admissible for tracking according to Definition 3. The reverse direction follows from the equivalence of all steps.

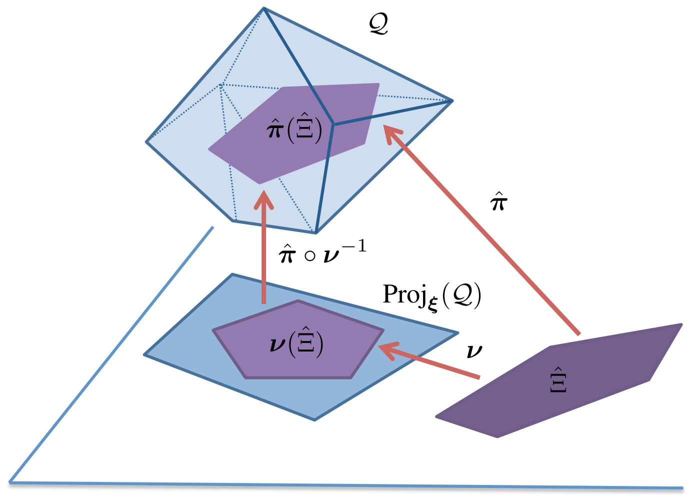


Fig. 1: Conceptual sketch of the relationships between uncertainty sets and applied functions. The initial uncertainty set $\hat{\Xi}$ is not necessarily a subset of the projection of $\mathcal{Q}$, therefore might not be admissible according to Lemma 1. However once we find a feasible lifting of this set into $\mathcal{Q}$, we can take its projection as an admissible uncertainty set, which is given by $\Xi = \boldsymbol{\nu}(\hat{\Xi})$. The corresponding admissible control policy can be obtained by letting $\boldsymbol{\pi} = \hat{\boldsymbol{\pi}} \circ \boldsymbol{\nu}$.

□

Remark 3. *Since the modifier function $\boldsymbol{\nu}$ is an arbitrary bijection, we do not lose generality when we consider uncertainty sets that can be characterized as the image of a given initial compact set $\hat{\Xi}$ with non-empty interior, under $\boldsymbol{\nu}$.*

According to Lemma 2 we can write an equivalent formulation of the family of admissible sets for a given initial set $\hat{\Xi}$

$$\Omega = \left\{ \Xi \subset \mathbb{R}^{Nn_\xi} \,\middle|\, \begin{array}{l} \exists \boldsymbol{\nu}, \hat{\boldsymbol{\pi}} : \Xi = \boldsymbol{\nu}(\hat{\Xi}), \\ \hat{\boldsymbol{\pi}} :\in \Delta_{\boldsymbol{\nu}}(\hat{\Xi}), \hat{\boldsymbol{\pi}} \circ \boldsymbol{\nu}^{-1} \in \mathcal{F}(\boldsymbol{\mathcal{I}}) \end{array} \right\} \tag{19}$$

When we look for a causally admissible set that belongs to Ω, the description (19) allows us to implicitly manipulate uncertainty sets and control policies simultaneously to verify admissibility, as will be seen in Section III-B. However, while searching for a modifier function $\boldsymbol{\nu}$, the condition $\hat{\boldsymbol{\pi}} \circ \boldsymbol{\nu}^{-1} \in \mathcal{F}(\boldsymbol{\mathcal{I}})$ is difficult to evaluate since it is a condition on a composite function that involves the inverse of $\boldsymbol{\nu}^{-1}$. In the following, we will propose a simple sufficient condition directly on $\boldsymbol{\nu}$, that is easy to evaluate and ensures causal admissibility of the modified uncertainty set. We start by splitting the causality conditions of the composite function $\hat{\boldsymbol{\pi}} \circ \boldsymbol{\nu}^{-1}$.

Lemma 3. *Let $(\mathcal{I}_k)_{k \in \mathbb{Z}_{[1,m]}}$ be a set of information structures and $f : \mathbb{R}^n \longrightarrow \mathbb{R}$. If for all k, $f \in \mathcal{F}(\mathcal{I}_k)$ then $f \in \mathcal{F}(\bigcap_k \mathcal{I}_k)$.*

The proof of Lemma 3, as well as other technical proofs in this section are grouped in Appendix B. The results will be briefly discussed in this section and we refer the reader to the appendices for more details. Lemma 3 states an intuitive fact, that is if the output of a function f depends only on inputs indexed by $\mathcal{I}_1$ and $\mathcal{I}_2$, then it actually depends only on inputs indexed by their intersection. This directly motivates the next lemma.

Lemma 4. *Let $\boldsymbol{g} : \mathbb{R}^n \to \mathbb{R}^n$, be a bijection. Given an information structure $\boldsymbol{\mathcal{I}}$, define $\hat{\boldsymbol{\mathcal{I}}}$ as*

$$\hat{\mathcal{I}}_j = \bigcap_{\{i | j \in \mathcal{I}_i\}} \mathcal{I}_i \tag{20}$$

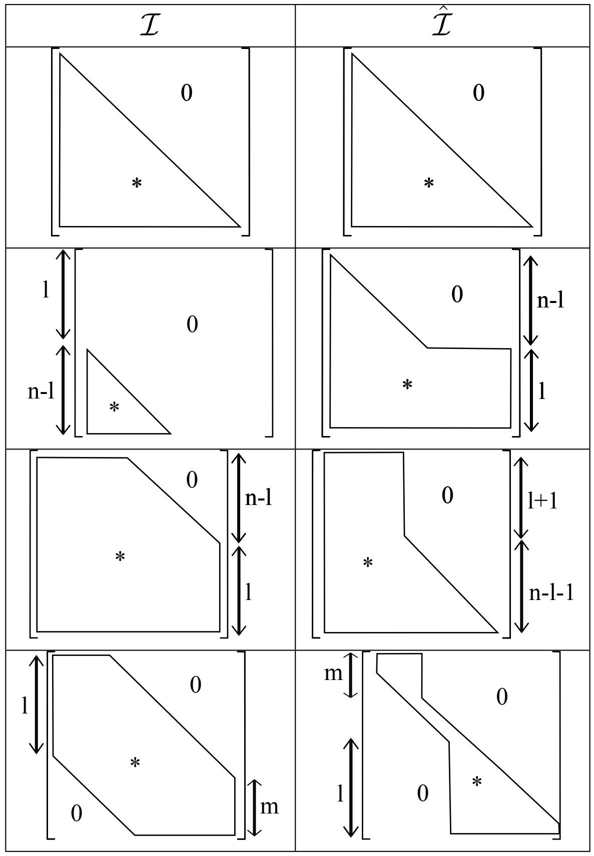


Fig. 2: For given information structures, the corresponding information structure of the uncertainty modifier function

The following equivalence holds

$$\forall \boldsymbol{f} \in \mathcal{F}(\boldsymbol{\mathcal{I}}), \boldsymbol{f} \circ \boldsymbol{g} \in \mathcal{F}(\boldsymbol{\mathcal{I}}) \Longleftrightarrow \boldsymbol{g} \in \mathcal{F}(\hat{\boldsymbol{\mathcal{I}}}) \tag{21}$$

Equation (20) characterizes a set of functions which do not change the information structure of $\boldsymbol{f}$. Loosely speaking, it states that if $\boldsymbol{f}_i$ depends on x_j then $\boldsymbol{g}_j$ should not depend on anything that $\boldsymbol{f}_i$ does not depend on. Notice that $\hat{\mathcal{I}}_j$ is always nonempty and in particular it contains j. This reflects the fact that a "diagonal" mapping (where $\boldsymbol{g}_j$ depends only on j for all j) does not change the information structure of any function it is composed with (for linear functions it means that multiplying by a diagonal matrix always preserves the sparsity pattern).

In Figure 2, the information structure $\hat{\boldsymbol{\mathcal{I}}}$ for different information structures $\boldsymbol{\mathcal{I}}$ are presented. The k^{th} row of the matrix represents the indicator vector of $\mathcal{I}_k$. These matrices can be thought of as sparsity patterns in the case that the control laws are linear. The first column shows the sparsity pattern of the control law $\boldsymbol{\pi}$ and the second column the corresponding sparsity-preserving sparsity pattern. In other words, multiplying the matrix from the first column by the matrix from the second column will result in the same sparsity pattern. This directly helps us select control laws and modifier functions such that their composition will still respect the required information structure. For example, as would be expected, the first row of the table tells us that a lower triangular control law composed with a lower triangular modifier will still be lower triangular. The second line tells us that if the information is known with a delay of l time steps, then the modifier function can be lower triangular and the last $l \times l$ block full.

In view of Lemma 2, simultaneous optimization over $\boldsymbol{\pi}$ and $\boldsymbol{\nu}$ would be beneficial for searching admissible uncertainty sets. Lemma 4 is instrumental in proving that from a control law $\hat{\boldsymbol{\pi}}$ defined on $\hat{\Xi}$

and an invertible mapping $\boldsymbol{\nu}$, a control law defined on $\boldsymbol{\nu}(\hat{\Xi})$ which has the desired information structure can be recovered. Indeed, $\hat{\boldsymbol{\pi}} \in \mathcal{F}(\boldsymbol{\mathcal{I}})$ and $\boldsymbol{\nu}^{-1} \in \mathcal{F}(\hat{\boldsymbol{\mathcal{I}}})$ ensures that $\hat{\boldsymbol{\pi}} \circ \boldsymbol{\nu}^{-1}$ defined on $\boldsymbol{\nu}(\hat{\Xi})$ belongs to $\mathcal{F}(\boldsymbol{\mathcal{I}})$ according to the lemma.

However, conditions on $\boldsymbol{\nu}^{-1}$ are inconvenient since the aim is to optimize directly over $\boldsymbol{\nu}$. Sufficient conditions on $\boldsymbol{\nu}$ are sought to replace the condition $\boldsymbol{\nu}^{-1} \in \mathcal{F}(\hat{\boldsymbol{\mathcal{I}}})$. Unfortunately a certain information structure for $\boldsymbol{\nu}^{-1}$ does not usually result in a specific information structure for $\boldsymbol{\nu}$. In particular, a sparse information structure for $\boldsymbol{\nu}^{-1}$ does not generally result in a sparse information structure for $\boldsymbol{\nu}$. For example, the inverse of a causal function is not generally causal. The following lemma gives sufficient conditions on $\boldsymbol{\nu}$.

Lemma 5. *Suppose $\boldsymbol{\nu} : \mathbb{R}^n \longrightarrow \mathbb{R}^n$ is a continuous bijection of $\mathbb{R}^n$ and $\boldsymbol{\nu} \in \mathcal{F}(\hat{\boldsymbol{\mathcal{I}}})$ as defined by equation* (20). *Define $\mathcal{G} = \{\boldsymbol{f} \circ \boldsymbol{\nu} \mid \boldsymbol{f} \in \mathcal{F}(\boldsymbol{\mathcal{I}})\}$. We have*

$$\mathcal{G} = \mathcal{F}(\boldsymbol{\mathcal{I}})$$

Under mild assumptions, Lemma 5 states that composing $\boldsymbol{f} \in \mathcal{F}(\boldsymbol{\mathcal{I}})$ with $\boldsymbol{\nu}$ results in a function with the same information structure.

Corollary 1. *Given an information structure $\boldsymbol{\mathcal{I}}$ and $\hat{\boldsymbol{\mathcal{I}}}$ as defined in equation* (20)*, if $\boldsymbol{\nu}$ is a continuous bijection and $\boldsymbol{\nu} \in \mathcal{F}(\hat{\boldsymbol{\mathcal{I}}})$, then $\boldsymbol{\nu}^{-1} \in \mathcal{F}(\hat{\boldsymbol{\mathcal{I}}})$.*

Proof. According to Lemma 5, $\mathcal{F}(\boldsymbol{\mathcal{I}}) = \{\boldsymbol{f} \circ \boldsymbol{\nu}^{-1} \mid \boldsymbol{f} \in \mathcal{G}\} = \{\boldsymbol{f} \circ \boldsymbol{\nu}^{-1} \mid \boldsymbol{f} \in \mathcal{F}(\boldsymbol{\mathcal{I}})\}$. Hence, for any $\boldsymbol{f} \in \mathcal{F}(\boldsymbol{\mathcal{I}})$, it holds that $\boldsymbol{f} \circ \boldsymbol{\nu}^{-1} \in \mathcal{F}(\boldsymbol{\mathcal{I}})$. The fact that $\boldsymbol{\nu}^{-1} \in \mathcal{F}(\hat{\boldsymbol{\mathcal{I}}})$ follows from Lemma 4. $\square$

Theorem 1. *Let $\boldsymbol{\nu} : \mathbb{R}^{Nn_\xi} \to \mathbb{R}^{Nn_\xi}$, be a continuous bijection and $\boldsymbol{\mathcal{I}}$ an information structure, $\hat{\boldsymbol{\mathcal{I}}}$ defined by equation* (20) *and $\Delta_{\boldsymbol{\nu}}$ in equation* (16). *$\boldsymbol{\nu}(\hat{\Xi})$ is causally admissible for tracking with respect to the information structure $\boldsymbol{\mathcal{I}}$ if*

$$\begin{aligned} &\mathcal{F}(\boldsymbol{\mathcal{I}}) \cap \Delta_{\boldsymbol{\nu}}(\hat{\Xi}) \neq \emptyset \\ &\boldsymbol{\nu} \in \mathcal{F}(\hat{\boldsymbol{\mathcal{I}}}) \end{aligned} \tag{22}$$

Proof. Suppose there exists $\hat{\boldsymbol{\pi}} \in \mathcal{F}(\boldsymbol{\mathcal{I}}) \cap \Delta_{\boldsymbol{\nu}}(\hat{\Xi})$. Since $\boldsymbol{\nu}$ is a continuous bijection, $\boldsymbol{\nu} \in \mathcal{F}(\hat{\boldsymbol{\mathcal{I}}})$ implies that $\boldsymbol{\nu}^{-1} \in \mathcal{F}(\hat{\boldsymbol{\mathcal{I}}})$ by Corollary 1. Lemma 4 in turn ensures that $\hat{\boldsymbol{\pi}} \circ \boldsymbol{\nu}^{-1} \in \mathcal{F}(\boldsymbol{\mathcal{I}})$. Finally, application of Lemma 2 concludes the proof. $\square$

Theorem 1 provides sufficient conditions for causal admissibility of an uncertainty set for tracking. We can define the family of admissible sets that comply with these sufficient conditions as

$$\tilde{\Omega}(\hat{\Xi}) = \left\{ \Xi \subset \mathbb{R}^{Nn_\xi} \;\middle|\; \begin{aligned} &\exists \boldsymbol{\nu}, \hat{\boldsymbol{\pi}} \\ &\Xi = \boldsymbol{\nu}(\hat{\Xi}),\; \boldsymbol{\nu} \in \mathcal{F}(\hat{\boldsymbol{\mathcal{I}}}) \\ &\hat{\boldsymbol{\pi}} \in \mathcal{F}(\boldsymbol{\mathcal{I}}) \cap \Delta_{\boldsymbol{\nu}}(\hat{\Xi}) \end{aligned} \right\} \tag{23}$$

For the definition of $\tilde{\Omega}$ we have replaced the condition $\hat{\boldsymbol{\pi}} \circ \boldsymbol{\nu}^{-1} \in \mathcal{F}(\boldsymbol{\mathcal{I}})$ with the sufficient but simpler conditions $\hat{\boldsymbol{\pi}} \in \mathcal{F}(\boldsymbol{\mathcal{I}})$ and $\boldsymbol{\nu} \in \mathcal{F}(\hat{\boldsymbol{\mathcal{I}}})$. Therefore $\tilde{\Omega}$ is a restriction of the original family of admissible sets Ω.

$$\tilde{\Omega}(\hat{\Xi}) \subseteq \Omega \tag{24}$$

The restriction will depend on the initial set $\hat{\Xi}$ and thus the argument of $\tilde{\Omega}$ is added to reflect this fact. However, this restriction leads to tractable formulations based on the available robust programming literature, as we will show in the following section.

Finally, we write the modified robust tracking commitment problem that is based on sufficient conditions (22) as

$$\text{find } \mathcal{R} : \mathcal{R} \times \mathcal{W} \in \tilde{\Omega}(\hat{\Xi}) \tag{25}$$

B. Tractable solutions

The problem formulation (25) allows us to search over uncertainty sets implicitly by means of modifier functions. However the problem is still difficult in its general form, due to the infinite dimension of the search space and the infinite number of constraints. Therefore we will look for finite dimensional and tractable approximations of the tracking commitment problem in order to solve it efficiently.

Using the definitions of $\tilde{\Omega}$ and $\Delta_{\boldsymbol{\nu}}$, we can rewrite the modified robust tracking commitment problem as

$$\begin{array}{ll} \text{find} & \hat{\boldsymbol{\pi}}, \boldsymbol{\nu} \\ \text{subject to} & \forall \hat{\boldsymbol{\xi}} \in \hat{\Xi} \\ & (\hat{\boldsymbol{\pi}}(\hat{\boldsymbol{\xi}}), \boldsymbol{\nu}(\hat{\boldsymbol{\xi}})) \in \mathcal{Q} \\ & \hat{\boldsymbol{\pi}} \in \mathcal{F}(\boldsymbol{\mathcal{I}}) \\ & \boldsymbol{\nu} \in \mathcal{F}(\hat{\boldsymbol{\mathcal{I}}}) \ . \end{array} \tag{26}$$

Note that (26) is an *adjustable robust optimization* (ARO) problem [6], that allows decisions to be taken after the realization of the uncertainty. In our case the uncertainty dependent decision rule is characterized by the control policy $\boldsymbol{\pi}$. In the standard form of ARO, the uncertainty set is fixed, whereas the tracking commitment problem requires optimization over possible uncertainty sets. However, even with the additional modification of the uncertainty set, the tracking commitment problem (26) still fits into the standard ARO framework, because the uncertainty modifier $\boldsymbol{\nu}$ can also be treated as a decision rule. Consequently, we can directly utilize results from adjustable robust programming [6], which shows that restricting the search space to linear (or affine) functions leads to finite dimensional and tractable formulations, referred as the *affinely adjustable robust counterpart* (AARC). For notational simplicity, we use linear functions, as any affine term that does not depend on the realization of the uncertainty can be defined as a separate non-adjustable decision variable in the optimization problem.

Firstly, we consider polytopic tracking sets, defined as

$$\mathcal{E} := \{\boldsymbol{e} \mid G\boldsymbol{e} \leqslant g\}$$

The feasibility set $\mathcal{Q}$ also becomes polytopic and can be written as

$$\mathcal{Q} = \{(\boldsymbol{u}, \boldsymbol{\xi}) \mid H\boldsymbol{u} + Q\boldsymbol{\xi} \leqslant q\}$$

For the derivation of $Q,\ H$, and q see Appendix A. Note that q is an affine function of the initial condition x, and g, which determines the size of the tracking set.

Let us further define the linear versions of the control policy and the uncertainty modifier

$$\boldsymbol{\pi}_{lin}(\boldsymbol{\xi}) := \mathbf{M}\boldsymbol{\xi}, \quad \boldsymbol{\nu}_{lin}(\boldsymbol{\xi}) = \mathbf{L}\boldsymbol{\xi} \tag{27}$$

where $\mathbf{M} \in \mathbb{R}^{Nn_u \times Nn_\xi}$ and $\mathbf{L} \in \mathbb{R}^{Nn_\xi \times Nn_\xi}$. We can describe the causality conditions by constraints on $\mathbf{M}$ and $\mathbf{L}$

$$\begin{array}{lll} \mathbf{M}(k, \mathbb{Z}_{[1,Nn_\xi]} \backslash \mathcal{I}_k) = 0, & k \in \mathbb{Z}_{[1,N]} & \Leftrightarrow \boldsymbol{\pi} \in \mathcal{F}(\boldsymbol{\mathcal{I}}) \\ \mathbf{L}(k, \mathbb{Z}_{[1,Nn_\xi]} \backslash \hat{\mathcal{I}}_k) = 0, & k \in \mathbb{Z}_{[1,N]} & \Leftrightarrow \boldsymbol{\nu} \in \mathcal{F}(\hat{\boldsymbol{\mathcal{I}}}) \end{array} \tag{28}$$

Note that the constraints (28) impose that the elements of $\mathbf{M}$ and $\mathbf{L}$ multiplying the elements of the uncertain variable which are not included in the information structure at step k to be zero, thus enforcing causality of the linear functions $\boldsymbol{\pi}_{lin}$ and $\boldsymbol{\nu}_{lin}$.

Furthermore, we restrict the class of allowed initial uncertainty sets to intersections of convex cones

$$\hat{\Xi} = \left\{ \boldsymbol{\xi} \mid F_i \boldsymbol{\xi} + f_i \in \mathbf{K}_i,\ i \in \mathbb{Z}_{[1,m]} \right\} \tag{29}$$

where the cone $\mathbf{K}_i$ is assumed to be proper. Note that, the considered class of uncertainty sets is very extensive, as it allows the description of well known cones such as the non-negative orthant, the Lorentz cone or the positive semi-definite cone as well as their intersections and products.

Let us now formulate the robust tracking commitment problem (26) with linear policies given in (27) and conic uncertainty sets described by (29)

$$\begin{aligned} \text{find} \quad & \hat{\mathbf{M}}, \mathbf{L} \\ \text{subject to} \quad & \forall \hat{\boldsymbol{\xi}} : F_i \hat{\boldsymbol{\xi}} + f_i \in \mathbf{K}_i, \quad i \in \mathbb{Z}_{[1,m]} \\ & H \hat{\mathbf{M}} \hat{\boldsymbol{\xi}} + Q \mathbf{L} \hat{\boldsymbol{\xi}} \leqslant q \\ & (\hat{\mathbf{M}}, \mathbf{L}) \text{ satisfies (28)} \end{aligned} \tag{30}$$

Once the problem is solved, a feasible solution $\hat{\mathbf{M}}^*$ and $\mathbf{L}^*$ can be used to construct the uncertainty set that is causally admissible for tracking and the corresponding control policy

$$\Xi = \mathbf{L}^* \hat{\Xi}, \quad \boldsymbol{\pi}(\boldsymbol{\xi}) = \mathbf{M} \boldsymbol{\xi}, \quad \mathbf{M} = \hat{\mathbf{M}}^* \mathbf{L}^{*-1} \tag{31}$$

The optimization problem (30) is still difficult, due to the infinite number of constraints for every possible realization of the uncertain variable. However this issue can be tackled by considering an equivalent formulation, where the worst case realizations of the uncertainty is considered by enforcing the constraint; $\max\limits_{\hat{\boldsymbol{\xi}} \in \hat{\Xi}} \left\{ H \mathbf{M} \hat{\boldsymbol{\xi}} + Q \mathbf{V} \hat{\boldsymbol{\xi}} \right\} \leqslant q$. Applying conic duality thereafter and stacking dual variables in matrix $\mathbf{Z}$, the robust counterpart can be formulated as

$$\begin{aligned} \text{find} \quad & \mathbf{Z}, \hat{\mathbf{M}}, \mathbf{L} \\ \text{subject to} \quad & \mathbf{Z}_i^T \in \mathbf{K}_i^*,\ i \in \mathbb{Z}_{[1,m]} \\ & \sum_{i=1}^{m} \mathbf{Z}_i f_i \leqslant q \\ & \sum_{i=1}^{m} \mathbf{Z}_i F_i = -\left(H \hat{\mathbf{M}} + Q \mathbf{L} \right) \\ & (\hat{\mathbf{M}}, \mathbf{L}) \text{ satisfies (28)} \end{aligned} \tag{32}$$

We refer the reader to [6] for the derivation of the robust counterpart. The robust counterpart for the tracking commitment is convex in linear control policies parameterized by $\hat{\mathbf{M}}$ and linear uncertainty modifiers parameterized by $\mathbf{L}$. Therefore, when sets $\mathbf{K}_i$ are polyhedral, second order or semi-definite cones, the problem formulation (32) allows tractable computations of feasible reference sets admissible for tracking with respect to the information structure $\boldsymbol{\mathcal{I}}$ and the tracking error set $\mathcal{E}$, by the system (1). Table I gives a summary of problem complexity in case of most common uncertainty sets for the reference and disturbance.

Remark 4. *Any additional decision variable that is independent of the uncertainty, $\bar{\mathbf{u}}$, as well as non zero nominal values for the disturbance and reference, $\bar{\boldsymbol{\xi}}$, can be easily incorporated in the above formulation as*

$$H \bar{\boldsymbol{u}} + Q \bar{\boldsymbol{\xi}} + \sum_{i=1}^{m} \mathbf{Z}_i f_i \leqslant q$$

$\mathcal{R}$	$\mathcal{W}$	Robust Counterpart
$F_r\boldsymbol{r} \leqslant f_r$	$F_w\boldsymbol{w} \leqslant f_w$	LP
$F_r\boldsymbol{r} + f_r : \|\boldsymbol{r}\|_2 \leqslant 1$	$F_w\boldsymbol{w} \leqslant f_w$	SOCP
$F_r\boldsymbol{r} \leqslant f_r$	$F_w\boldsymbol{w} + f_w : \|\boldsymbol{w}\|_2 \leqslant 1$	SOCP
$F_r\boldsymbol{r} + f_r : \|\boldsymbol{r}\|_2 \leqslant 1$	$F_w\boldsymbol{w} + f_w : \|\boldsymbol{w}\|_2 \leqslant 1$	SOCP

TABLE I: Optimization type for the robust counterpart formulation (32) of the tracking commitment problem, considering combinations of polytopes and ellipsoids as uncertainty sets. Note that the polytopic representation also covers 1 and ∞ norm balls.

C. Nonlinear policy and uncertainty modifiers

Since Theorem 1 and related Lemmata apply to generic functions and sets, all presented methods can be applied directly to nonlinear policies and modifier functions. Even though the formulation (32) is limited to linear functions, it is possible to incorporate nonlinear policies (or modifier functions) in a computationally tractable manner. The key principle, introduced in [6] and studied in greater detail in [11] is to consider a modified uncertainty set which is the image of the original uncertainty under a nonlinear lifting.

Consider the following constraint with non-linear control policy

$$H\boldsymbol{\pi}(\boldsymbol{\xi}) + Q\boldsymbol{\nu}(\boldsymbol{\xi}) \leqslant q, \quad \forall \boldsymbol{\xi} \in \Xi \tag{33}$$

where the policies $\boldsymbol{\pi}$ and $\boldsymbol{\nu}$ are linear combinations of a set of possibly nonlinear basis functions

$$\boldsymbol{\pi}_i(\boldsymbol{\xi}) := \sum_{j=1}^{n} a_{i,j} f_j(\boldsymbol{\xi}) \text{ and } \boldsymbol{\nu}_i(\boldsymbol{\xi}) := \sum_{j=1}^{n} b_{i,j} f_j(\boldsymbol{\xi}) \tag{34}$$

We define the lifted uncertainty variable, and the corresponding uncertainty set as

$$\mathcal{Z} := \{\boldsymbol{\zeta} = (f_1(\boldsymbol{\xi}), \dots, f_n(\boldsymbol{\xi})), \mid \boldsymbol{\xi} \in \Xi\} \tag{35}$$

with properly defined matrices, the inequality (33) can be converted into a linear one

$$H\tilde{\mathbf{M}}\boldsymbol{\zeta} + Q\tilde{\mathbf{L}}\boldsymbol{\zeta} \leqslant q \tag{36}$$

If the new extended uncertainty set $\mathcal{Z}$ or its convex hull conv($\mathcal{Z}$), can be represented in conic form (29), then the tractable formulation (32) can be to used solve the problem and obtain the nonlinear control policy that is composed of the basis function and linear decision rule. For details and possible nonlinear mappings that result in tractable formulations, we refer the reader to [6] and [11]. As an example, we will briefly summarize results from [6] showing that quadratic control policies can be handled with ellipsoidal uncertainty sets. Consider the policy

$$\boldsymbol{\pi}_i(\boldsymbol{\xi}) := \boldsymbol{\xi}^T Y_i \boldsymbol{\xi} + v_i^T \boldsymbol{\xi} + u_i \tag{37}$$

and the ellipsoidal uncertainty set

$$\Xi = \{\boldsymbol{\xi} \mid \|\mathcal{T}\boldsymbol{\xi}\|_2 \leqslant 1\} \tag{38}$$

with invertible $\mathcal{T}$. The lifted uncertainty set can be described as:

$$\boldsymbol{\zeta} = (1, \boldsymbol{\xi}, \{\boldsymbol{\xi}_i\boldsymbol{\xi}_j, \ \forall i, j\}) \tag{39}$$

Equivalently we can use a matrix notation

$$\boldsymbol{\zeta} = \begin{bmatrix} 1 & \boldsymbol{\xi}^T \\ \boldsymbol{\xi} & \boldsymbol{\xi}\boldsymbol{\xi}^T \end{bmatrix} \tag{40}$$

As shown in [6], the convex hull of the lifted uncertainty set $\mathcal{Z}$ is given by

$$\text{conv}(\mathcal{Z}) = \left\{ \boldsymbol{\zeta} = \begin{bmatrix} 1 & \boldsymbol{\xi}^T \\ \boldsymbol{\xi} & W \end{bmatrix} \middle| \boldsymbol{\zeta} \succeq 0, \ \text{tr}(\mathcal{T}W\mathcal{T}^T) \leqslant 1 \right\} \tag{41}$$

where $\boldsymbol{\zeta} \succeq 0$ means that $\boldsymbol{\zeta}$ is a symmetric positive semi-definite matrix. This representation can be put in the standard conic form of (29), and therefore allows a tractable robust counterpart formulation like (32). In [23], the authors use quadratic liftings to find the largest volume inner approximations of polytope projections. This can be seen as a specific case of the work presented here, with full information knowledge and quadratic lifting law $\boldsymbol{\pi}$ but linear modifier function $\boldsymbol{\nu}$.

D. Modulating the tracking error set

In the previous section, we have formulated a tractable version of the robust tracking commitment problem (12) which seeks a causally admissible reference set with respect to a fixed tracking error set $\mathcal{E}$. However for some applications, it might be preferable to modulate the reference set together with the associated tracking error set, since the relative sizes of the two sets will indicate the tracking performance. For example, for frequency regulation service to the power grid in Switzerland, the service providers are allowed to deviate from the reference up to a certain percentage of the total service capacity committed, therefore a service provider who is committing a larger reference set is allowed to have a larger error set [24].

Notice that the problem (32) is convex in q which parameterizes the polytopic feasibility set $\mathcal{Q}$. Therefore one can freely optimize over modifications of the feasibility set. For clarity let us write the system feasibility and tracking constraints separately as

$$\mathcal{Q} = \left\{ (\boldsymbol{u}, \boldsymbol{r}, \boldsymbol{w}) \middle| \begin{array}{l} T\boldsymbol{u} + V\boldsymbol{w} \leqslant h \\ G\boldsymbol{r} - (P\boldsymbol{u} + S\boldsymbol{w} + \bar{\boldsymbol{y}}) \leqslant g \end{array} \right\} \tag{42}$$

where $\bar{\boldsymbol{y}}$ is the nominal output of the system without control action and the derivation of matrices T, V, P, S can be found in Appendix A.

From (42), we can immediately observe that the problem (32) is also convex in g which parameterizes the tracking error set $\mathcal{E}$, therefore allows modulation in a tractable manner. The sizes of the tracking error set and the reference set can be related by enforcing a joint constraint on the uncertainty modifier function parameter $\mathbf{L}$ and the error set parameter g.

E. Optimal tracking commitment

As mentioned earlier, the robust tracking commitment (12) is a feasibility problem. On the other hand, the optimal robust tracking commitment problem is finding the control policy, reference and tracking error sets, that minimize a cost function.

$$\underset{\pi \in \mathcal{F}(\boldsymbol{\mathcal{I}}) \cap \Delta(\mathcal{R} \times \mathcal{W})}{\text{minimize}} \; J(\boldsymbol{\pi}, \mathcal{R}, \mathcal{E}) \tag{43}$$

Relying on the tractable formulation with linear control policies and uncertainty modifiers (30), we can solve the tractable version of the optimal robust tracking commitment.

$$\begin{array}{ll} \text{minimize} & J(\boldsymbol{u}, \mathbf{L}_r, g) \\ \text{subject to} & \forall \hat{\boldsymbol{r}} : F_r \hat{\boldsymbol{r}} + f_r \in \mathbf{K}_r, \\ & \forall \boldsymbol{w} : F_r \boldsymbol{w} + f_w \in \mathbf{K}_w, \\ & T\boldsymbol{u} + V\boldsymbol{w} \leqslant h \\ & G\mathbf{L}_r \hat{\boldsymbol{r}} - (P\boldsymbol{u} + S\boldsymbol{w} + \bar{\boldsymbol{y}}) \leqslant g \\ & \boldsymbol{u} = \hat{\mathbf{M}}_r \hat{\boldsymbol{r}} + \mathbf{M}_w \boldsymbol{w} + \bar{\boldsymbol{u}} \\ & (\hat{\mathbf{M}}, \mathbf{L}) \text{ satisfies (28)} \end{array} \tag{44}$$

where $\hat{\mathbf{M}}$ and $\mathbf{L}$ are defined as

$$\hat{\mathbf{M}} = \begin{bmatrix} \hat{\mathbf{M}}_r & \\ & \mathbf{M}_w \end{bmatrix}, \quad \mathbf{L} = \begin{bmatrix} \mathbf{L}_r & \\ & \mathbf{I}_{Nn_x} \end{bmatrix} \tag{45}$$

For notational simplicity, the reference and disturbance sets are represented as single conic sets, but they can also be defined as intersection of several conic sets as in (29).

With a suitable cost function, the optimal commitment problem (44) can be solved as explained in section III-B. The cost function usually depends on the uncertain realization of the reference and disturbance. However this dependence can be qualified out by either considering the worst-case or expectation of the possible cost realizations [25]. Furthermore, as long as J is bilinear in the uncertain variables and decision variables, the tractable robust formulation for the minimization of an upper bound, that constitutes an equivalent problem with certain cost function, can be obtained [6].

After solving (44), the optimal control policy, reference and tracking error sets can be obtained as

$$\begin{aligned} &\mathcal{R}^* = \{\boldsymbol{r} = \mathbf{L}_r^* \hat{\mathbf{r}} \mid F_r \hat{\boldsymbol{r}} + f_r \in \mathbf{K}_r\} \\ &\boldsymbol{\pi}^*(\boldsymbol{r}, \boldsymbol{w}) = \hat{\mathbf{M}}_r^* \mathbf{L}_r^{*-1} \boldsymbol{r} + \mathbf{M}_w^* \boldsymbol{w} + \bar{\boldsymbol{u}}^* \\ &\mathcal{E}^* = \{\boldsymbol{e} \mid G\boldsymbol{e} \leqslant g^*\} \end{aligned} \tag{46}$$

where * indicates that the variable is an optimizer of (44).

F. Collective tracking

In this section, we will consider the collective tracking commitment problem, where the reference signal is to be tracked by the summation (or average) of the outputs of several subsystems that are not coupled via constraints or dynamics. In this case, the aggregate tracking error can be written as

$$\boldsymbol{e} = \boldsymbol{r} - \sum_{j=1}^{n} \boldsymbol{y}^j \tag{47}$$

where superscript j indicates that the associated variable belongs to subsystem j.

In order to compute a causally admissible reference set for the collection of subsystems, it is necessary to compute the aggregated tracking capability simultaneously. An obvious option is to treat the collective system as a single system with block diagonal system matrices, and solve the problem (30) centrally. However this requires collecting the knowledge of detailed subsystem models by an aggregator and will possibly result in a very large number of decision variables. Therefore it is desirable to distribute the problem, such that it can be solved without central knowledge of the models and using limited communication between the agents.

Let us briefly show that the problem (30) in fact easily lends itself to distributed solution methods. The subsystems are supposed to track a single reference by collective action. This is equivalent to saying that the subsystems are tracking separate reference signals, which sum up to the central reference. Using a common nominal reference set and a linear uncertainty modifier function for each subsystem, the reference can be split as

$$\boldsymbol{r}^j = \mathbf{L}^j \hat{\boldsymbol{r}}, \quad \sum_{j=1}^{n} \mathbf{L}^j = \mathbf{L} \tag{48}$$

where $\mathbf{L}$ parameterizes the global linear modifier function. Therefore the aggregate reference set can be described as

$$\mathcal{R} = \sum_{j=1}^{n} \mathbf{L}^j \hat{\mathcal{R}} \tag{49}$$

Many distribution schemes are possible, given the cost-reward framework of the collective tracking task. As an example, we consider the case where the error set is fixed, the reward is split between agents according to their contributions to tracking characterized by parameter $\mathbf{L}^j$ and the objective is to minimize the total cost

$$\sum_{j=1}^{n} J^j(\boldsymbol{\pi}^j, \mathbf{L}^j) \tag{50}$$

Subsystem feasibility constraints $T^j \boldsymbol{u}^j + V^j \boldsymbol{w}^j \leqslant h^j$ and causality conditions on the local control policies can be treated separately by each subsystem. However (48) and the collective tracking constraint (47) introduces a constraint coupling among all subsystems. The collective tracking constraint can be written as

$$\begin{aligned}
&\forall \hat{\boldsymbol{r}} : F_r \hat{\boldsymbol{r}} + f_r \in \mathbf{K}_r, \quad \forall \boldsymbol{w}^j : F_w^j \boldsymbol{w}^j + f_w^j \in \mathbf{K}_w^j \\
&\sum_{j=1}^{n} \left(G\mathbf{L}^j \hat{\boldsymbol{r}} - (P^j \boldsymbol{u}^j + S^j \boldsymbol{w}^j + \bar{\boldsymbol{y}}^j) \right) \leqslant g \\
&\boldsymbol{u}^j = \hat{\mathbf{M}}_r^j \hat{\boldsymbol{r}} + \mathbf{M}_w^j \boldsymbol{w}^j + \bar{\boldsymbol{u}}^j
\end{aligned} \tag{51}$$

Utilizing linear control policies $\boldsymbol{u}^j = \hat{\mathbf{M}}_r^j \hat{\boldsymbol{r}} + \mathbf{M}_w^j \boldsymbol{w}^j + \bar{\boldsymbol{u}}^j$ and formulating the dual of the robust counterpart as in (32), we obtain a tractable formulation for the collective tracking constraint

$$\mathbf{Z}_r^T \in \mathbf{K}_r^*, \quad {\mathbf{Z}_w^j}^T \in {\mathbf{K}^j}_w^*, \qquad \forall j \in \mathbb{Z}_{[1,n]} \tag{52a}$$

$$\mathbf{Z}_r f_r + \sum_{j=1}^{n} \left(\mathbf{Z}_w^j f_w^j - P^j \bar{\boldsymbol{u}}^j - \bar{\boldsymbol{y}}^j \right) \leqslant g \tag{52b}$$

$$\mathbf{Z}_r F_r = \sum_{j=1}^{n} (P^j \hat{\mathbf{M}}_r^j - G\mathbf{L}^j), \tag{52c}$$

$$\mathbf{Z}_w^j F_w^j = P^j \mathbf{M}_w^j + S^j, \qquad \forall j \in \mathbb{Z}_{[1,n]} \tag{52d}$$

We observe that the coupling constraints are (52b) and (52c). The partial Lagrangian with the coupling constraints can be written as

$$\sum_{j=1}^{n} J^j(\pi^j, \mathbf{L}^j) + \lambda^T \mathrm{vec}(\mathbf{Z}_r F_r - \sum_{j=1}^{n} (P^j \hat{\mathbf{M}}_r^j + G\mathbf{L}^j)) + \mu^T (\mathbf{Z}_r f_r + \sum_{j=1}^{n} \left(\mathbf{Z}_w^j f_w^j - P^j \bar{\boldsymbol{u}}^j - \bar{\boldsymbol{y}}^j \right) - g) \tag{53}$$

which is separable given the variables λ, μ and $\mathbf{Z}_r$. Therefore the collective tracking commitment problem can be solved in a distributed manner with global updates of these variables or enforcing consensus on their local copies [26]. Note that, after the commitment of the aggregate reference set, it is not necessary to further communicate for guaranteed tracking if all subsystems have access to the aggregate reference $\boldsymbol{r}$, since collective tracking is robustly guaranteed by independent local control policies. However, the performance can be improved by repetitively solving the problem (52) online for redistributing the tracking task among subsystems according to the available information on the disturbance and reference as time progresses. Adjusting the error set parameter g is also possible with a suitable constraint on the aggregate modifier $\mathbf{L}$ and g. In case of dynamic couplings, inter-system constraints or common disturbances that couples the state and inputs of several subsystems, the same methodology can be applied to derive the tractable robust counterpart formulation and distribute the computation.

IV. Applications

A. *Power tracking with a building*

In this section, we will illustrate most of the theoretical concepts put forward in this article on a realistic example of a building providing power tracking to the grid operator. The problem of interest is the commitment of secondary frequency control provision. Secondary frequency control providers in the Swiss electricity grid have to commit regulation with a weekly bid. When their capacity bid is accepted, they have committed to track a power consumption signal called the *Area Control Signal (ACS) signal*, within the bounds specified by the bids. Deviations in the power consumption tracking are allowed within a error margin proportional to the bid. For details on frequency control, refer to [2].

We consider an office building with three controlled zones served by individual air handling units that we assume can control the heat fluxes to the zones. A linear state-space model of the building was extracted and validated against EnergyPlus simulation data using the toolbox OpenBuild [27]. The toolbox builds a thermal model of the building based on first principle modeling and collects realistic data for occupancy and equipment schedules, as well as weather. One week of typical summer weather for the city of Chicago is used in this study. The model of the building is a model of the form (1) with state dimension $n_x = 10$ and input dimension $n_u = 3$. The disturbance captures the effect of internal gains, solar radiations and outdoor temperature, and the input vector represents the negative thermal input power to the zones (it is cooling season). In this study, y is a scalar that represents the total electricity consumption so that $y_k = \alpha \sum_{i=1}^{n_u} u_i$ with α the electric to thermal conversion factor. For simplicity, a linear relationship is assumed here but a more detailed model could be used depending on the heating system, provided it is linearized. The peak thermal cooling load of the building is of 45kW for the summer. The input constraint set $\mathcal{U}$ specifies maximum and minimum cooling levels in the rooms so that $0 = u_{i,\text{min}} \leq u_i \leq u_{i,\text{max}}$ for each thermal zone input, reflecting the sizing of the equipment. The state constraints $\mathcal{X}$ specifies temperature zones in the constraints so that the temperature is maintained between 20°C and 25°C.

The bidding process is as follows: at time $t_0 = 0$, the building starts in initial condition x_0. The tracking period starts at time t_1 and ends at time t_2, therefore leaving a "preparation" period for the building controller. The capacity bid consists of the commitment of a baseline consumption during the tracking period u_{nom} and up-down regulation limits around that baseline for power tracking. Up-down regulation bids result in a "box" uncertainty set. We therefore fix the basic uncertainty shape as the unit box

$$\hat{\mathcal{R}}_{box} = \{\boldsymbol{r} \mid \|\boldsymbol{r}\|_\infty \leq 1\} \tag{54}$$

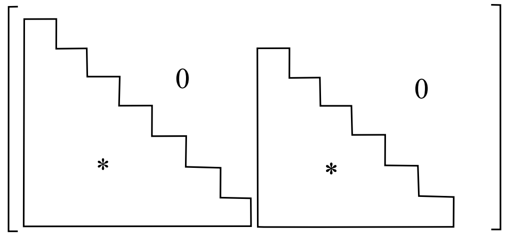
It is expected that a load like a building cannot easily increase or decrease its power consumption for a long period of time. It is therefore interesting to introduce the notion of a maximum "integral" capacity. This can be modeled by considering an uncertainty set with integral limits on the power consumption. This gives an uncertainty set of the form

$$\hat{\mathcal{R}}_{batt} = \left\{ \mathbf{r} \;\middle|\; \begin{array}{ll} s_0 = \bar{s}_0 & \\ 0 \leq s_t \leq s_{max}, & \forall t \in \mathbb{Z}_{[1,n]} \\ s_{t+1} = s_t + r_t & \forall t \in \mathbb{Z}_{[1,n]} \\ -1 \leq r_t \leq 1 & \forall t \in \mathbb{Z}_{[1,n]} \end{array} \right\} \tag{55}$$

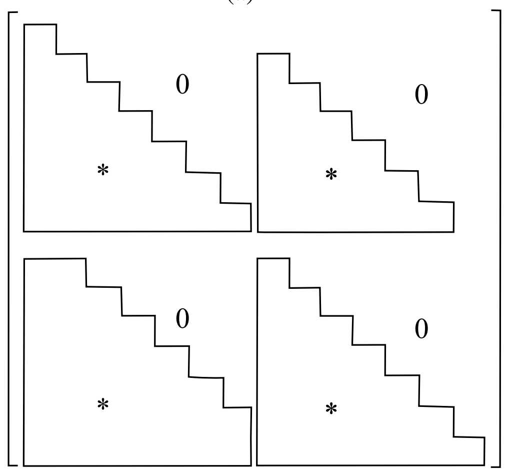
By analogy with the feasible set of a simplified battery model, we will refer to this uncertainty set as the "battery" reference set. For simplicity here indices are omitted but it is implied that the reference tracking set is defined only during the tracking period. For the external disturbance from weather and internal gains, the disturbance set is defined as follows

$$\mathcal{W} = \{\boldsymbol{w}_{\text{nom}} + \boldsymbol{w}_{\text{stoch}} \mid \boldsymbol{w}_{\text{stoch,i}}^T Q_i \boldsymbol{w}_{\text{stoch,i}} \leq 1,\; i = 1, 2, 3\} \tag{56}$$

As such, $\mathcal{W}$ is the direct product of three "uncorrelated" ellipsoidal uncertainty sets so that $\mathcal{W} = \mathcal{W}_{\text{sun}} \times \mathcal{W}_{\text{gains}} \times \mathcal{W}_{\text{temp}}$. $\boldsymbol{w}_{\text{nom}}$ is the nominal prediction of the uncertainty over the prediction horizon and the three ellipsoids represent confidence sets that should cover a reasonable part of the possible outcomes for the disturbance. To dimension properly the set $\mathcal{W}$, that is to choose the matrices Q_i, several methods have been proposed. Probabilistic data-driven methods will aim at selecting a reference set that will contain the realization of the uncertainty with some level of probability $1 - \epsilon$ with high confidence. For example, [28] exploits the scenario approach presented in [29] to derive a set that contains some level of the probability mass of the uncertainty with high confidence. The set is obtained as the solution of a convex scenario problem where the scenarios are taken from measured data and the number of scenarios determines the probabilistic bound. A similar method can be used in our case to characterize the typical variability of the weather and internal gains around predicted values. Data from historical prediction



(a) $\boldsymbol{\mathcal{I}}$



(b) $\hat{\boldsymbol{\mathcal{I}}}$

Fig. 3: Information structure for the example. It shows that decisions at time step t can depend on the reference up to time t and disturbance up to time $t-1$. Below is the resulting information structure for the modifier function.

error are easily available for weather. Generally speaking the selection of good uncertainty sets in robust optimization are a subject of active research [30] and fall outside the scope of the present work. It is assumed here that the maximum variability of the disturbance is 10% of its nominal value. Finally we have $\hat{\Xi} = \hat{\mathcal{R}} \times \mathcal{W}$.

We consider here an affine controller and modifier function as in (27). Our aim is to characterize the largest tracking reference set. Following the rules of the Swiss ancillary market, the bid is a fixed up/down capacity over the tracking period. This means the allowable modifier function is a uniform scaling of the uncertainty set (that is, time-varying capacity is not allowed). For clarity we keep the description of the uncertainty split between the reference to track and the external disturbance, so that: $\boldsymbol{\xi} = (\boldsymbol{r}, \boldsymbol{w})$ and $\boldsymbol{\nu} = (\boldsymbol{\nu}_{\mathcal{R}}, \boldsymbol{\nu}_{\mathcal{W}})$. We assume the weather uncertainty is unknown at the time of the decision whereas the reference is revealed as it needs to be tracked: this results in an information structure that is depicted in Figure 3. We see that the modifier function could theoretically modify the uncertainty set so as to "mix" the external disturbance and the reference. In this application, it would not have physical sense so it is preferable to keep a block diagonal structure for the modifier's information structure. The disturbance uncertainty set is fixed a priori while the reference set can be modified. Furthermore, in the case that the reference set is a fixed up/down box along the horizon then the reference tracking set can only be scaled uniformly so that the modifier function will reduce to the simpler form:

$$\mathbf{L} = \begin{pmatrix} \lambda \mathbb{I}_N & 0_{N,Nn_u} \\ 0_{Nn_w,N} & \mathbb{I}_{Nn_w} \end{pmatrix} \tag{57}$$

To maximize the up/down capacity bid, it suffices to maximize the scaling factor λ. Notice that enforcing (57) implicitly enforces the requirement that $\boldsymbol{\nu} \in \hat{\boldsymbol{\mathcal{I}}}$. The description of the uncertainty set $\hat{\Xi} = \hat{\mathcal{R}} \times \mathcal{W}$

can easily be put under the form of equation (29) since it is the direct product of a polyhedron with three ellipsoids. Details are skipped for brevity.

The optimal tracking commitment problem (44) is solved with cost function $J = c_e^t \bar{\mathbf{u}} - c_{comm}\lambda$ where c_e^t is the time-varying price of electricity, $\bar{\mathbf{u}}$ is the baseline consumption in the absence of disturbance and c_{comm} is the unit reward price of the power tracking commitment (hence promising to track $\pm 1kW$ for the commitment period is rewarded at the price c_{comm}).

The tracking error is sized proportionally to the tracking requirement so that tracking errors amounting up to 10% of the maximum tracking requirement are allowed. This yields:

$$\mathcal{E} := \{\boldsymbol{e} \mid \|\boldsymbol{e}\|_\infty \leqslant 0.1\lambda\}$$

A horizon of one day with a timestep of one hour is considered. For the sake of illustration, we take $c_e << c_{comm}$ in order to favour participation in the tracking commitment. The problem solved is a second-order cone problem with 240,000 non-zero variables and 900 second-order cone constraints. Solving time on a 2.7GHz i-Core 7 platform was 10 seconds. The optimal value of λ is 6.1, meaning that the building can offer a 6.1kW up/down power tracking capacity for a period of 10h with an integral energy limit of 30.5kWh and maximum tracking error of ± 0.61 kW. This represents 13.5% of the peak heating power and about 30% of the average power consumption for that particular day.

Figure 4 shows the open-loop trajectories generated in response to randomly generated weather and reference signals inside the prescribed sets. In each of the plots, the shaded band shows the reference tracking times. The different plots shows the average temperature in the building as well as in individual zones, the total power consumption in the building, the requested power consumption to be tracked on top of the nominal consumption, the state of the virtual battery and the tracking error.

B. Influence of the integral constraint in the reference set

As outlined previously, frequency control bids theoretically impose the providers to be able to offer up or down regulation for long periods of time, which appears to be limiting for loads. In this section, the influence of the addition of the integral constraint is studied. Notice that contrary to the box reference set, the battery reference set is time-correlated, and our approach directly accounts for that.

We respectively consider a box reference set (54) and a battery reference set (55). When optimizing over the set, a uniform scaling over time is considered as described in equation (57). This does not allow separate optimization of the maximum power rate and maximum capacity of the battery. A prototype battery shape is obtained by using actual measured data for the tracking signal used by SwissGrid during the year 2013. The following procedure has been applied:

- the historic ACS signal is normalized by the maximum observed ACS signal requested over the whole year.
- the empirical integral value of the signal over any one day period is considered as the maximum capacity of the battery
- the initial state of the battery is set at half of the maximum capacity $s_{max}/2$. This is justified by the symmetric nature of the tracking bid. Empirical data also suggest that up and down regulation are close to equally likely.

The power to capacity ratio of the corresponding battery is 5.6 (that is for 1kW of up/down power offered, a maximum capacity of 5.6kWh is considered). This suggests that the ACS tracking signal is relatively well-behaved in the sense that it does not typically ask for maximum positive or negative power tracking for long periods of time.

To study the influence of the integral limit in the reference set, the tracking bid is evaluated as a function of the duration of the tracking commitment. the preparation time is kept at 8 hours. The weather is considered known perfectly in advance in this case to rule out other factors of uncertainty in the computation. Beyond 66h, the computational burden becomes prohibitive. The maximum scalings for the battery and the box uncertainty sets are reported on Figure 5. Introducing an integral limit for the tracking

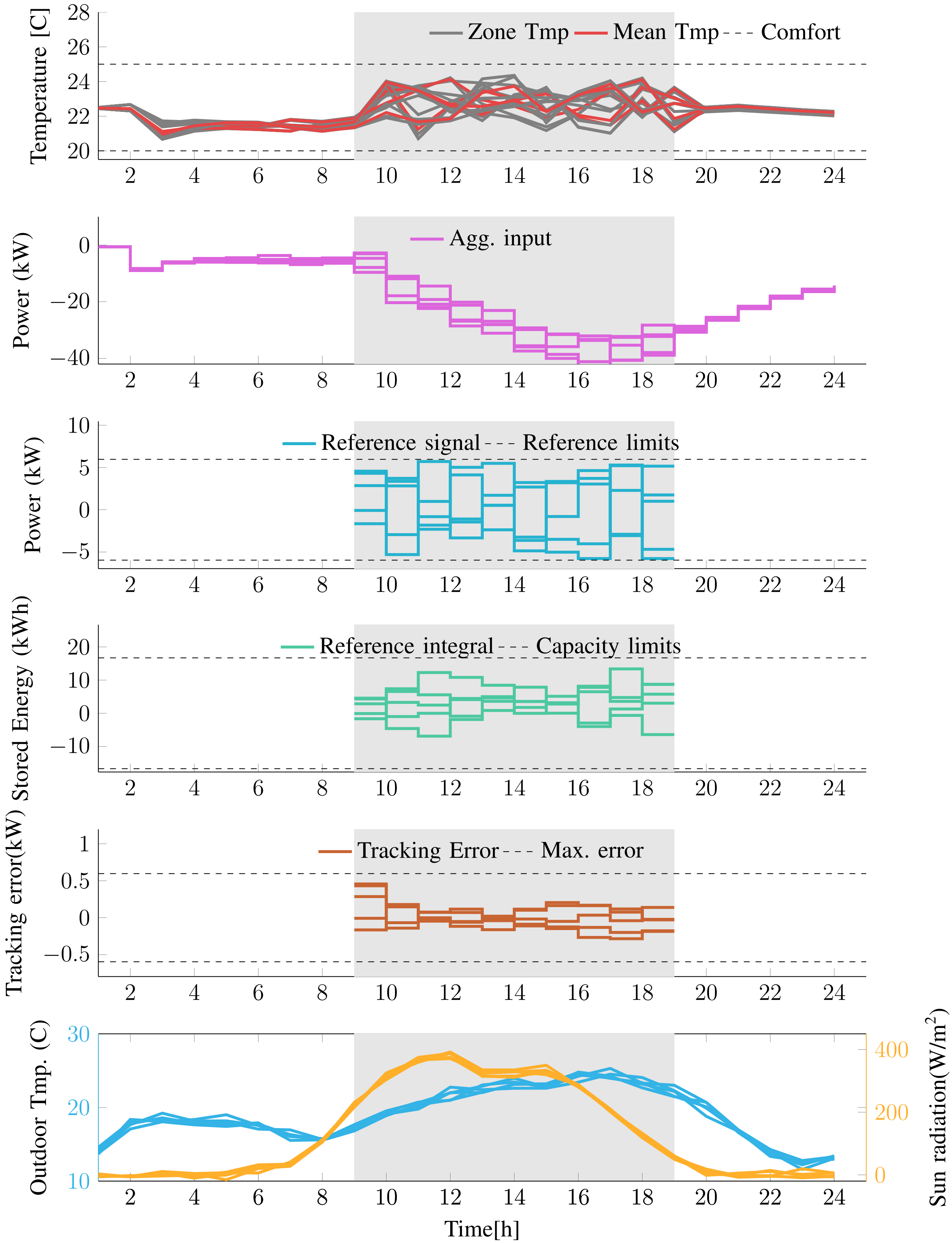


Fig. 4: Open-loop trajectories for randomly generated weather and reference trajectories. The shaded region corresponds to the tracking commitment period.

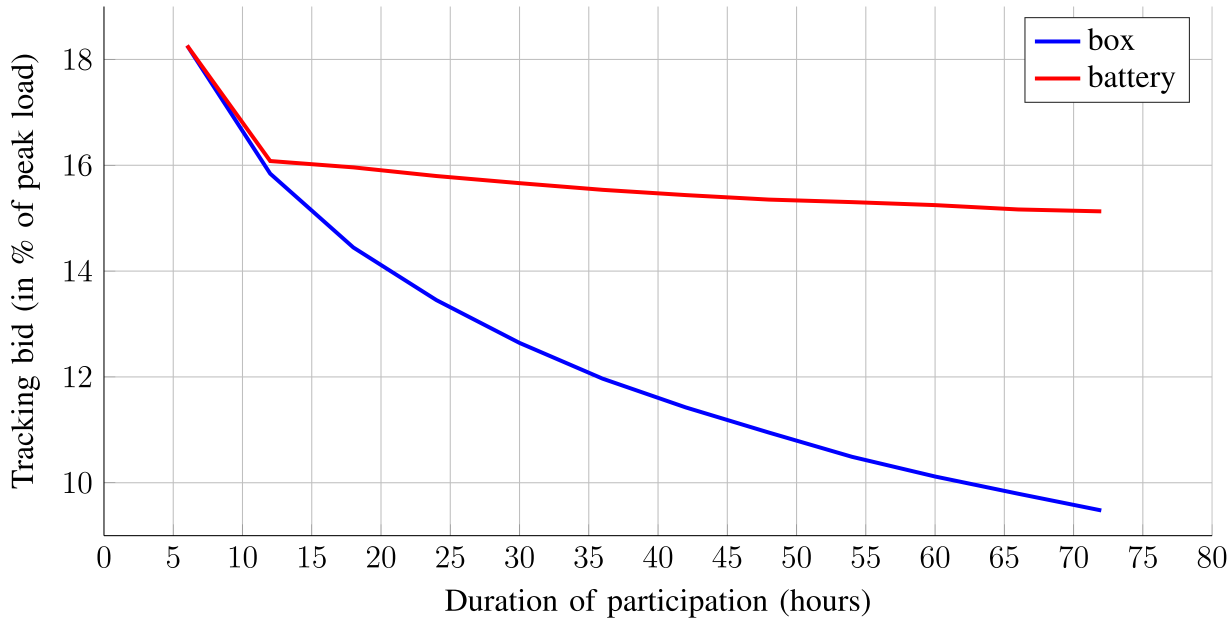


Fig. 5: Flexibility versus duration of participation

commitment allows to improve the tracking bid significantly as the duration of participation time increases. Indeed, situations of long lasting positive or negative tracking request are ruled out, thus relieving the tracking requirements on the building, and leading to less conservative solutions.

V. Conclusion

In this paper, we have formulated the problem of optimal robust commitment tracking and proposed a computationally tractable solution method. By implicit modification of uncertainty sets, the set of possible reference trajectories that can be tracked under additive disturbance with a guaranteed error bound can be efficiently computed over a finite prediction horizon. The presented tracking commitment framework is representative of many practical problems encountered in the hierarchical control of complex systems, that requires communication of tracking capability of subsystems to an upper level control layer. We have illustrated the description capability of the framework and the solution method with a practical example that investigates ancillary service provision to the power grid by a commercial building.

Appendix A
Polytopic description of the feasibility set $\mathcal{Q}$

The dense form of the system equations (1), which describes the evolution of the system for N steps, is given by

$$\boldsymbol{x} = \mathbf{A}x_0 + \mathbf{B}\boldsymbol{u} + \mathbf{E}\boldsymbol{\omega}$$
$$\boldsymbol{y} = \mathbf{C}\boldsymbol{x} + \mathbf{D}\boldsymbol{u}$$

The matrices $\mathbf{A} \in \mathbb{R}^{(N+1)n_x \times n_x}$, $\mathbf{B} \in \mathbb{R}^{(N+1)n_x \times Nn_u}$, $\mathbf{E} \in \mathbb{R}^{(N+1)n_x \times Nn_x}$, $\mathbf{C} \in \mathbb{R}^{Nn_y \times (N+1)n_x}$ and $\mathbf{D} \in \mathbb{R}^{Nn_y \times Nn_u}$ are defined as

$$\mathbf{A} := \begin{bmatrix} I_{n_x} \\ A \\ A^2 \\ \vdots \\ A^N \end{bmatrix}, \mathbf{E} := \begin{bmatrix} 0 & \cdots & \cdots & 0 \\ I_{n_x} & 0 & \cdots & \vdots \\ A & I_{n_x} & \cdots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ A^{N-1} & A^{N-2} & \cdots & I_{n_x} \end{bmatrix}$$

$$\mathbf{B} := \mathbf{E}(I_N \otimes B), \ \mathbf{C} := [I_N \otimes C \ \ \mathbf{0}], \ \mathbf{D} := I_N \otimes D$$

The polytopic state, input constraints and the disturbance set can be described as

$$\mathcal{X} := \{\boldsymbol{\phi} \in \mathbb{R}^{Nn_x} : \mathbf{F}_x\boldsymbol{\phi} \leqslant \mathbf{f}_x\}$$
$$\mathcal{U} := \{\boldsymbol{u} \in \mathbb{R}^{Nn_u} : \mathbf{F}_u\boldsymbol{u} \leqslant \mathbf{f}_u\}$$
$$\mathcal{W} := \{\boldsymbol{w} \in \mathbb{R}^{Nn_x} : \mathbf{F}_w\boldsymbol{w} \leqslant \mathbf{f}_w\}$$

The matrices used in the descriptions (3) and (42) of the feasibility set $\mathcal{Q}$ are given by

$$T = \begin{bmatrix} [\mathbf{0}\ \ \mathbf{F}_x]\mathbf{B} \\ \mathbf{F}_u \end{bmatrix},\ V = \begin{bmatrix} [\mathbf{0}\ \ \mathbf{F}_x]\mathbf{E} \\ \mathbf{0} \end{bmatrix},\ h = \begin{bmatrix} \mathbf{f}_x \\ \mathbf{f}_u \end{bmatrix} - \begin{bmatrix} [\mathbf{0}\ \ \mathbf{F}_x]\mathbf{A} \\ \mathbf{0} \end{bmatrix} x_0$$
$$P := \mathbf{CB} + \mathbf{D}, \quad S = \mathbf{CE}, \quad \bar{\boldsymbol{y}} = \mathbf{CA}x_0$$
$$H := \begin{bmatrix} T \\ -P \end{bmatrix}, \quad Q = \begin{bmatrix} \mathbf{0} & V \\ -S & G \end{bmatrix}, \quad q = \begin{bmatrix} h \\ g + \bar{\boldsymbol{y}} \end{bmatrix}$$

where $\mathbf{0}$'s are matrices of zeros with proper dimensions.

Appendix B
Proofs for theorems of Section II-B

Notations: Given a set of indices $\mathcal{J}$, let $\bar{\mathcal{J}}$ be the complementary of $\mathcal{J}$ in $\mathbb{Z}_{[1,n]}$. Denote m the cardinality of $\mathcal{J}$. As $x_{\mathcal{J}}$ denotes the entries of x indexed by $\mathcal{J}$, $\boldsymbol{\nu}_{\mathcal{J}}$ denotes the function from $\mathbb{R}^n$ into $\mathbb{R}^m$ formed by the outputs of $\boldsymbol{\nu}$ indexed by $\mathcal{J}$. Given $\mathcal{J}$, we also overload notations and denote $\boldsymbol{\nu}(x_{\mathcal{J}}, x_{\bar{\mathcal{J}}})$ to make explicit the respective dependency of $\boldsymbol{\nu}$ on $x_{\mathcal{J}}$ and $x_{\bar{\mathcal{J}}}$. Accordingly, denote $\boldsymbol{\nu}(x_{\mathcal{J}}, .)$ the restriction of $\boldsymbol{\nu}$ to $\{x_{\mathcal{J}}\} \times \mathbb{R}^{n-m}$.

Proof of Lemma 3. Consider two information structures $\mathcal{I}_1$ and $\mathcal{I}_2$. Suppose $f \in \mathcal{F}(\mathcal{I}_1), \mathcal{F}(\mathcal{I}_2)$. Let x, x' be such that $x_{\mathcal{I}_1\cap\mathcal{I}_2} = x'_{\mathcal{I}_1\cap\mathcal{I}_2}$. Choose y such that $y_{\mathcal{I}_1} = x_{\mathcal{I}_1}$ and $y_{\mathcal{I}_2} = x'_{\mathcal{I}_2}$ (this is possible because $x_{\mathcal{I}_1\cap\mathcal{I}_2} = x'_{\mathcal{I}_1\cap\mathcal{I}_2}$). Since $f \in \mathcal{F}(\mathcal{I}_1)$, we have that $f(x) = f(y)$. Similarly, $f \in \mathcal{F}(\mathcal{I}_2)$ implies that $f(x') = f(y)$. Together this gives $f(x) = f(y) = f(x')$ for all x, x' such that $x_{\mathcal{I}_1\cap\mathcal{I}_2} = x'_{\mathcal{I}_1\cap\mathcal{I}_2}$ *i.e.* $f \in \mathcal{F}(\mathcal{I}_1 \cap \mathcal{I}_2)$. Noticing that $\bigcap_k \mathcal{I}_k = \mathcal{I}_1 \cap (\bigcap_{k\neq 1} \mathcal{I}_k)$, it is straightforward to extend the argument above to the intersection of finitely many information structures. ☐

Proof of Lemma 4. By convention, $\hat{\mathcal{I}}_k = \mathbb{Z}_{[1,n]}$ if $\{i|k \in \mathcal{I}_i\}$ is empty.
Direction $\Longleftarrow$: Assume $\boldsymbol{g} \in \mathcal{F}(\hat{\boldsymbol{\mathcal{I}}})$. Consider $(x, \hat{x})$ such that $x_{\mathcal{I}_j} = x'_{\mathcal{I}_j}$ and $\boldsymbol{f} \in \mathcal{F}(\boldsymbol{\mathcal{I}})$. Let us prove that $\boldsymbol{f}\circ\boldsymbol{g}(x) = \boldsymbol{f}\circ\boldsymbol{g}(x')$. Let us denote $y = \boldsymbol{g}(x)$ and $y' = \boldsymbol{g}(x')$. Let us consider any $k \in \mathcal{I}_j$. Then according to equation (20), we have $\hat{\mathcal{I}}_k \subseteq \mathcal{I}_j$ and hence $x_{\hat{\mathcal{I}}_k} = x'_{\hat{\mathcal{I}}_k}$. In turn this implies $y_k = y'_k$ by definition of $\mathcal{F}(\hat{\boldsymbol{\mathcal{I}}})$. Since this holds for all $k \in \mathcal{I}_j$, it holds that $y_{\mathcal{I}_j} = y'_{\mathcal{I}_j}$ and therefore $\boldsymbol{f} \circ \boldsymbol{g}(x) = \boldsymbol{f}(y) = \boldsymbol{f}(y') = \boldsymbol{f} \circ \boldsymbol{g}(x')$ since $\boldsymbol{f} \in \mathcal{F}(\boldsymbol{\mathcal{I}})$.
Direction $\Longrightarrow$: Assume $\boldsymbol{g} \notin \mathcal{F}(\hat{\boldsymbol{\mathcal{I}}})$. There exists an index j such that $g_j \notin \mathcal{F}(\hat{\mathcal{I}}_j)$. Since $\hat{\mathcal{I}}_j = \bigcap_{\{i|j\in\mathcal{I}_i\}} \mathcal{I}_i$ we can use Lemma 3 to conclude that there exists i such that $g_j \notin \mathcal{F}(\mathcal{I}_i)$ and $j \in \mathcal{I}_i$. (The intersection is non-empty since if it was then $\hat{\mathcal{I}}_j = \mathbb{Z}_{[1,n]}$, which contradicts the possibility that $g_j(x) \neq g_j(x')$). Then there exist x and x' such that $x_{\mathcal{I}_i} = x'_{\mathcal{I}_i}$ and $g_j(x) \neq g_j(x')$. Consider the function $\boldsymbol{f}$ defined as follows: $\forall k \neq i, f_k$ is identically 0. This trivially implies $f_k \in \mathcal{F}(\mathcal{I}_k)$ no matter what $\boldsymbol{\mathcal{I}}$ is. Define f_i as:

$$\begin{cases} f_i(y) = 1 \text{ if } y_j = g_j(x') \\ f_i(y) = 0 \text{ otherwise} \end{cases}$$

Consider y, y' such that $y_{\mathcal{I}_i} = y'_{\mathcal{I}_i}$. Since $j \in \mathcal{I}_i$, we have $y_j = y'_j$ and hence $f_i(y) = f_i(y')$. Therefore $f_i \in \mathcal{F}(\mathcal{I}_i)$ and $\boldsymbol{f} \in \mathcal{F}(\boldsymbol{\mathcal{I}})$. However, $f_i\circ\boldsymbol{g}(x) = 0$ and $f_i\circ\boldsymbol{g}(x') = 1$ by definition of f_i. Putting everything together, we can conclude that $x_{\mathcal{I}_i} = x'_{\mathcal{I}_i}$ and $\boldsymbol{f} \circ \boldsymbol{g}(x) \neq \boldsymbol{f} \circ \boldsymbol{g}(x')$, therefore $\boldsymbol{f} \circ \boldsymbol{g} \notin \mathcal{F}(\boldsymbol{\mathcal{I}})$. ☐

Proof of Lemma 5. $\mathcal{G} \subseteq \mathcal{F}(\boldsymbol{\mathcal{I}})$: It directly follows from Lemma 4.

$\mathcal{F}(\boldsymbol{\mathcal{I}}) \subseteq \mathcal{G}$: Consider $\boldsymbol{g} \in \mathcal{F}(\boldsymbol{\mathcal{I}})$. Showing that there exists $\boldsymbol{f} \in \mathcal{F}(\boldsymbol{\mathcal{I}})$ such that $\boldsymbol{g} = \boldsymbol{f} \circ \boldsymbol{\nu}$ is equivalent to showing that $\boldsymbol{f} = \boldsymbol{g} \circ \boldsymbol{\nu}^{-1} \in \mathcal{F}(\boldsymbol{\mathcal{I}})$ ($\boldsymbol{\nu}$ is a bijection). It is done by contradiction. Suppose $\boldsymbol{f} \notin \mathcal{F}(\boldsymbol{\mathcal{I}})$. This means that for some k, $f_k \notin \mathcal{F}(\mathcal{I}_k)$. To lighten notation, let $\mathcal{I}_k = \mathcal{J}$. There exist y, y' such that $y_{\mathcal{J}} = y'_{\mathcal{J}}$ and $f_k(y) \neq f_k(y')$. By definition of $\hat{\boldsymbol{\mathcal{I}}}$, $\boldsymbol{\nu}_{\mathcal{J}}$ cannot depend on elements of $\bar{\mathcal{J}}$, *i.e.* $x_{\mathcal{J}} = x'_{\mathcal{J}} \Longrightarrow \boldsymbol{\nu}_{\mathcal{J}}(x) = \boldsymbol{\nu}_{\mathcal{J}}(x')$. Fix $x \in \mathbb{R}^n$. We divide the remainder of the proof in intermediate steps for clarity.

Bijectivity of $\boldsymbol{\nu}_{\mathcal{J}}(., x_{\bar{\mathcal{J}}})$: Notice that $\boldsymbol{\nu}_{\bar{\mathcal{J}}}(x_{\mathcal{J}}, .)$ is injective in $\mathbb{R}^{n-m}$ since $\boldsymbol{\nu}$ is injective. Denoting $V(x_{\mathcal{J}}) = \boldsymbol{\nu}_{\bar{\mathcal{J}}}(x_{\mathcal{J}}, \mathbb{R}^{n-m})$, by continuity of $\boldsymbol{\nu}$, $V(x_{\mathcal{J}})$ is an open set. By injectivity of $\boldsymbol{\nu}$, if $\boldsymbol{\nu}_{\mathcal{J}}(x) = \boldsymbol{\nu}_{\mathcal{J}}(x')$ with $x_{\mathcal{J}} \neq x'_{\mathcal{J}}$, then $V(x)$ and $V(x')$ are disjoint. By surjectivity of $\boldsymbol{\nu}$, it also holds that $\cup_{\{x'_{\mathcal{J}} | \boldsymbol{\nu}_{\mathcal{J}}(x_{\mathcal{J}}, x_{\bar{\mathcal{J}}}) = \boldsymbol{\nu}_{\mathcal{J}}(x'_{\mathcal{J}}, x_{\bar{\mathcal{J}}})\}} V(x'_{\mathcal{J}}) = \mathbb{R}^{n-m}$. Since $\mathbb{R}^{n-m}$ is connected, it cannot be covered by a non-trivial union of disjoint open sets, which implies that $\{x'_{\mathcal{J}} | \boldsymbol{\nu}_{\mathcal{J}}(x_{\mathcal{J}}, x_{\bar{\mathcal{J}}}) = \boldsymbol{\nu}_{\mathcal{J}}(x'_{\mathcal{J}}, x_{\bar{\mathcal{J}}})\}$ is reduced to $\{x_{\mathcal{J}}\}$, which in other words means injectivity of $\boldsymbol{\nu}_{\mathcal{J}}(., x_{\bar{\mathcal{J}}})$.
Surjectivity of $\boldsymbol{\nu}_{\mathcal{J}}(., x_{\bar{\mathcal{J}}})$ directly follows from the surjectivity of $\boldsymbol{\nu}$. Indeed, $\forall y \in \mathbb{R}^m$ there exist x' such that $\boldsymbol{\nu}_{\mathcal{J}}(x') = y$. Then $\boldsymbol{\nu}_{\mathcal{J}}(x'_{\mathcal{J}}, x_{\bar{\mathcal{J}}}) = y$. Together, this proves the bijectivity of $\boldsymbol{\nu}_{\mathcal{J}}(., x_{\bar{\mathcal{J}}})$ for all $x_{\bar{\mathcal{J}}}$.

Bijectivity of $\boldsymbol{\nu}_{\bar{\mathcal{J}}}(x_{\mathcal{J}}, .)$: Injectivity directly follows from the injectivity of $\boldsymbol{\nu}$. For $x_{\mathcal{J}}$ fixed, by injectivity of $\boldsymbol{\nu}_{\mathcal{J}}(., x_{\bar{\mathcal{J}}})$ there does not exist any other $x'_{\mathcal{J}}$ such that $\boldsymbol{\nu}_{\mathcal{J}}(x_{\mathcal{J}}, x_{\bar{\mathcal{J}}}) = \boldsymbol{\nu}_{\mathcal{J}}(x'_{\mathcal{J}}, x_{\bar{\mathcal{J}}})$. Therefore, surjectivity of $\boldsymbol{\nu}$ implies that $\boldsymbol{\nu}_{\bar{\mathcal{J}}}(x_{\mathcal{J}}, \mathbb{R}^{n-m}) = \mathbb{R}^{n-m}$, *i.e.* surjectivity of $\boldsymbol{\nu}_{\bar{\mathcal{J}}}(x_{\mathcal{J}}, .)$.

Contradiction : Consider $x_{\mathcal{J}}$ such that $\boldsymbol{\nu}_{\mathcal{J}}(x_{\mathcal{J}}, x_{\bar{\mathcal{J}}}) = y_{\mathcal{J}}$. Bijectivity of $\boldsymbol{\nu}_{\mathcal{J}}(.,, x_{\bar{\mathcal{J}}})$ ensures its existence. In turn, bijectivity of $\boldsymbol{\nu}_{\bar{\mathcal{J}}}(x_{\mathcal{J}}, .)$ ensures that there exists $x_{\bar{\mathcal{J}}}, x'_{\bar{\mathcal{J}}}$ such that $\boldsymbol{\nu}_{\bar{\mathcal{J}}}(x_{\mathcal{J}}, x_{\bar{\mathcal{J}}}) = y_{\bar{\mathcal{J}}}$ and $\boldsymbol{\nu}_{\bar{\mathcal{J}}}(x_{\mathcal{J}}, x'_{\bar{\mathcal{J}}}) = y'_{\bar{\mathcal{J}}}$. Combining the results above gives $\boldsymbol{\nu}(x_{\mathcal{J}}, x_{\bar{\mathcal{J}}}) = y$ and $\boldsymbol{\nu}(x_{\mathcal{J}}, x'_{\bar{\mathcal{J}}}) = y'$. Then, $g_k(x) = f_k \circ \boldsymbol{\nu}(x) = f_k(y)$ and similarly $g_k(x') = f_k \circ \boldsymbol{\nu}(x') = f_k(y')$. Finally, this shows that $g_k(x) \neq g_k(x')$ which implies $g_k \notin \mathcal{F}(\mathcal{I}_k)$. this contradicts the assumption that $\boldsymbol{g} \in \mathcal{F}(\boldsymbol{\mathcal{I}})$. Finally, this confirms that $\boldsymbol{f} \in \mathcal{F}(\boldsymbol{\mathcal{I}})$. $\square$

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REFERENCES

[1] Y. Rebours, D. Kirschen, M. Trotignon, and S. Rossignol, "A Survey of Frequency and Voltage Control Ancillary Services mdash;Part I: Technical Features," *IEEE Transactions on Power Systems*, vol. 22, no. 1, pp. 350–357, Feb. 2007.

[2] Swissgrid, "Overview of ancillary service products." [Online]. Available: https://www.swissgrid.ch/dam/swissgrid/experts/ancillary_services/Dokumente/D100412_AS-concept_V1R0_en.pdf

[3] J. MacDonald, P. Cappers, D. S. Callaway, and S. Kiliccote, "Demand response providing ancillary services a comparison of opportunities and challenges in the us wholesale markets," in *Grid-Interop 2012*, Irving, TX, December 2012 2012.

[4] T. Gorecki, A. Bitlislioğlu, G. Stathopoulos, and C. N. Jones, "Guaranteeing input tracking for constrained systems: theory and application to demand response," in *the 2015 American Control Conference (ACC) (to appear)*, 2015. [Online]. Available: http://arxiv.org/abs/1410.7297

[5] M. Maasoumy, C. Rosenberg, A. Sangiovanni-Vincentelli, and D. Callaway, "Model predictive control approach to online computation of demand-side flexibility of commercial buildings HVAC systems for Supply Following," in *American Control Conference (ACC), 2014*, Jun. 2014, pp. 1082–1089.

[6] A. Ben-Tal, L. El Ghaoui, and A. Nemirovski, *Robust Optimization*. Princeton University Press, 2009.

[7] S. Raković, "Invention of Prediction Structures and Categorization of Robust MPC Syntheses," M. Lazar, Ed., Aug. 2012, pp. 245–273.

[8] D. Mayne, M. Seron, and S. Raković, "Robust model predictive control of constrained linear systems with bounded disturbances," *Automatica*, vol. 41, no. 2, pp. 219–224, Feb. 2005.

[9] D. Q. Mayne, E. C. Kerrigan, E. J. van Wyk, and P. Falugi, "Tube-based robust nonlinear model predictive control," *International Journal of Robust and Nonlinear Control*, vol. 21, no. 11, pp. 1341–1353, Jul. 2011.

[10] P. J. Goulart, E. C. Kerrigan, and J. M. Maciejowski, "Optimization over state feedback policies for robust control with constraints," *Automatica*, vol. 42, no. 4, pp. 523–533, Apr. 2006.

[11] A. Georghiou, W. Wiesemann, and D. Kuhn, "Generalized decision rule approximations for stochastic programming via liftings," *Mathematical Programming*, pp. 1–38, May 2014.

[12] J. Warrington, P. Goulart, S. Mariethoz, and M. Morari, “Policy-Based Reserves for Power Systems,” *IEEE Transactions on Power Systems*, vol. 28, no. 4, pp. 4427–4437, Nov. 2013.

[13] F. Oldewurtel, T. Borsche, M. Bucher, P. Fortenbacher, M. G. V. T. Haring, J. L. Mathieu, O. Megel, E. Vrettos, and G. Andersson, “A framework for and assessment of demand response and energy storage in power systems,” in *Bulk Power System Dynamics and Control - IX Optimization, Security and Control of the Emerging Power Grid (IREP), 2013 IREP Symposium*, 2013, pp. 1–24.

[14] E. Vrettos, F. Oldewurtel, G. Andersson, and F. Zhu, “Robust Provision of Frequency Reserves by Office Building Aggregations,” in *Proceedings of the 19th IFAC World Congress*, Cape Town, South Africa, 2014.

[15] M. Balandat, F. Oldewurtel, M. Chen, and C. Tomlin, “Contract design for frequency regulation by aggregations of commercial buildings,” in *2014 52nd Annual Allerton Conference on Communication, Control, and Computing (Allerton)*, Sep. 2014, pp. 38–45.

[16] X. Zhang, M. Kamgarpour, P. Goulart, and J. Lygeros, “Selling robustness margins: A framework for optimizing reserve capacities for linear systems,” in *2014 IEEE 53rd Annual Conference on Decision and Control (CDC)*, Dec. 2014, pp. 6419–6424.

[17] E. Vrettos, F. Oldewurtel, and G. Anderson, “Robust reserve provision by commercial buildings considering energy-constrained frequency signals,” 2014. [Online]. Available: https://www.eeh.ee.ethz.ch/uploads/tx_ethpublications/WorkingPaperEV2014.pdf

[18] B. Francis, “The Linear Multivariable Regulator Problem,” *SIAM Journal on Control and Optimization*, vol. 15, no. 3, pp. 486–505, May 1977.

[19] U. Maeder and M. Morari, “Offset-free reference tracking with model predictive control,” *Automatica*, vol. 46, no. 9, pp. 1469–1476, Sep. 2010.

[20] P. Falugi and D. Mayne, “Model predictive control for tracking random references,” in *Control Conference (ECC), 2013 European*, Jul. 2013, pp. 518–523.

[21] S. Di Cairano and F. Borrelli, “Constrained tracking with guaranteed error bounds,” in *2013 IEEE 52nd Annual Conference on Decision and Control (CDC)*, Dec. 2013, pp. 3800–3805.

[22] S. V. Raković, E. C. Kerrigan, D. Q. Mayne, and K. I. Kouramas, “Optimized robust control invariance for linear discrete-time systems: Theoretical foundations,” *Automatica*, vol. 43, no. 5, pp. 831–841, May 2007.

[23] J. Zhen and D. den Hertog, “Computing the Maximum Volume Inscribed Ellipsoid of a Polytopic Projection,” *CentER Discussion Paper Series*, vol. 2015-004, Jan. 2015.

[24] Swissgrid, “Test for secondary control capability.” [Online]. Available: https://www.swissgrid.ch/dam/swissgrid/experts/ancillary_services/prequalification/D130422_Test-for-secondary-control-capability_V2R1_EN.pdf

[25] A. Shapiro, D. Dentcheva, and A. P. Ruszczyński, *Lectures on Stochastic Programming: Modeling and Theory*. Society for Industrial and Applied Mathematics, Philadelphia, 2009.

[26] S. Boyd, “Distributed Optimization and Statistical Learning via the Alternating Direction Method of Multipliers,” *Foundations and Trends in Machine Learning*, vol. 3, no. 1, pp. 1–122, 2010.

[27] T. T. Gorecki, F. A. Qureshi, and C. Jones, “Openbuild : An integrated simulation environment for building control,” 2014, submitted. [Online]. Available: https://infoscience.epfl.ch/record/197863

[28] K. Margellos, P. Goulart, and J. Lygeros, “On the Road Between Robust Optimization and the Scenario Approach for Chance Constrained Optimization Problems,” *IEEE Transactions on Automatic Control*, vol. 59, no. 8, pp. 2258–2263, Aug. 2014.

[29] M. Campi and S. Garatti, “The Exact Feasibility of Randomized Solutions of Uncertain Convex Programs,” *SIAM Journal on Optimization*, vol. 19, no. 3, pp. 1211–1230, Jan. 2008.

[30] D. Bertsimas, V. Gupta, and N. Kallus, “Data-driven robust optimization,” *arXiv preprint arXiv:1401.0212*, 2013.