Absolute and convective instabilities of heated coaxial jet flow

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This study investigates the inviscid, linear spatio-temporal stability of heated, compressible and incompressible coaxial jet flows. The influence of the temperature ratio and the velocity ratio between the core jet and the bypass stream on the transition from convectively to absolutely unstable flows is studied numerically. The investigation shows that for coaxial jets absolute instability can occur for considerably lower core-stream temperatures than for single jets. The reason for this modified stability character is the appearance of an additional unstable mode as a result of the outer velocity shear layer between the bypass stream and the ambient flow. The presence of two shear layers enables the interaction between otherwise free waves to give rise to new instabilities. When the bypass-stream velocity is increased, the classical absolute mode known from single jets (inner mode) is first stabilized and then destabilized for high bypass-stream velocities, whereas the outer mode reaches maximum spatio-temporal growth rates when the core-stream velocity is approximately equal to twice the bypass-stream velocity. Additionally, it is demonstrated that the spatio-temporal character of the modes is very sensitive to the shear-layer thickness and to the distance separating the two layers. Increasing the Mach number strongly dampens the onset of an absolute instability for both modes.

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I. INTRODUCTION

The study of axisymmetric jet flows gained considerable importance since the arrival of jet engines for the propulsion of civil and military aircraft[1]. One of the main reasons for a detailed fluid-mechanical analysis is the aerodynamic noise-generation of such flows[2], which leads to a significant environmental impact in the vicinity of airports[3]. It has been suggested that the noise generation mechanisms in jet flows can be separated into two sources: one from large turbulent structures and the other from fine-scale turbulence[4]. Large-scale (coherent) structures are thought to be mainly responsible for the low-frequency noise[3,5] and can be investigated by linear stability theory[6,7].

The majority of linear stability studies of *single jets* considered the response of the flow to harmonic forcing (see for example the review articles by Michalke[1] and Morris[8]). However, such a spatial stability analysis is meaningful only if the flow acts as a noise amplifier, i.e. if it is sensitive to external periodical perturbations. Nevertheless, open shear flows such as jets or wakes may also act as oscillators that exhibit a distinct disturbance frequency[9], especially if density differences or counterflow are present. To identify the character of the flow, a spatio-temporal stability analysis as proposed by Huerre and Monkewitz[10] can be performed. If the flow is convectively unstable (CI), it is appropriate to investigate the response to harmonic forcing, but if an absolute instability (AI) is present, a temporal analysis should be applied. Furthermore, Monkewitz *et al.*[11] have shown that self-excited global oscillations, which can create large-scale turbulent structures, are a result of a region of local absolute instability in heated jets. These global modes may be responsible for a significant amount of the aerodynamic noise. Therefore, local spatio-temporal stability studies are also relevant for global stability properties and can help to improve the understanding of the noise-generation mechanism.

In the last two decades, several studies have been performed to investigate the absolute/convective instability (AI/CI) characteristics of single heated jets[6,12–16]. Monkewitz and Sohn[6] investigated the spatio-temporal stability properties of an evolving jet flow at successive downstream locations. They found a critical temperature ratio between the surrounding flow and the core flow of $T_\infty/T_0 = 0.72$ below which absolute instability is observed. The critical value for T_∞/T_0 is lowered if the co-flow velocity is increased. Furthermore, they demonstrated the stabilizing effect of increasing the Mach number or the azimuthal disturbance wave number. The influence of density differences between the jet and the freestream was investigated experimentally by Sreenivasan, Raghu, and Kyle [12] and Monkewitz *et al.* [11]. They discovered self-excited side jets if density differences are present and associated this with an absolute instability of the jet flow. Such large-scale structures drastically change the characteristics of the jet and the mixing properties. Jendoubi and Strykowski [13] considered the influence of an external flow on the absolute/convective transition and showed that instability growth rates increased for higher velocity differences. They distinguished between two types of modes: shear-layer modes with strong pressure disturbances at the shear-layer locations and jet-column modes which have intense perturbations along the jet centerline. In low-density jets, the latter mode, which is not present in spatially developing shear layers, is the most unstable one. Juniper [15] also discussed the connections between these two types of modes. Srinivasan, Hallberg, and Strykowski [16] analyzed the sensitivity of this transition to the base-flow profile shapes. In addition to these studies, Lesshafft and Huerre [14] computed the full impulse response of a viscous compressible heated single jet. They showed that even if the absolute mode in jets without counterflow has been found to be of the axisymmetric jet-column type, the linear impulse response for thin-shear-layer jets is dominated by shear-layer modes with high group velocities. Moreover, they found that the onset of absolute instability in heated single jets arises from the action of the baroclinic torque and that viscosity has a purely stabilizing effect. The influence of confinement on the spatio-temporal stability of jet flows was investigated by Juniper [15].

The goal of the present study is to extend previous results on single heated jets to *heated coaxial jets*. In fact, a major reduction of the aerodynamic noise, and an improvement of fuel efficiency has been achieved by the use of turbofan jet engines with a large bypass ratio[3], which are now used for the majority of commercial aircraft. Indeed, the aerodynamic noise scales with a high power of the jet velocity[17]. To achieve a given thrust the jet velocity can be lowered thanks to the contribution of the bypass stream. In these engines, a cold bypass flow, which is accelerated by the fan, surrounds the heated core stream which flows through the fan, compressor, combustion chamber and turbine. Therefore, the exhaust flow is not a single heated jet but a coaxial jet with a heated core and an unheated bypass. Coaxial jet profiles have two shear layers: the *inner* between the heated core flow and the cold bypass stream, and the *outer* between the bypass stream and the ambient flow. The addition of the second shear layer opens the door to new absolutely unstable modes which are not present in heated single jets. For the base flows considered in the following, temperature gradients are only present along the inner velocity shear layer where the temperature drops from the primary-jet temperature to the ambient one, while the temperature across the outer shear layer remains constant. This flow configuration together with a uniform temperature in the core region are good approximations for the mean flow behind a turbofan jet engine. In fact, the bypass stream is only slightly accelerated so that the temperature does not differ significantly from the one of the surrounding. Furthermore, because of good mixing, the temperature in the bypass stream and in the heated core can be assumed as uniform.

A *spatial* viscous linear stability analysis has been carried out for compressible, subsonic coaxial jet flows by Gloor, Obrist, and Kleiser[18] and an inviscid study was performed by Perrault-Joncas and Maslowe[2]. The former authors showed that viscous effects are essential only below a Reynolds number of order 10^5 (based on the jet radius and centerline conditions). Furthermore, it was found that low-frequency disturbances have a wider support so that synchronized disturbances in both layers are possible. They also found a larger spatial growth rate for the outer mode, but a wider unstable frequency range for the inner mode. Talamelli and Gavarini[19] carried out a local spatial and *spatio-temporal* inviscid linear stability analysis of incompressible, isothermal coaxial jets, taking into account of the duct wall separating the two streams. The present work extends their results to the compressible regime and also investigates the influence of temperature differences between the core jet and ambient fluid, but neglects the velocity deficit that would be present in the wake of a nozzle wall. The linear spatio-temporal stability properties of parallel base-flow profiles are analyzed by systematically varying the relevant base-flow parameters. Variations of the temperature ratio, the velocity ratio, the Mach number, the shear-layer thickness and their relative distance shed light on the parameter sensitivity of the onset of absolute instability in coaxial jet flows. The results help to predict fundamental changes of the flow development and the mixing properties in coaxial jet flows.

To our knowledge, no other spatio-temporal studies have been performed on heated coaxial jets. However, such a study is of fundamental importance to verify the applicability of spatial instability studies when the flow is convectively unstable, and to find parameter configurations giving rise to an absolute instability. This may then lead to self-sustained global oscillations with a significant impact on the aerodynamic noise generation mechanisms. Aeroacoustics is, however, not the only field of application of this work. In fact, there are many other applications where an improved understanding of the transition mechanism for such flows is essential. Dual-stream nozzles can also be found, for example, in mixing devices where an increase of the turbulence intensity is desirable to improve mixing characteristics[20].

The use of linear theory can be justified by the finding that, for the types of flows investigated herein, the velocity of the perturbation front that separates the basic unperturbed state from the bifurcated state, i.e. where a nonlinearly saturated perturbation exists, is determined by a linear selection criterion[21,22]. Recently, fully nonlinear numerical simulations of synthetic wake flows[23–25], hot jets[26,27] and swirling jets[28] have confirmed that the linear spatio-temporal instability characteristics of a flow also hold in the nonlinear regime. The linear absolute marginal unstable frequency agrees well with the nonlinear global frequency, supporting the idea of the existence of a steep nonlinear global mode[29] in these flows. The existence of *nonlinear* self-excited global oscillations is therefore intrinsically related to the *linear* absolute instability[30,31].

This paper is structured as follows. Section II A presents the governing equations of the investigated problem along with the employed numerical methods (Sec. II B). The influence of a range of flow parameters on the spatio-temporal instability characteristics is discussed in Sec. III. Conclusions are drawn in Sec. IV.

II. MATHEMATICAL MODEL AND NUMERICAL METHODS

A. Governing equations and theoretical background

For this study, cylindrical coordinates $\underline{x} = [x, r, \phi]^T$ are used, where x corresponds to the axial direction, r the radial direction and ϕ the azimuthal direction. The corresponding velocity vector is given by $\underline{u} = [u, v, w]^T$. The flow is governed by the compressible Euler equations

$$\frac{\partial \rho}{\partial t} + \underline{\nabla} \cdot (\rho \underline{u}) = 0\,, \tag{1}$$

$$\frac{\partial \underline{u}}{\partial t} + (\underline{u} \cdot \underline{\nabla})\, \underline{u} = -\frac{1}{\rho} \underline{\nabla}\, p\,, \tag{2}$$

$$\frac{\partial}{\partial t}\left(\frac{p}{\rho}\right) + \underline{u} \cdot \underline{\nabla}\left(\frac{p}{\rho}\right) = -\,(\gamma - 1)\left(\frac{p}{\rho}\right) \underline{\nabla} \cdot \underline{u}\,, \tag{3}$$

where ρ is the fluid density, p is the pressure and $\gamma = 1.4$ is the heat capacity ratio. Length scales are non-dimensionalized by the radius of the primary jet r_1^*. Velocities and the density are scaled with the dimensional base-flow quantities u_0^* and ρ_0^* at $r = 0$, respectively. The pressure is non-dimensionalized by the the dynamic pressure $\rho_0^* u_0^{*2}$ along the jet centerline. The pressure p, density ρ and temperature T are related by the equation of state for a perfect gas

$$\gamma Ma^2 p = \rho T, \tag{4}$$

where the Mach number is defined as $Ma = u_0^*/a_0^*$ and $a_0^* = \sqrt{\gamma R^* T_0^*}$ is the speed of sound at $r = 0$.

In linear stability theory, the flow variables are decomposed into base flow quantities and disturbances,

$$[\rho, \underline{u}, p]^T = [\rho_b + \rho', \underline{u}_b + \underline{u}', p_b + p']^T . \tag{5}$$

The base flow is assumed to be parallel in the axial direction and swirl-free, i.e. $\underline{u}_b = [u_b, 0, 0]^T$. By extending the base-flow velocity profile for a single jet proposed by Michalke[1] to a coaxial jet, one obtains[2]

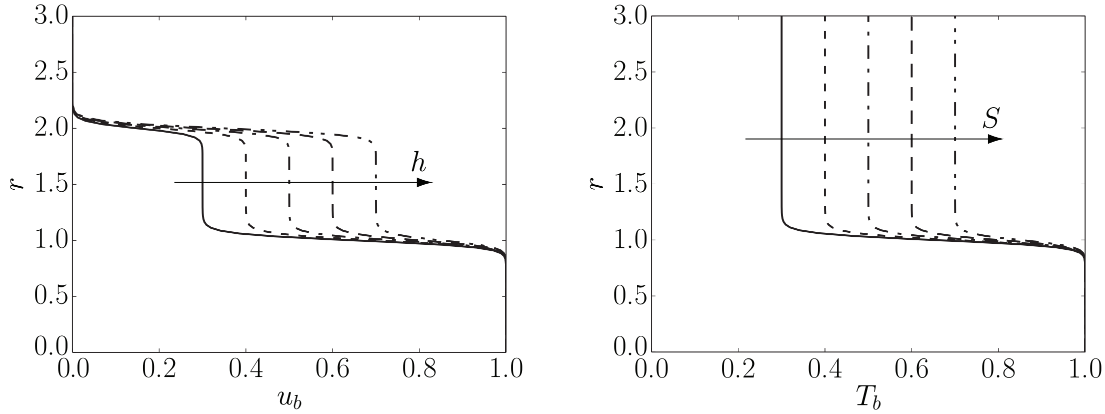
$$u_b(r) = (1 - h)\, u_1(r) + h\, u_2(r)\,, \tag{6a}$$

$$u_j(r) = \frac{1}{2}\left\{1 + \tanh\left[\frac{r_j}{4\theta_j}\left(\frac{r_j}{r} - \frac{r}{r_j}\right)\right]\right\}\,, \text{ for } j = 1, 2\,, \tag{6b}$$

where r_1 is the location of the inner shear layer, which lies between the heated core flow and the bypass stream, and r_2 is the radial location of the outer shear layer situated between the bypass stream and the surrounding (ambient) fluid, which is assumed to be stationary ($u_\infty = 0$). Throughout this work, unless specified otherwise, the shear-layer locations are $r_1 = 1$ and $r_2 = 2$ and their thicknesses are $\theta_1 = \theta_2 = \theta$. The parameter $h = u_s/u_p$ is called *bypass-velocity ratio* and defines the ratio between the secondary-stream velocity u_s and the primary-jet velocity u_p. Since in turbofan jet engines only the primary stream is significantly heated, the temperature profile of the coaxial jet is mainly related to the shape of the core-stream velocity profile $u_1(r)$ via the relation

$$T_b(r) = (1 - S)\, u_1(r) + S\,, \tag{7}$$

where $S = T_\infty/T_0$ is the *ambient-to-core temperature ratio.* The previously described non-dimensionalization and assumptions yield relations for the constant base-flow pressure p_b



(a) (b)

FIG. 1: Base-flow profiles for heated coaxial jet flows: $r_1 = 1$, $r_2 = 2$, $\theta_1 = \theta_2 = 0.03$. (a) Axial velocity u_b for different bypass-velocity ratios: —–, $h = 0.3$; - - -, $h = 0.4$; - · -, $h = 0.5$; — —, $h = 0.6$; - · · -, $h = 0.7$. (b) Temperature T_b for different ambient-to-core temperature ratios: —–, $S = 0.3$; - - -, $S = 0.4$; - · -, $S = 0.5$; — —, $S = 0.6$; - · · -, $S = 0.7$.

and the density ρ_b, namely

$$p_b = \frac{1}{\gamma Ma^2} \quad \text{and} \quad \rho_b(r) = \frac{1}{T_b(r)}, \tag{8}$$

respectively. Base-flow profiles for different bypass-velocity and temperature ratios are presented in Fig. 1.

A classical normal mode ansatz is used for the perturbations

$$\begin{bmatrix} \rho' \\ u' \\ v' \\ w' \\ p' \end{bmatrix} (\underline{x}, t) = \begin{bmatrix} \hat{\rho}(r) \\ \hat{u}(r) \\ i\hat{v}(r) \\ \hat{w}(r) \\ \hat{p}(r) \end{bmatrix} \exp\left[i\left(\alpha x + m\phi - \omega t\right)\right] + c.c.\,, \tag{9}$$

where α is the complex axial wavenumber, the integer m is the azimuthal wavenumber, ω the complex angular frequency and the quantities denoted by the symbol $\hat{(\cdot)}$ are complex amplitude functions. In the following, only axisymmetric modes are considered ($m = 0$). In fact, we have found that for thin velocity shear layers, typically with a thickness of $\theta = 0.03$, zero-group-velocity modes with higher azimuthal wave numbers have smaller growth rates[32]. This is in agreement with results for single jets[6,14].

By introducing the normal mode ansatz into the linearized governing equations one obtains the usual system of ordinary differential equations, see Ref.[32] for their explicit form. These disturbance equations correspond to the equations proposed by Lesshafft and Huerre[14] in the limit $Re \to \infty$. We would like to point out a sign error in the first term of their continuity equation A1. For vanishing Mach numbers, the energy equation reduces to the divergence-free condition for an incompressible flow. Because of the singular nature of the cylindrical coordinate system, the boundary conditions have to be defined carefully to ensure bounded solutions for all perturbations. The compatibility conditions[33] yield vanishing azimuthal derivatives for $r \to 0$. The far-field boundary conditions depend on the Mach number. For incompressible flows ($Ma = 0$), vanishing disturbances are imposed at the outer end r_{max} of the finite computational domain. The eigenfunctions always have to vanish for $r \to \infty$, but for increasing Mach numbers the support of the disturbances becomes wider. A radiation boundary condition according to Ref.[34] is therefore imposed at r_{max} to avoid spurious reflections for compressible-flow computations ($Ma > 0$). The system of stability equations can be written as a generalized eigenvalue problem and can be solved for a given angular frequency ω, yielding the axial wavenumber α as the eigenvalue.

The local spatio-temporal instability properties of a given flow are determined by the large-time impulse response of the instability eigenmode with zero group velocity, also called the absolute mode. The absolute mode is defined by the *absolute wavenumber* $\alpha_0 \in \mathbb{C}$ and the *absolute frequency* $\omega_0 \in \mathbb{C}$. According to the Briggs-Bers criterion different instability types can be distinguished, depending on the sign of the imaginary part of the absolute frequency[21]. If $\omega_{0,i} < 0$, the flow is either *stable* or *convectively unstable* whereas for $\omega_{0,i} > 0$ the flow is said to be *absolutely unstable*. An absolute mode can be identified by a saddle point in the complex wavenumber plane α. The Briggs-Bers criterion also states that the corresponding saddle point of an absolute mode must be formed by a pinching of two spatial branches α^+ and α^-, i.e. they must lie in the upper, resp. lower, half α-planes for high enough $\omega_{0,i}$. These branches are determined by the dispersion relation $D(\omega, \alpha) = 0$ derived from the linear disturbance equations which maps the complex frequency ω into the complex wavenumber plane α. To determine the spatio-temporal instability characteristics, it is therefore necessary to identify such saddle points. The aim of this study is to clarify the parametric sensitivity of the transition from convective to absolute instabilities. The stability of the mode with zero group velocity satisfying the pinch criterion is sufficient for determining the instability character of a coaxial jet with the external flow at rest. The study of the full impulse response as performed by Lesshafft and Huerre [14] was therefore not attempted (it was only used to validate the numerical solver, see Appendix).

B. Numerical methods

A Chebyshev collocation method[35,36] has been used for the discretization of the generalized eigenvalue problem. Because the employed Gauss-Lobatto discretization points are concentrated at the domain boundaries, a coordinate transform proposed by Bayliss, Class, and Matkowsky [37] has been used to refine the numerical grid in the vicinity of the two shear-layer regions. The locations of the refinements correspond to the shear-layer positions, whereas their widths are found with a convergence study for the most critical parameter configuration, namely a thin shear layer in combination with large velocity and temperature differences between the two layers. The discretized generalized eigenvalue problem reads

$$\left(\underline{\underline{\mathrm{A}}} - \omega\,\underline{\underline{\mathrm{B}}}\right)\,\underline{\hat{q}}(r) = \alpha\,\underline{\underline{\mathrm{C}}}\,\underline{\hat{q}}(r)\,, \tag{10}$$

where $\underline{\underline{\mathrm{A}}}$, $\underline{\underline{\mathrm{B}}}$ and $\underline{\underline{\mathrm{C}}}$ are the linear operator matrices derived from the governing equations (see Ref.[32] for their expressions). For the incompressible calculations, Chebyshev polynomials up to the order $N = 128$ were considered for a radial domain $0 \leq r \leq 25$. Imposing zero Dirichlet boundary conditions at $r_{max} = 25$ is a good approximation of the far-field boundary conditions because of the rapid radial decay of all eigenfunctions. For non-zero Mach numbers, the domain and the number of collocation points had to be increased to $r_{max} = 100$ and $N = 192$ because of the energy radiation to the far-field. Convergence studies have been performed to validate these choices.

For a given base flow, the absolute mode was determined by using Briggs' method[9] and then tracked with the computationally efficient algorithm proposed by Monkewitz and Sohn [6], which was also used by Lesshafft and Huerre [14]. It has to be pointed out that because two velocity shear layers exist in coaxial jet flows, two separate modes have to be considered. The full spectrum of the generalized eigenvalue problem, which is necessary for the initial localization of the saddle point using Briggs' method, was obtained by using the ZGGEV routine of the LAPACK library[38], whereas for tracking the saddle point location the computationally less expensive, implicitly restarted Arnoldi algorithm implemented in the

ARPACK library was sufficient to solve for a subregion of the eigenvalue spectrum[39]. The MATLAB routines `eig.m` and `eigs.m`, which implement the aforementioned libraries, were employed, respectively. A validation of the numerical solver is discussed in the Appendix.

III. RESULTS

A. Influence of the bypass-velocity ratio h

The most significant difference between the present paper and prior studies on absolute instability in jet flows[6,12–16] is the extension of the base flow to a coaxial-jet configuration with a heated core flow. As mentioned in section II A, the principal parameter characterizing the base-flow velocity field is the bypass-velocity ratio $h = u_s/u_p$, Eq. (6).

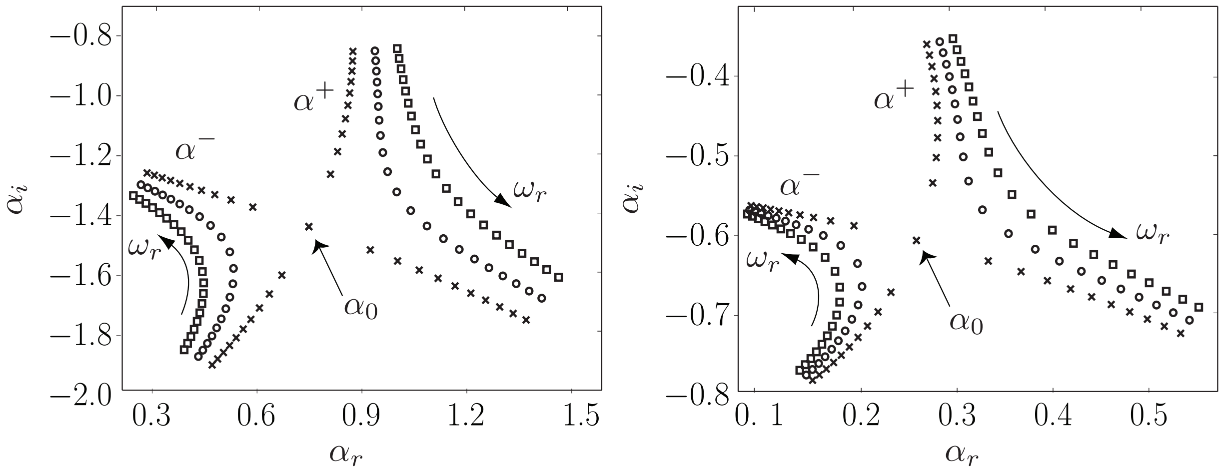
The coaxial base flow supports two unstable Kelvin-Helmholtz modes, which are coupled to the inner or outer velocity shear layer[18] and are denoted as the *inner* mode and the *outer* mode, respectively. Previous investigations on heated single jets considered only one unstable mode, which in our case corresponds to the inner mode for $h = 0$. To compute the influence of the velocity ratio h, the following algorithmic approach is chosen to track the absolute mode in an efficient and robust way. Each mode is initially found for a base flow which comprises only the respective shear layer (i.e. $h = 0$ or $h = 1$) by using Briggs' method. While decreasing the imaginary part of the angular frequency it is possible to visually verify that the saddle point results from the merging of an α^+ and an α^- branch and thus satisfies the Briggs-Bers criterion. For the outer mode, the computation is started for a base-flow profile with $h = 1$ (no velocity gradient along the inner shear layer). Then the velocity ratio h is decreased stepwise towards zero and the so-called outer mode saddle point is tracked. For the inner mode, the procedure is vice-versa, i.e. the algorithm is started for $h = 0$ (no bypass flow) and then h is increased towards unity. The corresponding spatial branches and pinch points for these initial conditions are shown in Fig. 2. It can already be anticipated that the inner mode for $h = 0$ is absolutely unstable ($\omega_{0,i} > 0$) because of the presence of velocity and temperature gradients at the inner shear layer, whereas the outer mode for $h = 1$ is convectively unstable ($\omega_{0,i} < 0$) since velocity and temperature gradients occur at different radial locations. All other base-flow parameters, e.g. the temperature ratio $S = 0.3$, the thickness $\theta = 0.03$ of the shear layers, their locations and the Mach number

$Ma = 0$ remain unchanged while varying h.

The absolute growth rate of both modes is plotted in Fig. 3(a) for $h \in [0, 1]$. It can be seen that the inner mode is absolutely unstable for low bypass-velocity ratios, whereas the outer mode exhibits highest absolute growth for $h \cong 0.5$. The inner mode is stabilized when the bypass-flow velocity increases. This stabilizing effect is similar to the effects observed for single jets exiting into an *ambient* co-flow, see e.g. Jendoubi and Strykowski [13].
For coaxial jets, however, the bypass stream, which has a finite radial extent, gives rise to an additional outer shear-layer instability. For the investigated base-flow configuration, the maximum $\omega_{0,i}$ of the inner mode is almost an order of magnitude larger than the one of the outer mode. In fact, because only the primary jet is heated, there are no temperature gradients along the outer shear layer and therefore the outer instability mode is solely due to the velocity shear at $r_2 = 2$. The outer mode is convectively unstable for $h = 1$, i.e. in the limit of a vanishing velocity gradient at $r_1 = 1$. For $h = 0$, on the other hand, both velocity and temperature gradients are large at $r_1 = 1$, yielding an absolute instability of the inner mode.

The real part of the angular frequency of the absolute modes $\omega_{0,r}$ is shown in Fig. 3(b) for different velocity ratios. For the inner mode it decreases almost linearly from $\omega_{0,r} = 0.94$ towards zero when h is increased, whereas for the outer mode $\omega_{0,r}$ tends to saturate for $h \gtrsim 0.5$. Fig. 3(c) shows the saddle-point location of the absolute mode for various bypass-velocity ratios. The axial wavenumber of the inner absolute mode shifts towards the origin of the complex α-plane when $h \to 1$. The interpretation of eigenmodes with vanishing axial wavenumber is still a subject of controversy because a mode of infinite axial wavelength $\lambda = 2\pi/\alpha_r$ violates the fundamental approximation of a parallel base flow over a distance of the order of a disturbance wavelength[13,40]. A possible explanation may be that for $h \to 1$ the inner shear, which creates the mode, ceases to exist.

Further insight into the character of the absolute modes can be gained by looking at the disturbance eigenfunctions. From the axial velocity disturbance component of the inner mode (see Fig. 4(a)) one can observe that for increasing bypass-velocity ratios h fluctuation amplitudes decrease along the inner shear layer and increase along the outer one (the eigenfunctions are normalized with the kinetic disturbance energy[41]). For the pressure disturbance eigenfunctions, the amplitude at the centerline decreases for increasing h, see Fig. 4(b). It becomes apparent that the absolute modes feature a wide radial support and



(a) (b)

FIG. 2: Discretization of the mapping $\omega \to \alpha^+, \alpha^-$ in the neighborhood of the absolute mode α_0 for $\omega_r \in \{\omega_{0,r} - 0.1, \omega_{0,r} + 0.1\}$ and $\square$, $\omega_i = \omega_{0,i} + 0.04$; $\circ$, $\omega_i = \omega_{0,i} + 0.02$; $\times$, $\omega_i = \omega_{0,i}$. $S = 0.3$, $\theta = 0.03$, $Ma = 0$. (a) Inner mode with $h = 0$ and $\omega_0 = 0.94 + 0.26i$. (b) Outer mode with $h = 1$ and $\omega_0 = 0.57 - 0.07i$.

are not confined to one of the shear layers but lead to disturbance growth in the jet column as well as along the two velocity shear layers at $r_1 = 1$ and $r_2 = 2$, respectively. Lesshafft and Huerre [14] and Jendoubi and Strykowski [13] distinguished between *shear-layer* and *jet-column* modes. The modes discussed in the present work feature a mixed character between these two types of modes. For example, when increasing the velocity of the bypass flow, it can be observed that the inner mode becomes more of the shear-layer type whereas the outer mode remains a mixed mode with strong perturbations also at the centerline, see Fig. 5.

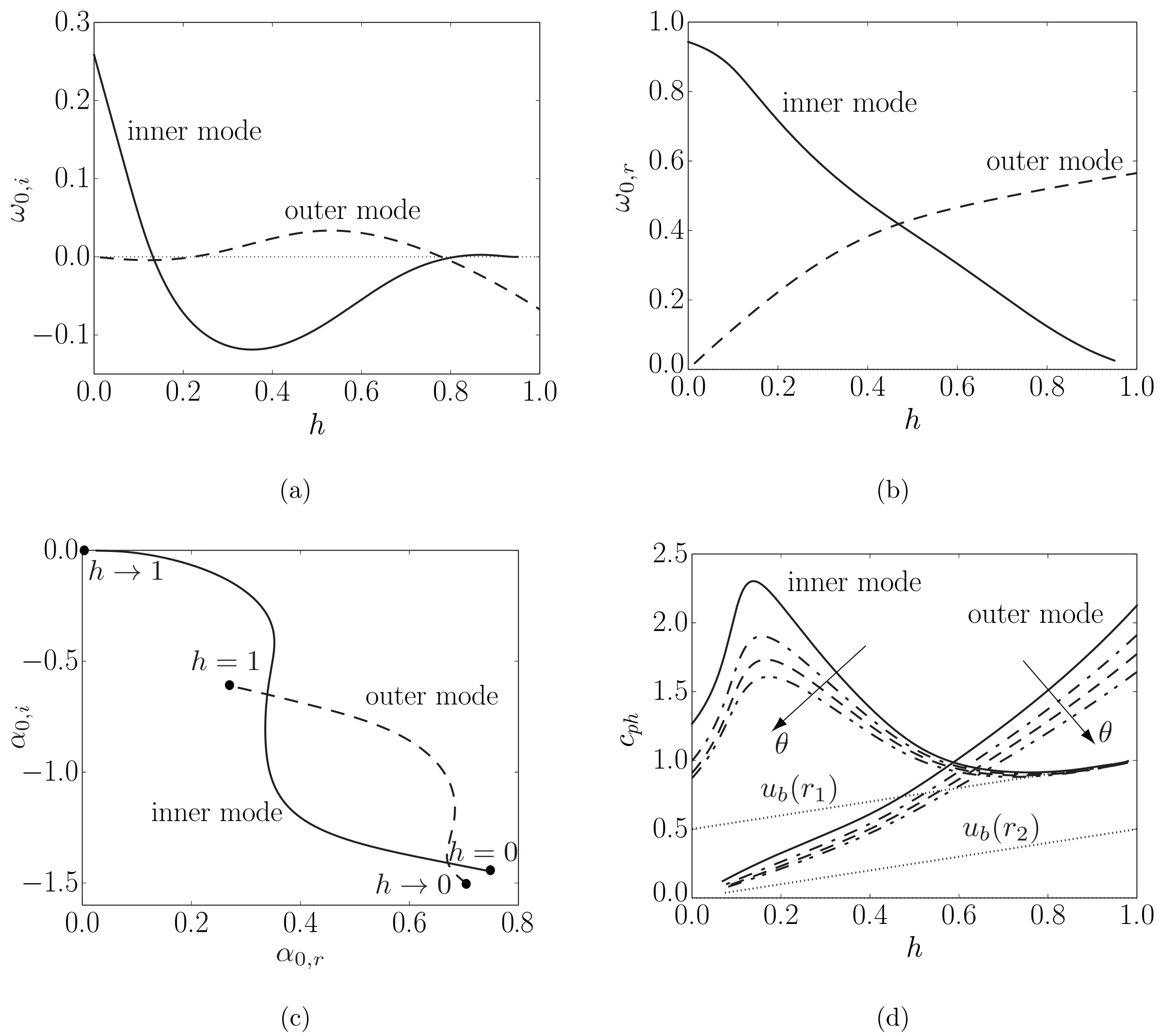
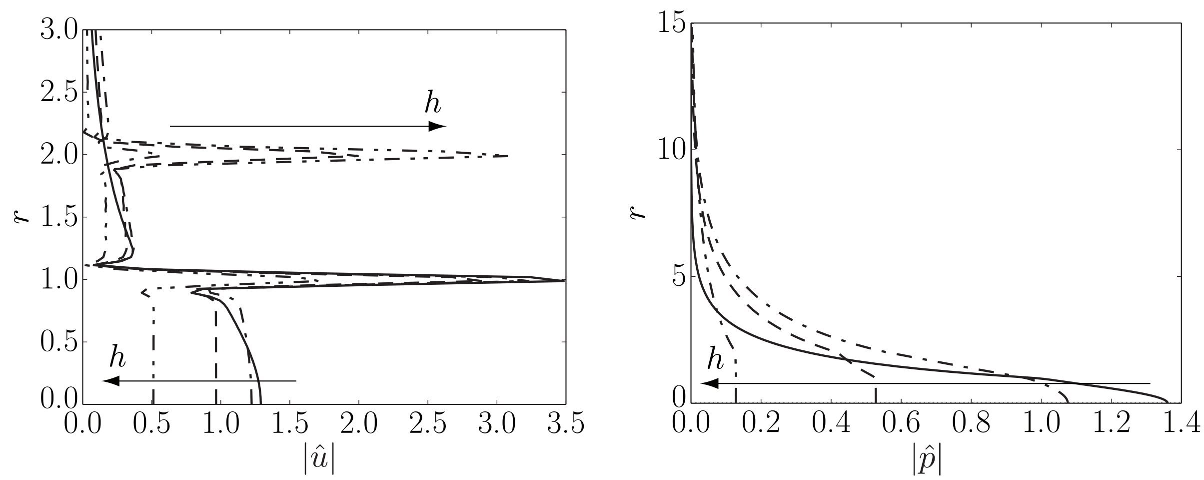


FIG. 3: Influence of the bypass-velocity ratio on the spatio-temporal instability for $\theta_1 = \theta_2 = 0.03$ (a)-(c), and on the phase velocity (d) for the inner and outer modes. $S = 0.3$, $Ma = 0$. (a) Absolute growth rate $\omega_{0,i}$. (b) Real part of absolute angular frequency $\omega_{0,r}$. (c) Pinch point location α_0 in the complex streamwise wavenumber plane. (d) Phase velocities c_{ph} of the mode with zero group velocity for different shear-layer thicknesses: ——, $\theta = 0.03$; - · -, $\theta = 0.06$; — —, $\theta = 0.08$; - · · -, $\theta = 0.1$. · · · · ·, axial base-flow velocity at inner ($r_1 = 1$) and outer ($r_2 = 2$) shear-layer locations.

B. Influence of the temperature ratio S

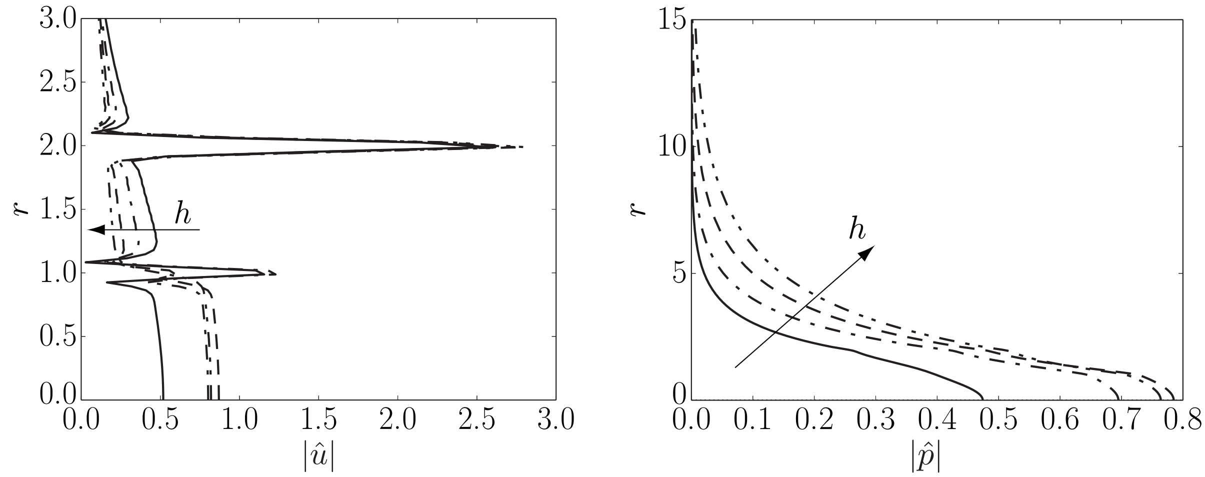
The influence of the core-jet temperature on the absolute instability is studied by varying the temperature ratio $S = T_\infty/T_0$ in the range $S \in [0.1, 1]$. The absolute marginal instability



(a) (b)

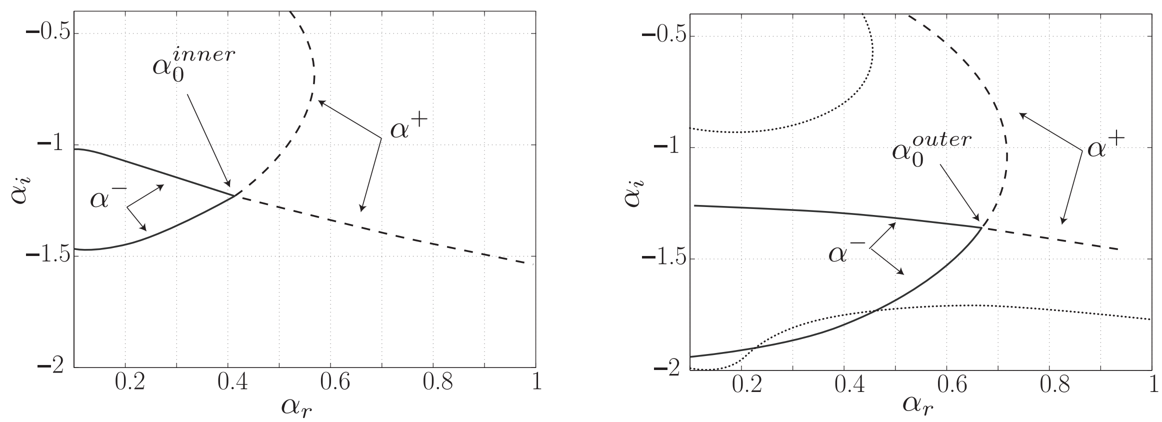
FIG. 4: Magnitude of disturbance eigenfunctions of the inner mode for different bypass-velocity ratios: ——, $h = 0$; - · -, $h = 0.25$; – –, $h = 0.5$; - · · -, $h = 0.75$. $S = 0.3$, $\theta = 0.03$, $Ma = 0$. (a) Axial velocity disturbances $\hat{u}$ and (b) pressure disturbances $\hat{p}$.

curves of both modes are drawn in Fig. 7. For the limiting cases in which either the inner or the outer shear layer vanishes, i.e. if $h \approx 1$ or $h \approx 0$, the absolute/convective instability boundary cannot be determined accurately because the respective instability modes cease to exist. In these limits, the stability of the flow is expected to be fully determined by the properties of the remaining mode. In the region where both the inner and the outer mode display an absolute instability, only the mode with the highest $\omega_{0,i}$ has to be considered. Indeed, the mode with the largest temporal growth rate will dominate the evolution of the absolute instability. However, as can be seen from Fig. 6, both the inner and the outer modes arise from the coalescence of two different α^+ branches with two distinct α^--branches, satisfying thus the additional requirement of the Briggs-Bers criterion[9]. This differs from the single-jet configuration where only one mode exists and thus no second physical saddle point can be found after the first pinching of the spatial branches. Note that even if both pinch points satisfy the previously mentioned requirement, only the one with the highest absolute growth rate is physically relevant. The absolutely unstable region of the inner mode which appears for low bypass velocities (see Fig. 3(a)) extends to higher h-values if the core-jet temperature is further increased in relation to the ambient temperature, i.e. when S is decreased. This is in agreement with the findings of Monkewitz and Sohn[6]. If the bypass-



(a) (b)

FIG. 5: Magnitude of disturbance eigenfunctions of the outer mode for different bypass-velocity ratios: ——, $h = 0.25$; - · -, $h = 0.5$; – –, $h = 0.75$; - · · -, $h = 1$. $S = 0.3$, $\theta = 0.03$, $Ma = 0$. (a) Axial velocity disturbances $\hat{u}$ and (b) pressure disturbances $\hat{p}$.



(a) (b)

FIG. 6: Complex α-plane for $h = 0.1$ where both inner (a) and outer (b) pinch points are visible. $\omega_r \in [0.005, 1]$ and (a) $\omega_i = \omega_{0,i}^{\text{inner}} = 0.05$; (b) $\omega_i = \omega_{0,i}^{\text{outer}} = -0.004$. $S = 0.3$, $r_1 = 1$, $r_2 = 2$, $\theta = 0.03$, $Ma = 0$. ——, α^--branch; – –, α^+-branch; ·····, spatial branches resulting from first pinching at $\omega_i = 0.05$ shown in (a).

stream velocity ratio is increased above 0.4, an absolute instability is observed already for lower temperature differences. Concerning the outer mode, the absolutely unstable parameter range remains centered around $h \cong 0.5$ but extends over a wider range of bypass-

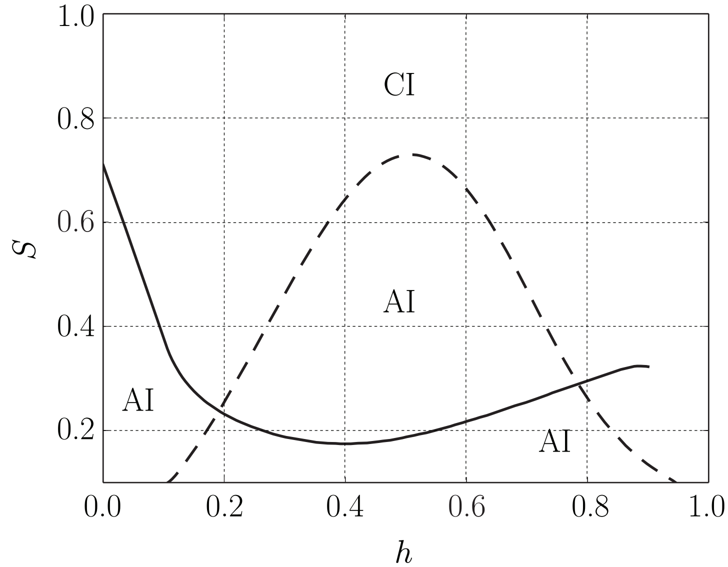


FIG. 7: Absolute marginal instability curve ($\omega_{0,i} = 0$) as a function of the bypass-velocity ratio h and the ambient-to-core temperature ratio S for the inner mode (—) and the outer mode (– –). $\theta = 0.03$, $Ma = 0$. Parameter pairs (h, S) located below the respective line correspond to configurations that lead to absolute instability (AI), whereas pairs above the lines indicate regions of convective instability (CI).

velocity ratios if S is decreased. A validation of the results is found by looking at the absolute marginal instability for $h = 0$. The temperature ratio for which the spatio-temporal instability characteristic changes from convective to absolute instability is $S = 0.71$. Keeping in mind that the employed shear-layer thicknesses are $\theta_1 = \theta_2 = 0.03$, which corresponds to $r_1/\theta_1 \cong 33$, this critical value for S agrees well with the value obtained by Lesshafft and Huerre[14] ($S = 0.713$) for $r_1/\theta_1 = 26$. Monkewitz and Sohn[6] obtained very similar results as well ($S = 0.72$) for a comparable base flow. Furthermore, the highest temperature ratio yielding an absolute marginal unstable outer mode is reached for $h = 0.51$ and is $S = 0.73$. An isothermal jet ($S = 1$) is always convectively unstable for $\theta_1 = \theta_2 = 0.03$ and $r_1 = 1$, $r_2 = 2$. The lowest temperature ratio S for which the flow is convectively unstable is found at $h = 0.19$ and $S = 0.23$. If the temperature ratio is lowered further, a mode will become absolutely unstable. As explained in the following sections, these values are very sensitive to the base-flow parameters.

C. Influence of the shear-layer thickness θ

It is worthwhile to investigate the influence of the shear-layer thicknesses of the base-flow profiles, assumed to be the same for both shear layers, on the absolute/convective instability character. Results for various values of θ are displayed in Fig. 8. It can be noticed that for smaller velocity gradients (i.e. for larger values of θ), the inner mode is stabilized over a wide range of bypass-velocity ratios and destabilized only for high values of h, see Fig. 8(a). For $\theta = 0.1$ and $h < 0.7$, the lower the bypass-velocity ratio the larger the temperature difference that is necessary to reach an absolute marginal state compared to the baseline case with $\theta = 0.03$. For a very thick shear layer with $\theta = 0.2$ the inner mode is absolutely unstable only if $h > 0.35$. A possible explanation for this phenomenon is given in Sec. III D. Concerning the outer mode, the effect of increasing the shear-layer thickness is more complex. In fact it is found that the absolutely unstable region extends to higher temperature ratios when the shear-layer thickness increases from $\theta = 0.02$ to $\theta = 0.07$ and reduces to smaller values of S again when θ is further increased, see Fig. 8(b). For $\theta \in [0.06, 0.08]$, the outer mode becomes absolutely unstable even for isothermal conditions ($S = 1$). It is, however, important to stress that the positive absolute growth rate of the outer mode is very small as indicated in Fig. 9. This partially explains the high sensitivity to θ of the absolute marginal curve ($\omega_{0,i} = 0$). Note that for $\theta = 0.2$ the absolutely unstable region is confined to small temperature and high bypass-velocity ratios. In general, the transition between convective and absolute instability becomes less dependent on θ for $h \gtrsim 0.6$, cf. Fig. 8(b). In this region $\omega_{0,i}$ varies rapidly with h (see Fig. 9) and changes of the shear-layer thickness have only little impact on the spatio-temporal stability.

The phase velocities $c_{ph} = \omega_{0,r}/\alpha_{0,r}$ of both modes are plotted in Fig. 3(d) for different shear-layer thicknesses and for $h \in [0, 1]$. Besides the clear distinction of the phase velocities between the two modes, it can also be observed that modes related to thinner shear-layers have higher phase velocities compared to modes for smoother base-flow velocity profiles. For $h \to 1$ the vanishing inner mode has a phase velocity corresponding to the base-flow velocity at $r = 1$, whereas for $h \to 0$ the phase velocity of the vanishing outer mode tends to $u_b(r = 2) = 0$. The peak of the phase velocity for the inner mode for $h \approx 0.2$ is due to the rapid decrease of the axial wavenumber $\alpha_{0,r}$ of the absolute mode in this region, cf. Fig. 3(b) and Fig. 3(c).

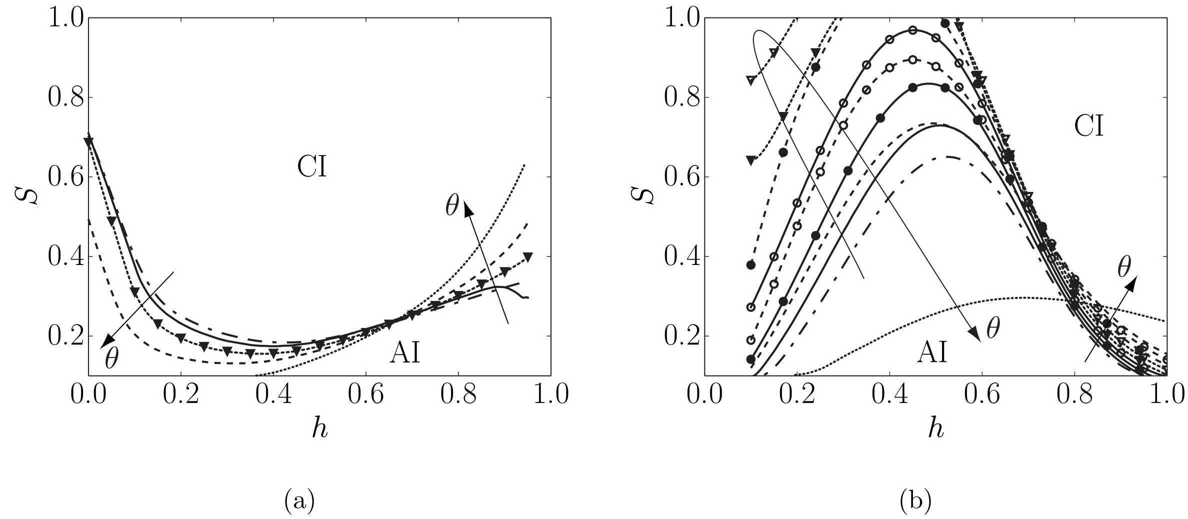


FIG. 8: Absolute marginal instability curve of (a) inner mode and (b) outer mode for different shear-layer thicknesses: - · -, $\theta = 0.02$; —, $\theta = 0.03$; —●—, $\theta = 0.04$; —○—, $\theta = 0.05$; ··▼··, $\theta = 0.06$; ··▽··, $\theta = 0.07$; - - -●- - -, $\theta = 0.08$; - - -○- - -, $\theta = 0.09$; - - -, $\theta = 0.1$; ·····, $\theta = 0.2$. $Ma = 0$.

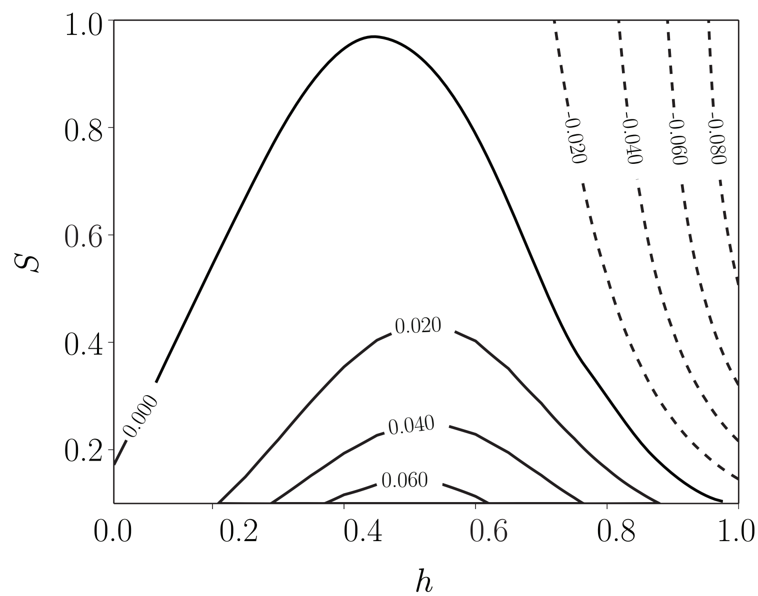


FIG. 9: Contour lines of the absolute growth rate $\omega_{0,i}$ of the outer mode as a function of the bypass-velocity ratio h and the ambient-to-core temperature ratio S. $\theta = 0.05$, $Ma = 0$. —, $\omega_{0,i} \geq 0$; - - -, $\omega_{0,i} < 0$.

D. Physical mechanism driving the absolute instability in isothermal coaxial jets

It may be surprising at first sight that for an isothermal jet, when the action of baroclinic torque is absent, an absolute instability still occurs. In Fig. 8(b) we observed a large absolutely unstable region for a shear-layer thickness around $\theta = 0.07$ for $r_2 = 2$ and $r_1 = 1$ that includes isothermal ($S = 1$) and even cold ($S > 1$) conditions. To better understand this result, we investigated the influence of the distance between the layers and their thicknesses on the spatio-temporal instability. The absolute growth rates for different θ-values as a function of the location of the second shear layer r_2 (while keeping $r_1 = 1$ fixed) are shown in Fig. 10(a) for $\theta = 0.03$, 0.07 and 0.1. It can be concluded that for small enough values of θ there is a range of outer-shear-layer locations r_2 for which a positive absolute growth rate is observed. The thinner the shear layer, the smaller the distance between the layers must be for an absolute instability to occur (see also Fig. 10(b)). At the same time the maximum absolute growth rate increases with decreasing values of θ. The possibility of an absolute instability in isothermal dual-stream jets therefore crucially depends on the respective location and thickness of the inner and outer shear layers.

Therefore, we believe that the absolute instability for the isothermal configuration arises because of an interaction mechanisms between waves that originate from the two shear layers in the base-flow velocity profile. Indeed, from the study of a model problem of a planar incompressible inviscid flow with step-wise constant vorticity, we learn that two so-called Rossby vorticity waves that propagate in opposite direction can be associated with each shear layer[42]. In the case of a coaxial jet with bypass ratio $0 < h < 1$ and linear velocity profiles, four Rossby waves are present. Depending on the base-flow parameters, an absolute instability can arise because of the interaction of some of these waves[43]. For a single jet the instability must result from the two vorticity waves associated with a single shear layer. If no temperature gradients are present, the resulting mode is known to be convectively unstable[14]. On the other hand, if two shear layers are present, as it is the case for a dual-stream jet, vorticity waves originating from the inner and outer shear layers may interact and give rise to a larger absolute growth rate, which can even be positive. For very large values of r_2 we

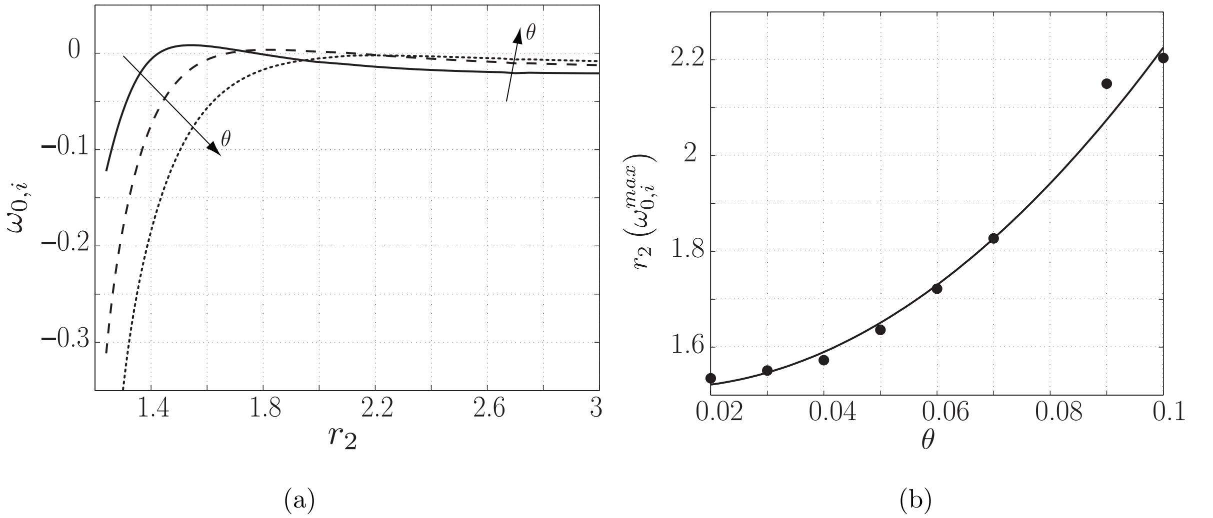


FIG. 10: Influence of outer shear-layer location and shear-layer thicknesses on the absolute growth rate for $r_1 = 1$, $h = 0.38$, $S = 1$, $Ma = 0$. (a) Absolute growth rate $\omega_{0,i}$ as a function of the location of the outer shear r_2 for different shear-layer thicknesses $\theta_1 = \theta_2 = \theta$. —, $\theta = 0.03$; − −, $\theta = 0.07$; $\cdots\cdots$, $\theta = 0.1$. (b) Location of the outer shear layer r_2 giving rise to the largest absolute growth rate $\omega_{0,i}^{max}$ as a function of the shear-layer thickness θ. •, numerical results; —, quadratic polynomial data fitting.

note that the absolute growth rate seems to converge towards a constant value, which in the limit $r_2 \to \infty$ corresponds to the instability of the isolated outer mode. Since the flow is isothermal, this absolute growth rate is negative. On the contrary, when the outer shear layer approaches the inner one, i.e. $r_2 \to r_1$, the absolute growth rate reduces drastically before the mode ceases to exist because the base-flow profile becomes a single jet with a thicker shear layer. This stabilizing effect for small separation lengths between the shear layers was also found in Ref.[18] for a spatial analysis. This physical mechanism based on the interaction of counter-propagating Rossby waves explains for example instabilities in stratified shear flows with statically stable density fields[44]. Since the mathematical description of this interaction goes beyond the scope of the present article, we refer to Carpenter *et al.*[44] for more details. Because of the richness of this phenomenon, further investigations should be undertaken in a separate study.

It is enlightening to look at the effect of varying r_2 on the absolute marginal instability curve in the $(h - S)$-plane for both modes, i.e. also for temperature ratios different from

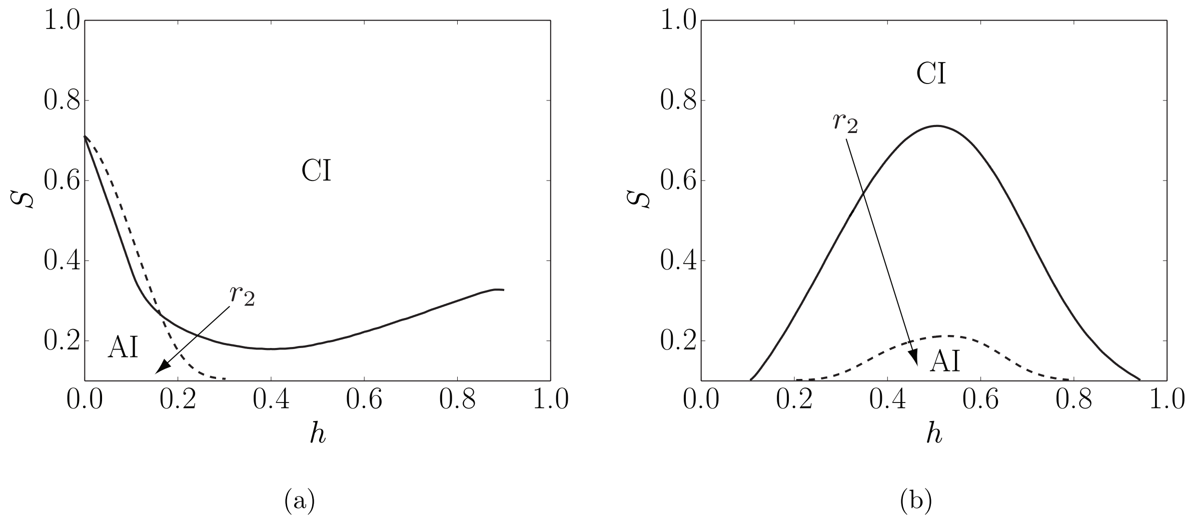


FIG. 11: Absolute marginal instability curve of (a) the inner mode and (b) the outer mode for different locations of the outer shear layer: ——, $r_2 = 2$; - - -, $r_2 = 3$. $r_1 = 1$, $\theta = 0.03$, $Ma = 0$.

unity. In Fig. 11, results for two different locations of the outer shear layer ($r_2 = 2$ and $r_2 = 3$) are compared. For the outer mode (Fig. 11(b)), stronger temperature gradients are needed to produce an absolute instability if the distance between the layers is increased because of the resulting weakening of the interaction mechanism. Furthermore, by looking at the marginal absolute instability curve for the inner mode (Fig. 11(a)), we see that for $r_2 = 3$ the absolutely unstable region is solely confined to low bypass ratios. The absolutely unstable region for higher temperature ratios observed for large bypass ratios when $r_2 = 2$ (see Fig. 8(a)) disappears when the distance between the shear layers is increased. This leads to the conclusion that the destabilization at higher values of h is also very likely due to an interaction of waves from the outer and inner shear-layers. When increasing the bypass ratio, the outer shear becomes stronger and for an appropriate distance and thickness of the shear layers an interaction between the waves can lead to an absolute instability.

If the shear-layer locations are fixed to $r_1 = 1$ and $r_2 = 2$, but their thicknesses are changed to $\theta_1 = 0.03$ and $\theta_2 = 0.1$, we observe that the inner mode is slightly more unstable for large bypass velocities (see Fig. 12(a)). The reason is the increasing interaction between the two shear layers if the outer layer becomes thicker. This effect mainly becomes apparent for large values of h where the base-flow-velocity gradients near r_2 are large. The influence

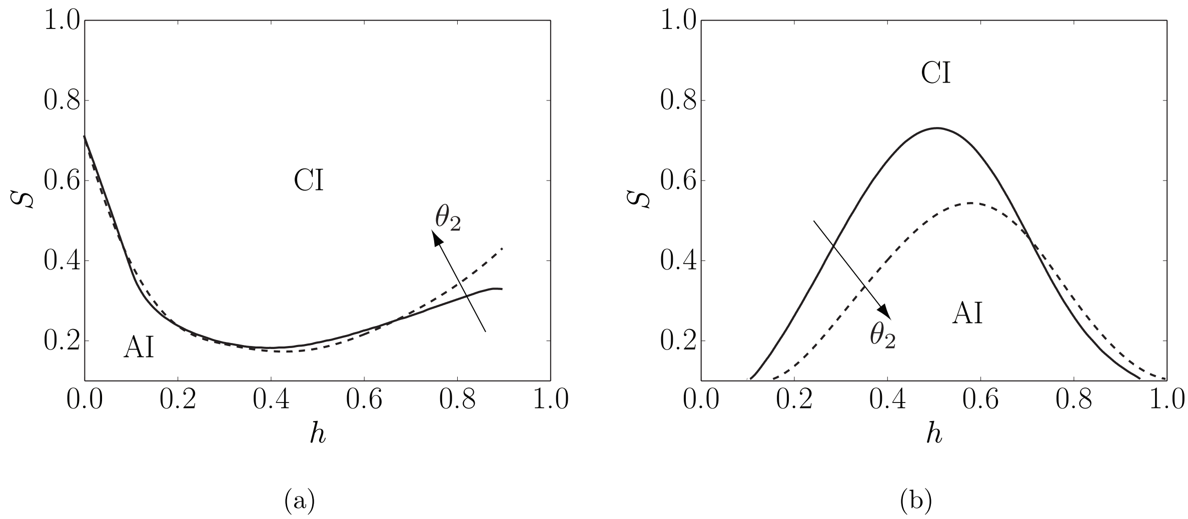


FIG. 12: Absolute marginal instability curve of (a) the inner mode and (b) the outer mode for different shear-layer thicknesses of the outer layer: ——, $\theta_2 = 0.03$; - - -, $\theta_2 = 0.1$. $r_1 = 1$, $r_2 = 2$, $\theta_1 = 0.03$, $Ma = 0$.

of increasing θ_2 on the marginal instability curve of the outer mode is shown in Fig. 12(b). A thicker outer shear layer reduces the absolutely unstable region to higher bypass-velocity ratios and lower temperature ratios.

E. Influence of the Mach number Ma

The results presented in the previous sections were computed for incompressible jet flows. In this section, the influence of compressibility is investigated by studying the dependence of the absolute growth rate on the Mach number Ma. Increasing the Mach number is known to have a strongly attenuating effect on the appearance of an absolute instability as was shown in previous studies on heated single jets[6,14]. The localization of the absolute mode, i.e. the determination of the saddle point in the wavenumber plane α, becomes delicate for high subsonic Mach numbers because the pinch point location of spatial branches α^+ and α^- is situated close to the imaginary axis. In this region many other eigenvalues are present, which leads to algorithmic difficulties in determining the correct saddle point in a robust way, as was already documented in Ref.[13] for single jets.

The absolute marginal instability curves for Mach numbers $Ma \in [0, 0.4]$ are displayed in Fig. 13. For both the inner and the outer mode it can be observed that when increasing

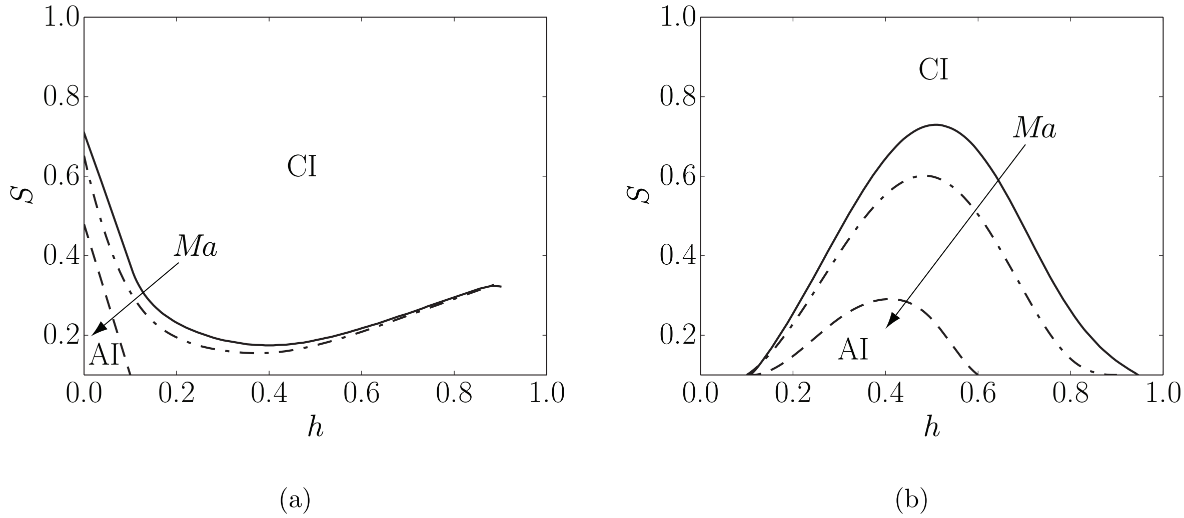


FIG. 13: Absolute marginal instability curve of (a) the inner mode and (b) the outer mode for different Mach numbers: —–, $Ma = 0$; - · -, $Ma = 0.2$; − −, $Ma = 0.4$. $\theta = 0.03$.

the Mach number, the ambient-to-core temperature ratio S that is needed for an absolute instability to occur decreases. Therefore, a higher core-jet temperature is needed for higher Mach numbers to observe a transition from convective to absolute instability. This is in agreement with Ref.[14]. Note that the strong sensitivity of the outer mode to compressibility effects is due to the small absolute growth rates in the absolutely unstable region (see Fig. 9). For the inner mode, increasing the Mach number has a less pronounced stabilizing effect.

A relation for the offset ΔS of the absolute marginal instability curve for a given Mach number proposed by Lesshafft and Huerre[14] can be extended to non-vanishing bypass-velocity ratios h for the inner mode. The formula $\Delta S = -1.4Ma^2(1-h)^2$ is consistent with our numerical results. Regarding the outer mode, no obvious analytical Mach-number relation for the prediction of the convective/absolute transition could be found.

For reasons of completeness, the influence of the shear-layer thickness on the absolute marginal instability for a non-vanishing Mach number was also investigated. The results shown in Fig. 14 suggest that the previously explained influence of the width of the shear-layers also holds for larger Ma, at least for values which still allow for an absolutely unstable region. As expected, the effects of compressibility (Fig. 13) and shear-layer thickness (Fig. 8)

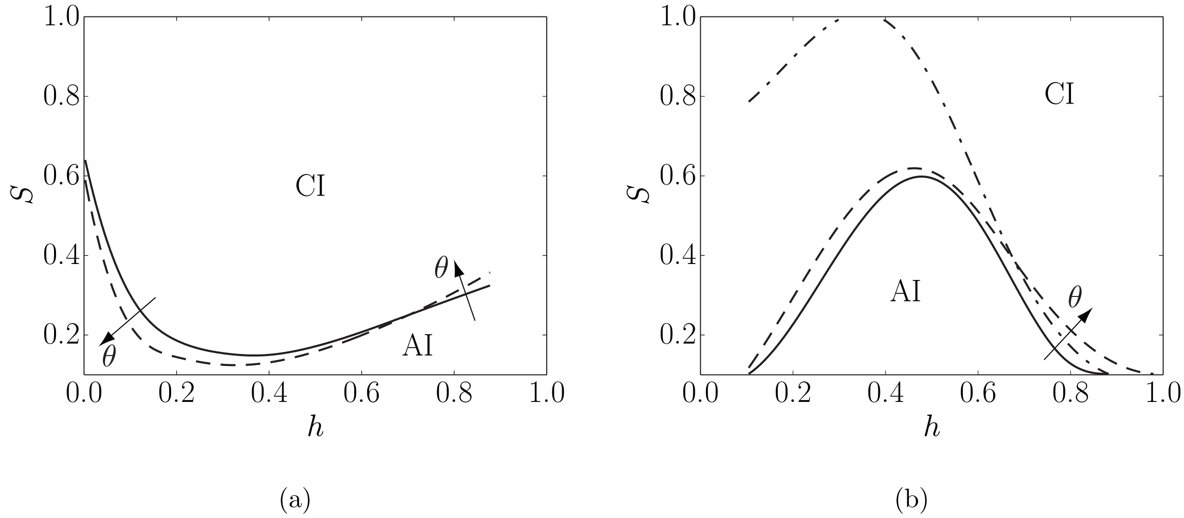


FIG. 14: Absolute marginal instability curve of (a) the inner mode and (b) the outer mode for different shear-layer thicknesses: ——, $\theta = 0.03$; - · -, $\theta = 0.07$; − −, $\theta = 0.1$. $Ma = 0.2$.

do not seem to be coupled.

IV. CONCLUSIONS

The local inviscid spatio-temporal instability characteristics of heated, coaxial jet flows, which model the exhaust conditions of turbofan jet engines, have been investigated for different base-flow conditions. A focus was laid on the influence of the velocity ratio $h = u_s/u_p$ between the secondary (bypass) stream and the primary (core) stream, and on the effects of the ambient-to-core temperature ratio $S = T_\infty/T_0$. Additionally, the effects of the shear-layer thickness θ and of compressibility are studied. The present study extends the single-jet results of Lesshafft and Huerre [14] to coaxial jet flows. We have shown that coaxial jet flows support an additional mode (*outer mode*), which is related to the presence of a secondary shear layer, and which can lead to absolute instability at lower core-stream temperatures than is the case for the primary shear-layer instability (*inner mode*).

The influence of the bypass-flow velocity on the absolute growth rate for both modes has been studied. It was found that the inner mode is stabilized as the bypass-velocity ratio h increases, but it may be destabilized for high values of h. The outer mode instead shows largest absolute growth when the core-flow velocity is about twice the bypass velocity. Increasing the core-jet temperature (or equivalently lowering the core-jet density) favors the transition from convective to absolute instability for both the inner and the outer mode. The critical temperature ratio for $h \to 0$, namely $S = T_\infty/T_0 = 0.71$ for a shear-layer thickness of $r_1/\theta_1 \cong 33$, agrees very well with results of Refs.[6,14]. The absolutely unstable region in the (h, S)-plane of the inner mode extends to higher bypass-velocity ratios if the temperature difference between the streams increases/decreases for h smaller/greater than approximately 0.4, whereas for the outer mode it remains centered around $h \cong 0.5$. However, the range of absolutely unstable velocity ratios increases for the outer modes if the core-jet temperature is increased. The inner mode was found to be always convectively unstable for isothermal jets, whereas the outer mode may present a positive absolute growth rate for $\theta \in [0.06, 0.08]$ if $r_2 = 2$, $r_1 = 1$ and $Ma = 0$. For higher shear-layer thicknesses, the primary-jet temperature needs to be increased (or equivalently S be decreased) in order to reach absolute instability of the inner mode for low values of h. For higher bypass-velocity ratios, an absolutely unstable region of the inner mode for smaller temperature differences has been found for thick shear layers. The absolutely unstable region of the outer mode reaches the highest temperature ratios for $\theta \cong 0.07$ (see Fig. 8) for $r_2/r_1 = 2$. However, it has to be kept in mind that the largest absolute growth rate of the outer mode is almost an order of magnitude smaller than for the inner mode.

A physical mechanism based on the interaction of vorticity waves originating from the two separate shear layers has been proposed to explain an absolute instability for an isothermal coaxial jet. To support this interpretation, the sensitivity of the absolute growth rate to the ratio between the distance and the width of the shear layers has been discussed.

In agreement with previous studies[6,14], finite Mach numbers have a monotonically stabilizing effect. The higher the Mach number, the larger the density difference needed for an absolute instability to occur. For high but still subsonic Mach numbers, the flow is dominated by a convective instability.

The most relevant finding of this study is the presence of an absolutely unstable outer mode, intrinsically related to the coaxial-jet configuration, with small absolute growth rate that can lead to an absolute instability at considerably lower core-jet temperatures than is the case for the classical inner-mode instability. Both the inner and the outer absolute instability may result in a self-excited global mode in coaxial jet flows. For a streamwise evolving flow, the global mode will most likely be selected by the dominating mode at the upstream edge of the absolute instability region.

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APPENDIX: VALIDATION OF THE NUMERICAL SOLVER

To validate the developed numerical solver we computed the linear impulse response for an axisymmetric, incompressible single jet with $S = 0.5$ and $\theta_1 = 0.05$ (see Fig. 15) and compared our results with those of Ref.[14] (their Fig. 1). It can be seen that the agreement is very good. In particular, we can confirm that a transition between jet-column modes to shear-layer modes occurs at a group velocity $v_g = 0.17$[13,14] (the computations presented in the previous sections correspond to $v_g = 0$). Note that the most amplified mode of the impulse response σ_{max} is the most unstable temporal mode ($\alpha_{0,i} = 0$) with a temporal growth rate $\omega_{i,max} = \sigma_{max} = 0.9$ for a group velocity of $v_g \cong 0.45$, angular frequency $\omega_{0,r} \cong 2$ and wavenumber $\alpha_{0,r} \cong 4.3$ (using the same notation as in Ref.[14]). For more detail on the impulse response of a single jet see Ref.[14]. Furthermore, we checked the correctness of our solver also in the compressible regime by computing the absolute growth rate for several parameter configurations of a single jet and by comparing our results with the ones of Ref.[14]. For example, with $Ma = 0.4$, $S \cong 0.3$ and $\theta \cong 0.1$ (i.e. $r_1/\theta \cong 10$) or with $Ma = 0.3$, $S \cong 0.55$ and $\theta \cong 0.067$ (i.e. $r_1/\theta \cong 15$) we obtain a vanishing absolute growth rate, corresponding to a marginal absolute instability, see Fig. 10 of Lesshafft and Huerre[14].

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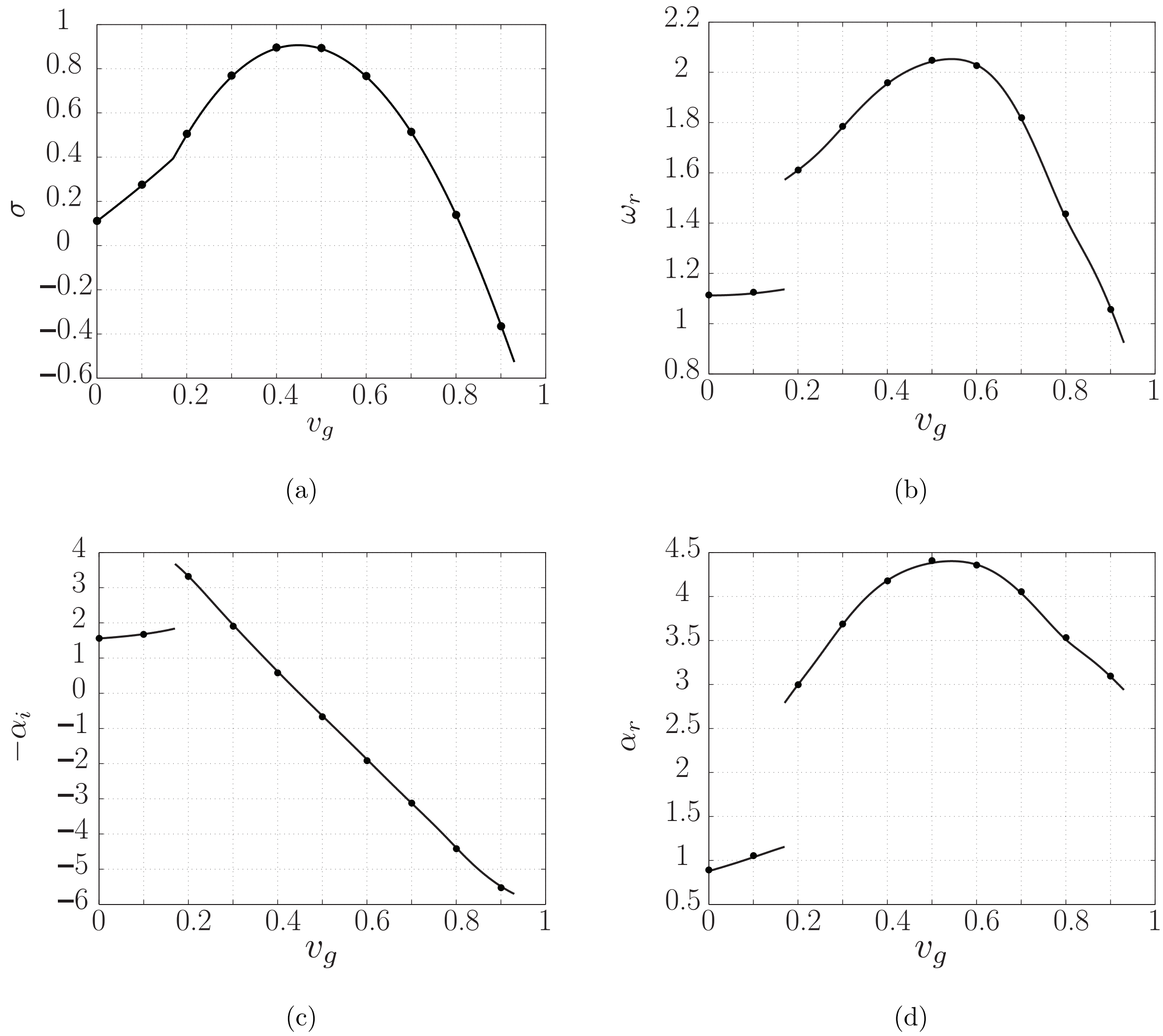

FIG. 15: Impulse response for an incompressible heated single jet: $h = 0$, $S = 0.5$, $\theta_1 = 0.05$, $Ma = 0$, $m = 0$. (a) Spatio-temporal growth rate $\sigma(v_g) = \omega_i(v_g) - \alpha_i(v_g)v_g$ in the frame of reference moving with velocity v_g, (b) real angular frequency ω_r, (c) spatial growth rate $-\alpha_i$, (d) real wavenumber α_r, all as a function of the group velocity $v_g = x/t$. •, data from Lesshafft and Huerre[14].

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