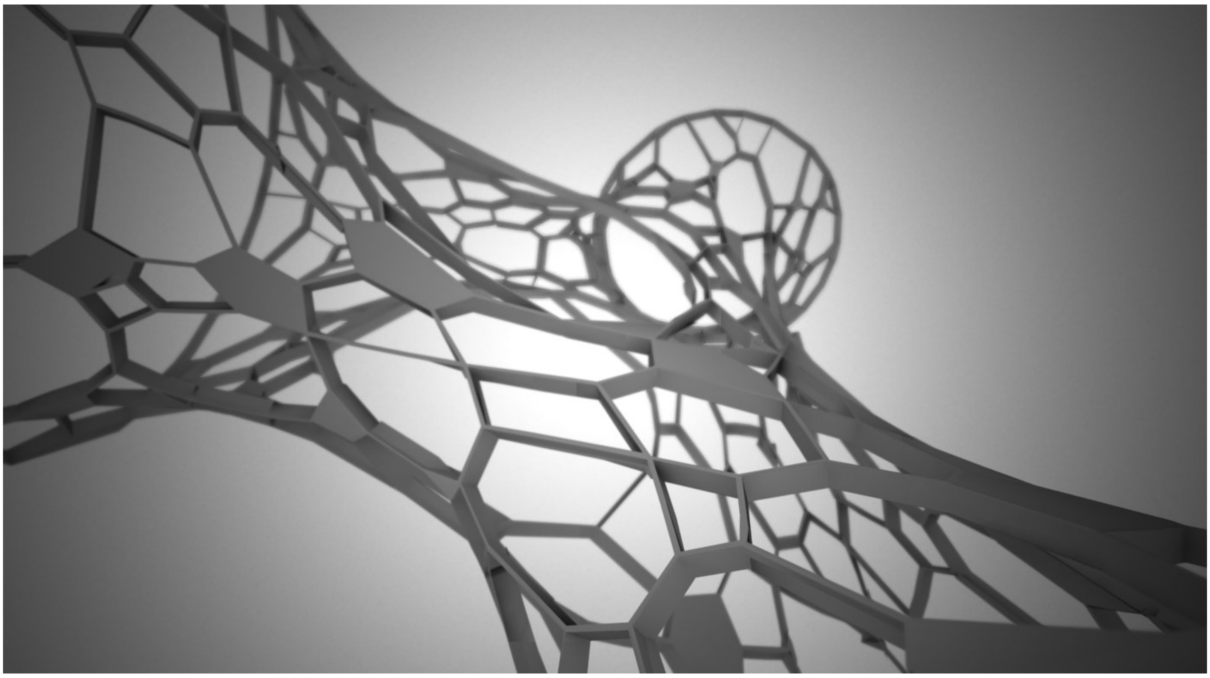

Pedestrian flow characterization based on spatio-temporal Voronoi tessellations

Marija Nikolić

Michel Bierlaire

Transport and Mobility Laboratory, EPFL

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Marija Nikolić, Michel Bierlaire
Transport and Mobility Laboratory
Civil and Environmental Engineering
École Polytechnique Fédérale de Lausanne
phone: +41 21 693 24 08
fax: +41 21 693 80 60
{marija.nikolic,michel.bierlaire}@epfl.ch

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Abstract

We propose a data-driven approach to pedestrian flow characterization. New definitions of density, flow and speed are derived by extending the ones existing in the literature through three-dimensional Voronoi diagrams. The proposed approach results in a set of definitions that are adjusted to the reality of the flow and independent from an arbitrarily chosen discretization. The advantages of this approach are illustrated empirically. Using data from simulation and walking experiments the approach is shown to: (i) reproduce the settings with uniform and non-uniform movement; (ii) reflect the self-organization phenomena typical for pedestrian traffic and pedestrian heterogeneity; (iii) allow for the analysis of congestion at the microscopic level; (iv) lead to smooth transitions in measured traffic characteristics.

Cover photo: `http://theprovingground.wdfiles.com/local--files/voronoi-structural-system/Voronoi%20Structure.jpg`

Keywords

pedestrian flow indicators, time and space discretization, three-dimensional Voronoi diagrams, individual trajectories

1 Introduction

Understanding and predicting of pedestrian flows are of the utmost importance for providing convenience and safety for pedestrians. In this respect a number of modeling approaches have been proposed in the literature (Duives *et al.,* 2013) addressing different levels of pedestrian movement and behavior (strategic, tactical or operational). In all cases speed (v, in m/s), density (k, in ped/m^2) and flow (q in ped/ms) are used as fundamental variables to describe pedestrian traffic. Nonetheless, the consistent and unified approach to the definitions of these variables is still missing.

Zhang (2012) has provided a comprehensive review of existing approaches to pedestrian flow characterization (denoted by A, B, C and D in the mentioned study). The approach A is concerned with a stream behavior on a small increment of a walkway over a longer period of time. A reference location in space (x) is considered and flow and (time-mean) speed are specified as the average over time (Δt):

$$q = \frac{n}{\Delta t} \tag{1}$$

$$v = \frac{1}{n}\sum_i v_i(t), \tag{2}$$

where n is the number of pedestrians passing the location x during Δt and $v_i(t)$ is the instantaneous speed of a pedestrian i observed at location x.

In the approach B density and (space-mean) speed are specified as the average over time (Δt) and space ($\Delta x \times \Delta y$):

$$k = \frac{1}{\Delta t}\int_t \frac{n}{\Delta x \Delta y} dt \tag{3}$$

$$v = \frac{\sum_i v_i}{n}, \tag{4}$$

where Δx and Δy are the length, respectively width of the considered discretization unit in space, and $v_i = \frac{\Delta x}{\Delta t}$ is the individual space-mean speed.

The approach C focuses on a stream behavior during a small increment of time over longer spatial units and defines density and (space-mean) speed per unit of space ($\Delta x \times \Delta y$) as:

$$k = \frac{n}{\Delta x \Delta y} \tag{5}$$

$$v = \frac{\sum_i v_i}{n}, \tag{6}$$

where Δx, Δy and v_i are the same as for the approach B.

The approach D (proposed by Steffen and Seyfried (2010)) is based on spatial discretization via Voronoi diagrams (Okabe *et al.,* 2009). Given the positions of the pedestrians at any time, Voronoi diagram for these positions is computed, giving cells A_i to each person i. Based on this discretization in space the density distribution is specified as:

$$k_i = \frac{1}{|A_i|}, \tag{7}$$

where $|A_i|$ is the area of the Voronoi cell A_i, with the unit m^2. Finally, density and speed are defined per unit of space ($\Delta x \times \Delta y$) as:

$$k = \frac{\int_x \int_y k_i dxdy}{\Delta x \Delta y}, \tag{8}$$

$$v = \frac{\int_x \int_y v_i dxdy}{\Delta x \Delta y}, \tag{9}$$

where v_i is the instantaneous speed of pedestrian i.

In the field of vehicular traffic the widely used definitions of traffic indicators are the ones proposed by Edie (1963). Edie's definitions are derived based on vehicular trajectories in the time-space region of duration Δt and length Δx as:

$$k = \frac{\sum_i t_i}{\Delta x \Delta t}, \tag{10}$$

$$q = \frac{\sum_i x_i}{\Delta x \Delta t}, \tag{11}$$

$$v = \frac{\sum_i x_i}{\sum_i t_i}, \tag{12}$$

where t_i is the time spent by vehicle i in the region $\Delta x \times \Delta t$ and x_i is the distance traversed by vehicle i in the region $\Delta x \times \Delta t$.

Saberi *et al.* (2014) have proposed a three-dimensional approach to pedestrian flow characterization by extending the definitions introduced by Edie (1963). Density and flow are defined as the flux of pedestrian trajectories through a plane $ax + by + ct + d = 0$ (with normal vector $n = (a, b, c)$) that is specified for the considered volume in the three-dimensional space-time diagram. Specifically, the density at time t is defined as the flux of trajectories through the plane parallel to the time axis:

$$k = \frac{Ndt}{|A|dt} = \frac{N}{|A|}, \tag{13}$$

where N is the total number of pedestrians passing through the plane and $|A|$ is the area of the specified plane with the unit square meters.

The flow is defined as the flux of trajectories through the plane vertical to the time axis:

$$q = \frac{Ndm}{|A|dm} = \frac{N}{|A|}, \tag{14}$$

where N is the total number of pedestrians passing through the plane, m is the normal vector of the specified plane, and $|A|$ is the area of the specified plane with the unit meters times seconds. The distinction is made between the backward and forward flow, thus accounting to some extent for the directional composition of pedestrian flow. A similar approach is presented in van Wageningen-Kessels *et al.* (2014), where definitions for speed and velocity are proposed additionally.

What all of the presented approaches appear to have in common is their discretization being chosen arbitrarily in at least one dimension. This may generate noise in the data, and results may be highly sensitive even to minor changes in the discretization in both time and space. If discretization units are too small, many of them are at risk of remaining empty, whereas a discretization with units too large may lead to the loss of heterogeneity. It is the approach D in which this issue has been partially addressed through a Voronoi-based spatial discretization. In addition, several empirical studies (Seyfried *et al.,* 2005, Zhang, 2012) have demonstrated that the results obtained by utilizing the presented approaches are incomparable due to the averaging performed over different degrees of freedom. Edie's definitions appeared to eliminate any ambiguity with respect to averaging and to lead to consistent results in observations and modeling (van Wageningen-Kessels *et al.,* 2014). What we suggest is combining the strengths of the approaches proposed by Edie (1963) and Steffen and Seyfried (2010) in order to address the issues with respect to pedestrian flow characterization. In particular, we propose a new set of definitions of pedestrian traffic variables derived by adapting Edie's definitions through data-driven spatio-temporal discretization.

The structure of the paper is as follows. Section 2 describes the data-driven discretization framework designed by using three-dimensional Voronoi diagrams. In Section 3, we formally define the characteristics of pedestrian traffic, that is density, flow and speed. Section 4 presents the preliminary empirical analysis of the proposed approach based on data from simulated and walking experiments. Finally, Section 5 summarizes the outcomes of the proposed methodology and determines future research directions.

2 Data-driven discretization framework

We consider a space-time representation, where triplet (x, y, t) represents a physical position (x, y) in space at a specific time t. An orthonormal basis for the spatial dimensions is considered, so that the distance along each of the two spatial axis is expressed in meters. Time is expressed in seconds.

We define the distance between two points $p_1 = (x_1, y_1, t_1)$ and $p_2 = (x_2, y_2, t_2)$ as:

$$d_\alpha(p_1, p_2) = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + \alpha(t_1 - t_2)^2}, \tag{15}$$

where α is a conversion constant expressed in meters per second. Note that using $\alpha = 1$m/s would give the interpretation that 1 second in time is equivalent to 1 meter in space.

The trajectory of pedestrian i is a curve in space and time, that is:

$$p_i(t) = (x_i(t), y_i(t), t), \tag{16}$$

such that $x_i(t)$ and $y_i(t)$ are the coordinates of the position of pedestrian i at time t. In practice, the analytical description of a trajectory is seldom available. Instead, the pedestrian trajectory data are collected by using an appropriate tracking technology (e.g. Daamen and Hoogendoorn (2003), Alahi *et al.* (2014)). In this case, a sample of points is available:

$$p_{is} = (x_{is}, y_{is}, t_s), \tag{17}$$

where i ($i = 1, ..., N$) is the pedestrian and $t_s = (t_0, t_1, ..., t_f)$ corresponds to the available sample.

We consider three-dimensional (3D) Voronoi diagram (Okabe *et al.,* 2009) associated with the points p_{is} for the distance d_α defined by (15). This spatio-temporal tessellation assigns a 3D cell V_{is} to each pedestrian i in such a way that each point $p = (x, y, t)$ in V_{is} is closer to i than to any other pedestrian, with respect of d_α:

$$V_{is} = \{p | d_\alpha(p, p_{is}) \leq d_\alpha(p, p_{js}), \forall j\}. \tag{18}$$

If p lies on the border between two or more Voronoi cells, exactly one of them is arbitrarily selected to be associated to p. The volume of a cell V_{is} is denoted by $Vol(V_{is})$ and has the unit square meters times seconds.

We define the set of all points in V_{is} corresponding to a specific time t, that is:

$$V_{is}(t) = \{(x, y, t) \in V_{is}\}. \tag{19}$$

It represents the set of dimension 2 or a physical area on the floor.

Similarly, we define the set of all points in V_{is} corresponding to a specific location (x, y), that is:

$$V_{is}(x, y) = \{(x, y, t) \in V_{is}\}. \tag{20}$$

It represents the set of dimension 1 or a time interval.

3 Traffic indicators

The definitions of traffic indicators are specified by extending the definitions of Edie (1963) through a data-driven discretization in space and time described in Section 2.

The density of the cell V_{is} around pedestrian i positioned at $p_{is} = (x_{is}, y_{is}, t_s)$ is defined as:

$$k_{is} = \frac{V_{is}(x_{is}, y_{is})}{Vol(V_{is})}. \tag{21}$$

The cell $V_{is}(x_{is}, y_{is})$ is the time interval 'occupied' by pedestrian i positioned at p_{is}. The unit of k_{is} is a number of pedestrians per square meter.

The flow for pedestrian i positioned at $p_{is} = (x_{is}, y_{is}, t_s)$ is defined as:

$$q_{is} = \frac{d_{is}}{Vol(V_{is})}, \tag{22}$$

where d_{is} is approximated by a maximum distance in $V_{is}(t_s)$ in the movement direction of pedestrian i positioned at p_{is}. Note that $V_{is}(t_s)$ is the region in space 'belonging' to pedestrian i at time t_s, with the unit square meter. The unit of q_{is} is a number of pedestrians per meter per second.

The speed of pedestrian i positioned at $p_{is} = (x_{is}, y_{is}, t_s)$ is defined as the ratio between the distance traversed and the time spent by a pedestrian in V_{is} :

$$v_{is} = \frac{d_{is}}{V_{is}(x_{is}, y_{is})}. \tag{23}$$

The unit of v_{is} is meters per second.

The three indicators are related as follows:

$$q_{is} = k_{is} v_{is}. \tag{24}$$

Note that upon taking the limit conditions ($t \to 0$, $x \to 0$, $y \to 0$) the definitions yield the classical ones (instantaneous and local).

The definitions given by (21), (22) and (23) are adjusted to the reality of the flow flow and are as much independent from the arbitrarily chosen discretization as possible. They provide the characterization of pedestrian traffic at the disaggregated level. If more aggregated indicators are of interest, then the indicators from multiple Voronoi cells are to be combined as follows:

$$k(V) = \frac{\sum_i k_{is} Vol(V_{is})}{\sum_i Vol(V_{is})} = \frac{\sum_i V_{is}(x_{is}, y_{is})}{\sum_i Vol(V_{is})}, \tag{25}$$

$$q(V) = \frac{\sum_i q_{is} Vol(V_{is})}{\sum_i Vol(V_{is})} = \frac{\sum_i d_{is}}{\sum_i Vol(V_{is})}, \tag{26}$$

$$v(V) = \frac{\sum_i d_{is}}{\sum_i V_{is}(x_{is}, y_{is})}. \tag{27}$$

Note that in specific circumstances the aggregation of speed and flow indicators can lead to their mutual cancellation. For instance, in a case of two equally sized groups of pedestrians who are walking at the same speed but in the opposite directions, the aggregation will lead to zero value of the resulting speed. Therefore, the proposed discretization framework is further enhanced by stream-based definitions of the variables so that they can be applied in the case of multi-directional flow composition. Since the focus of this paper is on data-driven discretization, for further details on stream-based definitions of traffic variables we refer to Nikolić and Bierlaire (2014).

4 Empirical analysis

In this section, we present the preliminary results of our analysis with an intention to illustrate the advantages of the proposed approach empirically. First, we consider pedestrian trajectory data that is generated in a simulated environment. In the second case, the data from a controlled walking experiment are employed. All analyses are performed with the parameter α fixed to the value $1m/s$.

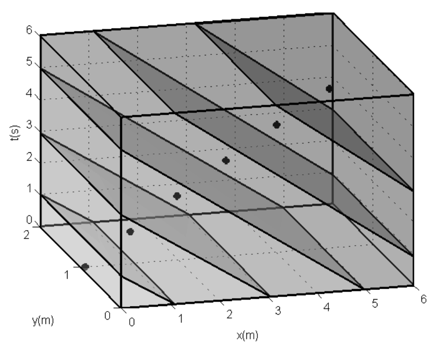
4.1 Simulation experiments

We first analyze the characteristics of pedestrian traffic defined in Section 3 in the case of a single pedestrian. Pedestrian is assumed to perform a straight line movement and his trajectory in an analytical form is given as:

$$p = (x(t), y(t), t) = (0.02t^2 + 0.9t + 0.1, 1, t). \tag{28}$$

We consider a three dimensional area of length 6 meters, width 2 meters and duration 6 seconds and sample points from pedestrian trajectory every 1 second. 3D Voronoi diagram obtained based on this sample is shown in Figure 1. Discretization based on Voronoi diagrams depends on the proximity of the points in the considered space-time region. Since a pedestrian in our experiment is alone, the Voronoi-based time intervals occupied by a pedestrian and regions in space belonging to him will be overestimated. Consequently this will lead to an inaccurate estimation of traffic indicators. However, if we bring more pedestrians into focus, it is expected that the accuracy of the results of the corresponding 3D Voronoi discretization increases. Indeed, this is the case of interest to be analyzed.

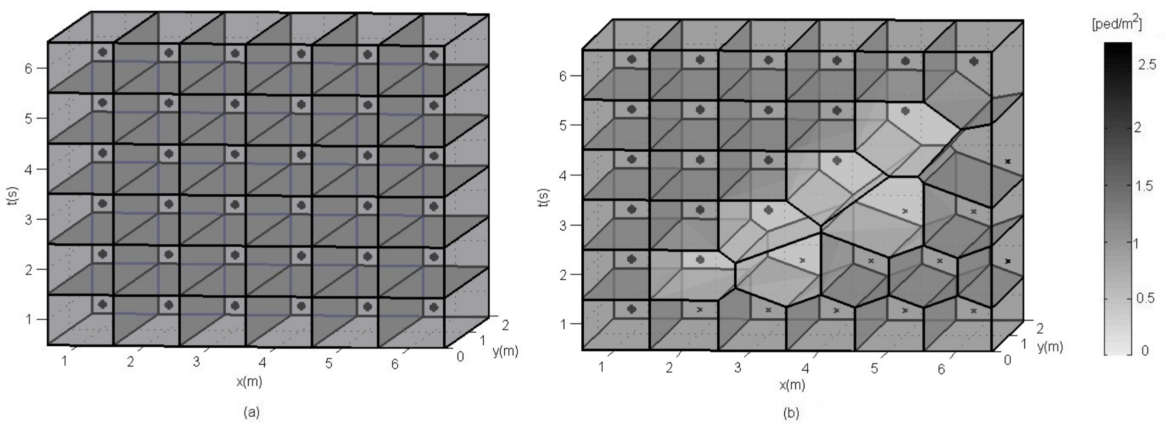
Figure 1: Three-dimensional Voronoi diagram for single pedestrian



To illustrate this we consider the experiment in which we sample from the trajectories of 11 pedestrians. In this case density is controlled by changing the speed of pedestrians over time. In the first case, we consider pedestrians who are walking at a constant speed, while in the second case, one group of pedestrians is made to walk at a faster speed. In both cases we consider a three dimensional area of length 6 meters, width 2 meters and duration 6 seconds.

The Voronoi-based density maps obtained for both the former and latter case respectively are shown in Figure 2(a) and Figure 2(b). In Figure 2(a) the black dots correspond to all of the positions of pedestrians over time. In Figure 2(b) the dots correlate with the positions of the slower-walking pedestrians while the crosses agree with the positions of those walking at a greater speed. The color corresponds to the density level in such a way that a darker gray color indicates a greater density. In the first case, the density is constant over space and time, whereas in the second one, inhomogeneous density conditions between the two groups of pedestrians are observable. This observation indicates the ability of our approach to reproduce the simulated settings with uniform and non-uniform movement.

Figure 2: Voronoi-based density maps for a sample from pedestrian trajectories: (a) uniform and (b) non-uniform movement



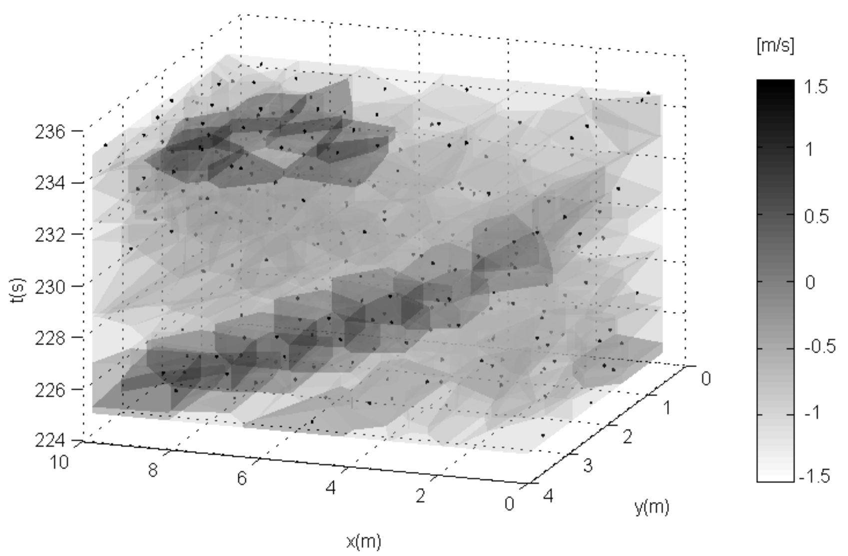
4.2 Controlled walking experiments

The second set of data has been collected during a controlled experiment at the Technical University of Delft in the Netherlands (Daamen and Hoogendoorn, 2003). The individuals participating in the experiment were instructed to walk at a normal speed along a corridor that is 10 meters long and 4 meters wide. The scene was filmed from above by digital cameras. The individual trajectories were extracted from the digital video sequences.

For the purposes of our analysis we use the subset of the data set corresponding to a bi-directional flow. The experiment lasted for about 7 minutes. A total of 709 trajectories were collected whereby the position of each individual was available every 0.1s. The average length of the trajectories is approximately 10 meters. The average time of the trajectories is 7.8 seconds.

The investigation that was performed on the basis of this data set has revealed the capability of the approach to reflect the lane formation for bi-directional flows (Figure 3). The Voronoi cells represented by darker gray color in Figure 3 correspond to a lane formed by pedestrians from a minor stream when confronted to a major stream from the opposite direction. Given that the approach allows to correlate the momentary speed of an individual pedestrian (or a group of pedestrians) with the availability of space, it is possible to get more insight into the formation of such patterns by analyzing the effects of flow composition and congestion at the microscopic level. This property makes the framework useful for the analysis of the self-organization phenomena typical for pedestrian traffic.

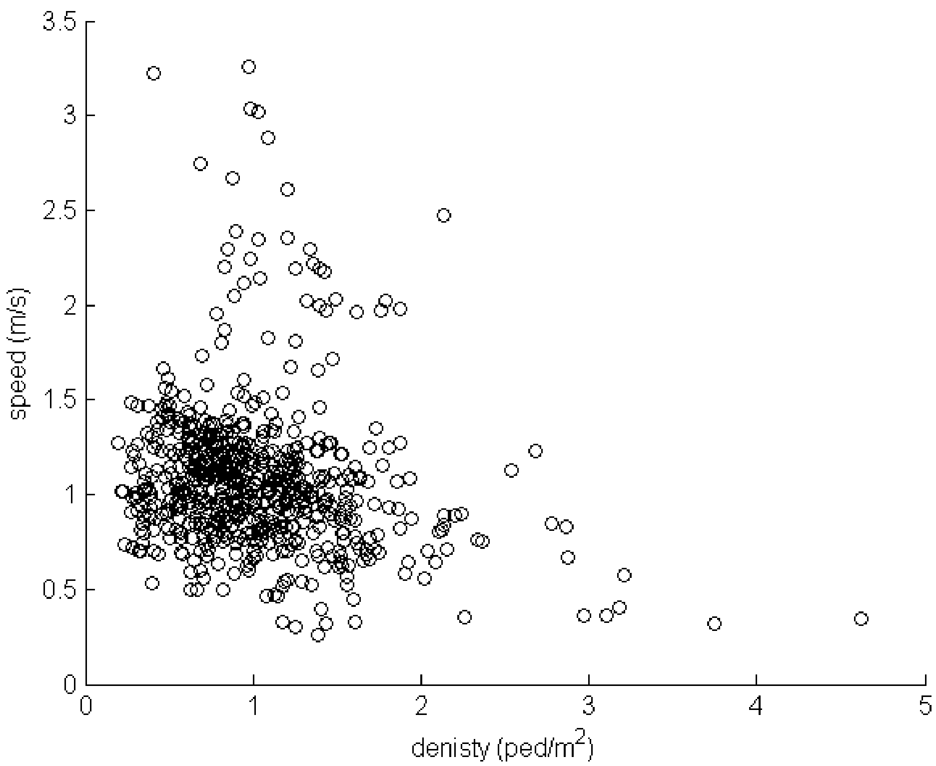
Figure 3: Self-organization phenomena revealed by three-dimensional Voronoi approach



The approach allows for the detailed representation of pedestrian traffic indicators at the disaggregated level. Figure 4 illustrates an empirical speed-density profile obtained through pedestrian trajectories from bi-directional experiment. The considered time interval ranges from 200 seconds to 235 seconds. Figure 4 shows 638 observations where each circle corresponds to one observation, that is, one pedestrian at one specific time in the horizon. The x coordinate of the circle corresponds to the density calculated from (21), and its y coordinate corresponds to the speed calculated from (23).

A scattering is observed with a higher level of variability at lower densities in comparison to greater densities where the distribution of speed is less spread and shifted more towards lower values This observation indicates the capability of the approach to reflect the heterogeneity of pedestrians. On the other hand, the methods with arbitrarily selected discretization may suffer from the loss of heterogeneity. Consequently, the models of the speed-density relationship derived from such methods may lack the realism in the representation of the observed phenomena.

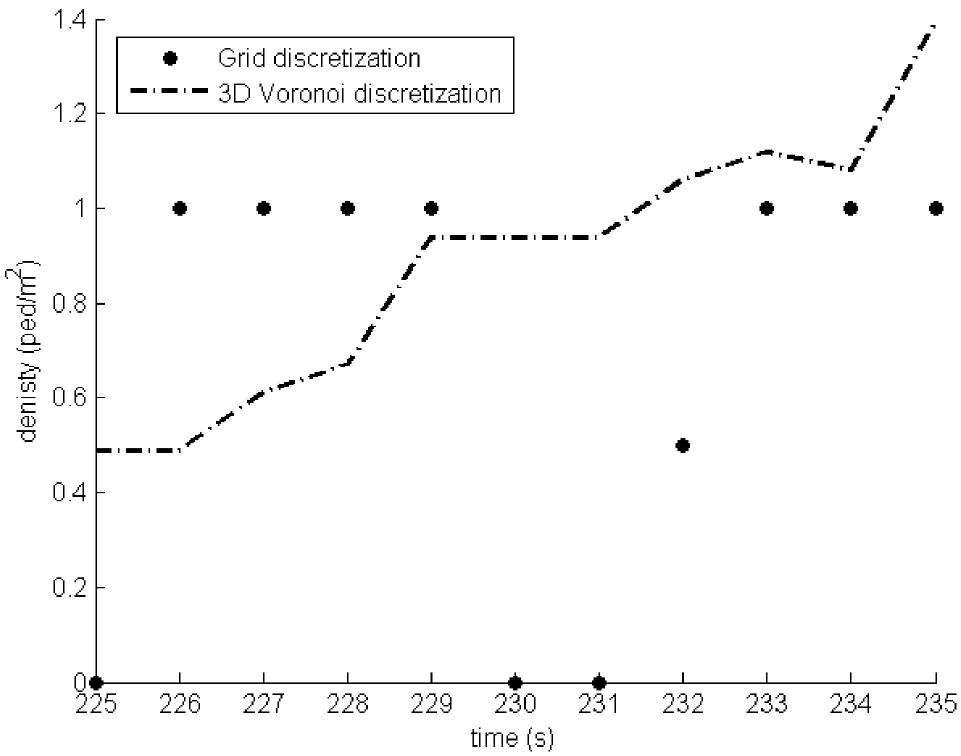
Figure 4: Speed-density profile obtained using three-dimensional Voronoi method



We also compare the properties of our approach with the traditional ones from the literature based on the data extracted from the bi-directional walking experiment. Figure 5 shows a time sequence of density measured at a specific point in space (x = 6.5m, y = 2.5m) using three-dimensional Voronoi (dash-dot line) and grid-based (black dots) discretization. The considered grid unit has the length and width of 1 meter.

The pattern in Figure 5 illustrates that the grid-based method leads to unrealistic features which are reflected through large fluctuations (discontinuities) in density. These fluctuations occur due to the fact that entering or exiting of the grid unit by a person affects the density indicator considerably. Our approach, however, is able to provide a detailed resolution in space and time and results in smooth (realistic) transitions in the measured density.

Figure 5: Time sequence of density values measured using three-dimensional Voronoi and grid based method



5 Conclusion

We proposed a methodology for pedestrian-oriented flow characterization by utilizing the potential of the data itself. The definitions of traffic indicators that we have put forward are based on data-driven 3D Voronoi partitioning in space and time, and therefore, have a capacity to eliminate the issues related to an arbitrary selection of discretization. We have shown empirically that this new approach leads to a number of desired features: (i) reproduces the settings with uniform and non-uniform movement; (ii) reflects the self-organization phenomena typical for pedestrian traffic and pedestrian heterogeneity; (iii) allows for the analysis of congestion at the microscopic level; (iv) leads to smooth transitions in measured traffic characteristics.

To further evaluate the performance and potential shortcomings of the proposed methodology the real case study will be used. It represents a dataset of pedestrian trajectories collected in the train stations in Lausanne and Basel by using technology provided by Alahi *et al.* (2014). The numerical analysis will be directed towards the investigation of the role of conversion constant α and the issues related to the potential numerical instability of results and flow characterization in the case of the presence of obstacles.

The presented approach can also be regarded as the basis for the specification of better or improvement of existing models of pedestrian traffic. In that manner, our further research will focus on the stream-based definitions of the flow indicators and on their mutual interaction. It will subsequently lead to the representation of the effects of multi-directional pedestrian flows through a stream-based model of fundamental relationships.

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6 References

Alahi, A., M. Bierlaire and P. Vandergheynst (2014) Robust real-time pedestrians detection in urban environments with a network of low resolution cameras, *Transportation Research Part C: Emerging Technologies*, **39**, 113–128.

Daamen, W. and S. P. Hoogendoorn (2003) Experimental research of pedestrian walking behavior, *Transportation Research Record: Journal of the Transportation Research Board*, **1828** (1) 20–30.

Duives, D. C., W. Daamen and S. P. Hoogendoorn (2013) State-of-the-art crowd motion simulation models, *Transportation Research Part C: Emerging Technologies*, **37**, 193–209.

Edie, L. C. (1963) *Discussion of traffic stream measurements and definitions*, Port of New York Authority.

Nikolić, M. and M. Bierlaire (2014) Pedestrian-oriented flow characterization, *Transportation Research Procedia*, **2**, 359–366.

Okabe, A., B. Boots, K. Sugihara and S. N. Chiu (2009) *Spatial tessellations: concepts and applications of Voronoi diagrams*, vol. 501, John Wiley & Sons.

Saberi, M., H. S. Mahmassani and T. C. Director (2014) Exploring area-wide dynamics of pedestrian crowds using a three-dimensional approach, paper presented at the *Transportation Research Board 93rd Annual Meeting*, no. 14-1609.

Seyfried, A., B. Steffen, W. Klingsch and M. Boltes (2005) The fundamental diagram of pedestrian movement revisited, *Journal of Statistical Mechanics: Theory and Experiment*, **2005** (10) P10002.

Steffen, B. and A. Seyfried (2010) Methods for measuring pedestrian density, flow, speed and direction with minimal scatter, *Physica A: Statistical mechanics and its applications*, **389** (9) 1902–1910.

van Wageningen-Kessels, F., S. P. Hoogendoorn and W. Daamen (2014) Extension of edie's definitions for pedestrian dynamics, *Transportation Research Procedia*, **2**, 507–512.

Zhang, J. (2012) Pedestrian fundamental diagrams: Comparative analysis of experiments in different geometries, Ph.D. Thesis, Forschungszentrum Jülich.