The Ideal Train Timetabling Problem

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Abstract
The aim of this paper is to analyze and to improve the current planning process of the passenger railway service. At first, the state-of-the-art in research is presented. Given the recent changes in legislature allowing competitors to enter the railway industry in Europe, also known as liberalization of railways, the current way of planning does not reflect the situation anymore. The original planning is based on the accessibility/mobility concept provided by one carrier, whereas the competitive market consists of several carriers that are driven by the profit.
Moreover, the current practice does not define the ideal timetables (the initial most profitable timetables) and thus it is assumed that the Train Operating Companies (TOCs) use their historical data (train occupation, ticket sales, etc.) in order to construct the ideal timetables. For the first time in this field, we tackle the problem of ideal timetables in railway industry from passenger behavior point of view. We propose the Ideal Train Timetabling Problem (ITTP) to create a list of train timetables for each TOC separately. The ITTP approach incorporates the passenger demand in the planning and its aim is to minimize the passengers' cost. The outcome of the ITTP is the ideal timetables (including connections between the trains), which then serve as inputs for the traditional Train Timetabling Problem (TTP).

Keywords
Ideal Railway Timetabling, Cyclic - Noncyclic timetable, Mixed Integer Linear Programming

1 Introduction

The time of dominance of one rail operating company (usually the national carrier) over the markets in Europe is reaching to an end. The new EU regulation (EU Directive 91/440) allows open access to the railway infrastructure to companies other than those who own the infrastructure, thus allowing the competition to exist in the market.

Up to this point, the national carriers were subsidized by local governments and their purpose was to offer the accessibility and mobility to the public (passengers). On the other hand, the goal of the private sector is to generate revenue, *i.e.* to maximize the captured demand.

However, the passenger demand is subject to the human behavior that incorporates several factors, to list a few: sensitivity to the time of the departure related to the trip purpose (weekday peak hours for work or school, weekends for leisure, *etc.*), comfort, perception, *etc.* Moreover the passenger service has to compete with other transportation modes (car, national air routes, *etc.*) and thus faces even higher pressure to create good quality timetables.

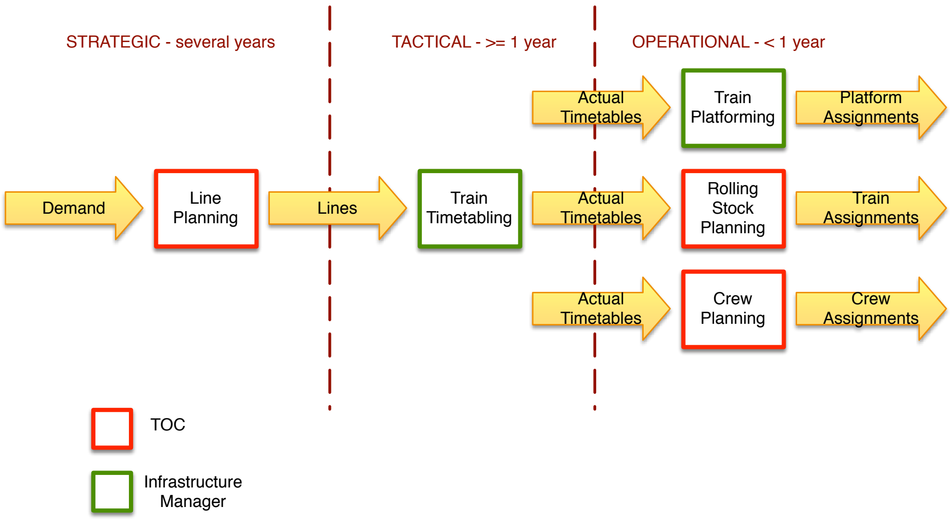

Figure 1: *Planning overview of railway operation*

If we have a closer look at the planning horizon of the railway passenger service (as described in Caprara et al. (2007) and visualized on Figure 1), we can see that the issue of ideal departure times has been neglected in the past. The Train Timetabling Problem (TTP) does take as input the ideal timetables (in its non-cyclic version), however the procedure of generating such timetables is missing. Similarly, in the cyclic version of the TTP, the objective function that would consider passenger demand as the driver is undefined.

We believe that the lack of the definition of the ideal timetables and how to create them, is a major gap, caused by the lack of a competition in the previous railway market settings. We assume that not taking the passengers' wishes into account, lead to the decrease of the railway mode share in the transportation market.

And thus we propose to insert an additional section in the planning horizon called the Ideal Train Timetabling Problem (ITTP). In the ITTP, we introduce a definition of the ideal timetable as follows: the ideal timetable, consists of train schedules, such that the cost associated with traveling by train, of all of the passengers is minimized. Such a timetable would benefit both, passengers and the TOC in the respective manner: it would fit passengers' wishes, which would lead to the increase of the demand and to increase the TOCs' profit.

The ITTP is using the output of the LPP and serves as an input to the traditional TTP and hence, it is placed between the two respective problems (Figure 2). The driver of this problem is the passenger demand. The model will allow timetables of the line to take the form of the non-cyclic or cyclic schedule. Moreover, we introduce a demand induced connections. The connections between the trains are not pre-defined, but are subject to the demand. In the literature the connections are handled only in the cyclic version of the TTP,

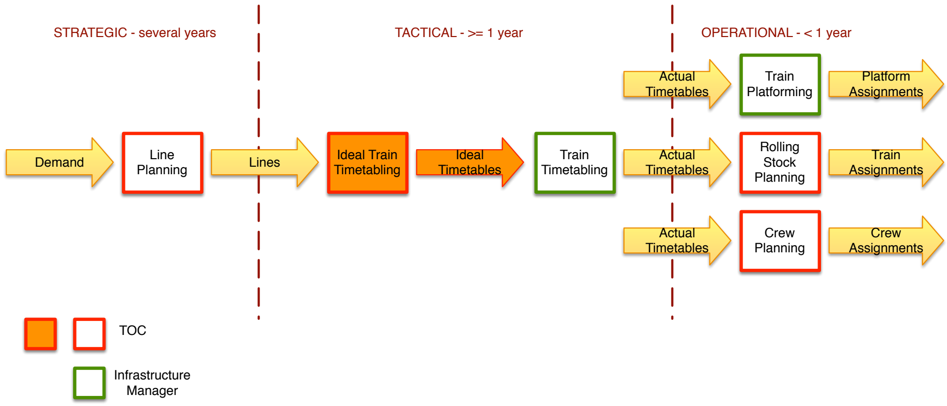

Figure 2: *Modified overview of railway operation*

where the connections are always induced, without a proper reasoning.

In this manuscript, we introduce the current literature on the topic (Section 2) and before we formulate the Ideal Train Timetabling Problem mathematically (Section 4), we discuss how to define quality of a timetable from the passenger point of view, *i.e.* how to form the objective function (Section 3). We finalize the paper by drawing some conclusions (Section 5).

2 Literature Review

The state-of-the-art literature is mostly focused on the traditional planning problems and considers the demand only in the initial phase (*i.e.* the LPP). Due to the extensiveness of the literature, we focus on reviewing of the classical TTP as the ITTP's goal is to provide better information for the TTP using the outcome of the LPP. The latest extensive literature review on LPP can be found in Schöbel (2012).

The aim of the TTP is to find a feasible (operational) timetable for a whole railway network, *i.e.* there are no conflicts of the trains using the tracks. In the non-cyclic version, ideal timetables with their respective profits or costs serve as the main input. The TTP then shifts the departures for conflicting trains, such that the losses of the profits or increase of the costs are minimized. In the cyclic version, the model searches for a first feasible timetable given the size of the cycle. The user can create his/her own objective function, otherwise arbitrary solution will be selected.

2.1 Non-Cyclic TTP

Most of the models, on the non-cyclic timetabling, in the published literature, formulate the problem either as Mixed Integer Linear Programming (MILP) or Integer Linear Programming (ILP). The MILP model uses continuous time, whereas the ILP model discretizes the time. Due to the complexity of the problem, many heuristic approaches are considered.

Brannlund et al. (1998) use discretized time and solve the problem with lagrangian relaxation of the track capacity constraints. The model is formulated as an ILP. Caprara et al.

(2002, 2006), Fischer et al. (2008) and Cacchiani et al. (2012) also use lagrangian relaxation of the same constraints to solve the problem. In Cacchiani et al. (2008), column generation approach is tested. The approach tends to find better bounds than the lagrangian relaxation. In Cacchiani et al. (2010a), several ILP re-formulations are tested and compared. In Cacchiani et al. (2010b), the ILP formulation is adjusted, in order to be able to schedule extra freight trains, whilst keeping the timetables of the passengers' trains fixed. In Cacchiani et al. (2013), dynamic programming, to solve the clique constraints, is used.

In Carey and Lockwood (1995), a heuristic, that considers one train at a time and solves a MILP, based on the already scheduled trains, is introduced. Higgins et al. (1997) then show several more heuristics to solve the MILP model.

Oliveira and Smith (2000) and Burdett and Kozan (2010), re-formulate the problem as a job-shop scheduling one. Erol (2009), Caprara (2010) and Harrod (2012), survey different types of models for the TTP.

None of the above formally defines the ideal timetable. The models focus on the feasibility of the solutions, *i.e.* the track occupation constraints. Demand is omitted in the formulations.

2.2 Cyclic TTP

One of the first papers, dealing with cyclic timetables is Serafini and Ukovich (1989). The paper brings up the topic of cyclic scheduling based on the Periodic Event Scheduling Problem (PESP). The problem is solved with an algorithm using implicit enumeration and network flow theory. In Nachtigall and Voget (1996) model for minimization of the waiting times in the railway network, whilst keeping the cyclic timetables (based on PESP), is solved using branch and bound. The same model is solved using genetic algorithms in Nachtigall (1996). The general PESP model is solved using constraint generation algorithm in Odijk (1996) and with branch and bound in Lindner and Zimmermann (2000).

In Kroon and Peeters (2003), variable trip times are considered. Peeters (2003) then further elaborates on PESP and discuss different forms of the objective function. In Liebchen and Mohring (2002), the PESP attributes are analyzed on the case study of Berlin's underground and in Liebchen (2004) implementation of the symmetry in the PESP model is presented. Lindner and Zimmermann (2005) propose to use decomposition based branch and bound algorithm to solve the PESP.

Kroon et al. (2007) and Shafia et al. (2012), deal with robustness of cyclic timetables. Liebchen and Mohring (2004) propose to integrate network planning, line planning and rolling stock scheduling into the one periodic timetabling model (based on PESP). Caimi et al. (2007) and Kroon et al. (2014) introduce flexible PESP – instead of the fixed times of the events, time windows are provided.

3 Quality of a Timetable

In order to find a good timetable from the passenger point of view, we need to take into account passenger behavior. Such a behavior can be modeled using discrete choice theory (Ben-Akiva and Lerman (1985)). The base assumption in discrete choice theory is that the passengers maximize their utility, *i.e.* minimize the cost associated with each alternative and select the best one.

We propose the following costs associated with passengers' ideal timetable:

- in-vehicle-time (VT)
- waiting time (WT)
- number of transfers (NT)
- scheduled delay (SD)

The **in-vehicle-time** is the (total) time passengers spend on board of (each) train. This time allows the passengers to distinguish between the "slow" and the "fast" services.

The **waiting time** is the time passengers spend waiting between two consecutive trains in their respective transfer points. The cost perception related to the waiting time is evaluated as double and a half of the in-vehicle-time (see Wardman (2004)).

The **transfer(s)** aim at distinguishing between direct and interchange services. In literature and practice, it is by adding extra travel (in-vehicle) time to the overall journey. In our case, we have followed the example of Dutch Railways (NS), where penalty of 10 minutes per transfer is applied (see de Keizer et al. (2012)). Even though variety of studies show that number of interchanges, distance walked, weather, *etc.* play effect in the process, it is rather difficult to incorporate in optimization models. Thus using the applied value (by NS) will bring this research closer to the industry.

The **scheduled delay** is indicating the time of the day passengers want to travel, *i.e.* following the assumption that the demand is time dependent. For example: most of the people have to be at their workplace at 8 a.m. Since it is impossible to provide service that would secure ideal arrival time to the destination for everyone, scheduled delay functions are applied (Figure 3).

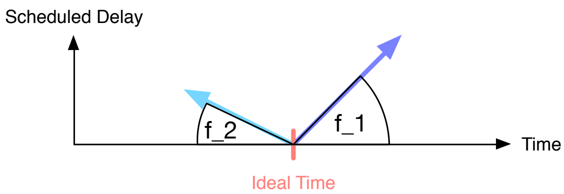

Figure 3: *Scheduled Delay Functions*

As shown in Small (1982), the passengers are willing to shift their arrival time by 1 to 2 minutes earlier, if it will save them 1 minute of the in-vehicle-time, similarly they would shift their arrival by 1/3 to 1 minute later for the same in-vehicle-time saving. If we would consider the boundary case, the lateness ($f_1 = 1$) is perceived equal to the in-vehicle-time and earliness ($f_2 = 0.5$) has half of the value (as seen on Figure 3).

To estimate the perceived cost (quality) of the selected itinerary in a given timetable for a single passenger, we sum up all the characteristics:

$$C = VT + 2.5 \cdot WT + 10 \cdot NT + SD \,[\text{min}] \tag{1}$$

For a better understanding, consider the following example using network on Figure 4: passenger's itinerary consists of taking 3 consecutive trains in order to go from his origin to

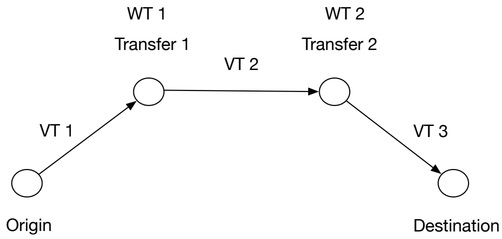

Figure 4: *Example Network*

his destination, he has to change train twice. If he arrives to his destination earlier than his ideal time, his SD will be:

$$SD_e = \text{argmax}\left(\frac{ideal\ time - arrival\ time}{2}\ ,\ 0\right) \tag{2}$$

We use argmax function as one train line has several trains per day scheduled and the passenger selects the one closest to his desired traveling time. On the other hand, if he arrives later than his ideal time, then his SD will be:

$$SD_l = \text{argmax}\,(0\ ,\ arrival\ time - ideal\ time) \tag{3}$$

The overall scheduled delay is then formed:

$$SD = \text{argmin}\,(SD_e\ ,\ SD_l) \tag{4}$$

His overall perceived cost will be the following:

$$C = \sum_{trains} VT + 2.5 \cdot \sum_{transfers} WT + 10 \cdot NT + SD\,[\text{min}] \tag{5}$$

The resulting value is in minutes, however it is often desirable to estimate the cost in monetary values for pricing purposes. In such a case, national surveys estimating respective nation's value of time (VOT) exist. The VOT is given in nation's currency per hour, for instance in Switzerland the VOT for commuters using public transport is 27.81 swiss francs per hour (Axhausen et al. (2008)). To make the cost in monetary units, simply multiply the whole Equation 1 by the VOT/60.

The aim of our research is not to calibrate the weights in Equation 1, but to provide better timetables in terms of the departure times. The weights serve as an input for our problem and thus can be changed at any time. Adding everything up, the ideal timetable from the passenger point of view can be defined as follows:

> The ideal timetable consists of train departure times that passengers' global costs are minimized, *i.e.* the most convenient path to go from an origin to a destination traded-off by a timely arrival to the destination for every passenger.

Similar concept, improving quality of timetables has been done in Vansteenwegen and Oudheusden (2006, 2007). Their approach has been focused on reliable connections for transferring passengers, whereas in our framework we focus on the overall satisfaction of every passenger.

Other concept similar to ours has been used in the delay management, namely in Kanai et al. (2011) and Sato et al. (2013). However their definition of dissatisfaction of passengers omits the scheduled delay.

4 Mathematical Formulation

In this section, we present a mixed integer programming formulation for the Ideal Train Timetabling Problem.

The aim of this problem is to define and to provide the ideal timetables as input for the traditional TTP. It is not well said in the TTP, what ideal means. It is only briefly mentioned, that supposedly, those are the timetables, that bring the most profit to the TOCs (this assumption is in line with the competitive market). Generally speaking, the more of the demand captured, the higher the profit. Thus the ITTP's goal is to design TOC's timetables, such that the captured passenger demand is maximized (objective, but not the form of the objective function).

The input of the ITTP is the demand that takes the form of the amount of passengers that want to travel between OD pair $i \in I$ and that want to arrive to their destination at their ideal time $t \in T_i$. Apart of that, there is a pool of lines $l \in L$ along with the lines' frequencies expressed as the available train units $v \in V^l$ (both results of the LPP) and the set of paths between every OD pair $p \in P_i$. The path is called an ordered sequence of lines to get from an origin to a destination including details such as the running time from the origin of the line to the origin of the OD pair h_i^{pl} (where $l = 1$), the running time from an origin of the OD pair to a transferring point between two lines r_i^{pl} (where $l = 1$), the running time from the origin of the line to the transferring point in the path h_i^{pl} (where $l > 1$), the running time from one transferring point to another r_i^{pl} (where $l > 1$ and $l < |L^p|$) and the running time from the last transferring point to a destination of the OD pair r_i^{pl} (where $l = |L^p|$). Note that the index p is always present as different lines using the same track might have different running times.

Part of the ITTP is the routing of the passengers through the railway network. Using a decision variable x_i^{tp}, we secure that each passenger (it) can use exactly one path. Similarly, within the path, passenger can use exactly one train on every line in the path (decision variable y_i^{tplv}). These decision variables, among others, allow us to backtrace the exact itinerary of every passenger. The timetable is understood as a set of departures for every train on every line (values of d_v^l). The timetable can take form of a non-cyclic or a cyclic version (depending if the cyclicity constraints are active, see below).

Since the input demand is deterministic, the objective function would be to minimize the total travel time of every passenger. However, as discussed in the previous section, passengers are not simply minimizing the total travel time, but the overall cost of the journey by train. Thus the objective function takes form of the Equation 5 weighted by the demand and the value of time.

It could be objected that since the demand is deterministic, the problem does not reflect the reality. However,the goal of the ITTP is to take the deterministic demand and design such timetables that would fit the estimated demand and avoid loss of the estimated number of passengers. In order to increase the forecasted deterministic demand, external factors like service on board, destinations served, renewed vehicle park, *etc.* would have to be changed.

We can formulate the ITTP as follows:

Sets Following is the list of sets used in the model:

I	–	set of origin-destination pairs
T_i	–	set of ideal times for OD pair i
P_i	–	set of possible paths between OD pair i
L	–	set of operated lines
L^p	–	set of lines in the path p
V^l	–	set of available vehicles on line l

The lines and the set of available vehicles per line V^l is an output from the Line Planning Problem based on the selected frequencies within the problem.

Input Parameters Following is the list of parameters used in the model:

M	–	sufficiently large number (for daily planning in minutes, the value can be 1440)
m	–	minimum transfer time
c	–	cycle
r_i^{pl}	–	running time between OD pair i on path p using line l
h_i^{pl}	–	time to arrive from the starting station of the line l to the first point (that involves this one) in the path p of the OD pair i
D_i^t	–	demand between OD i with ideal time t'
q_w	–	value of the waiting time (=2.5)
q_t	–	value of the in vehicle time (VOT)
f_1	–	coefficient of being early ($SD_e = 0.5$)
f_2	–	coefficient of being late ($SD_l = 1$)
a	–	penalty for having a train transfer (=10 min)

Decision Variables Following is the list of decision variables used in the model:

$\mathcal{C}_i^t$	–	the total cost of a passenger with ideal time t between OD pair i
w_i^t	–	the total waiting time of a passenger with ideal time t between OD pair i
w_i^{tp}	–	the total waiting time of a passenger with ideal time t between OD pair i using path p
w_i^{tpl}	–	the waiting time of a passenger with ideal time t between OD pair i on the line l that is part of the path p, i.e. the waiting time in the transferring point, when transferring to line l
x_i^{tp}	–	1 – if passenger with ideal time t between OD pair i chooses path p; 0 – otherwise
s_i^t	–	the scheduled delay of a passenger with ideal time t between OD pair i
s_i^{tp}	–	the scheduled delay of a passenger with ideal time t between OD pair i traveling on the path p
d_v^l	–	the departure time of a train v on the line l

y_i^{tplv}	–	1 – if a passenger with ideal time t between OD pair i on the path p takes the train v on the line l; 0 – otherwise
z_v^l	–	dummy variable to help modeling the cyclicity corresponding to a train v on the line l

Routing Model The ITTP model can be decomposed into 2 parts: routing and pricing. The routing takes care of the feasibility of the solution, whereas pricing takes care of the cost attributes. At first, we present the routing of the passengers – the Routing Model (RM):

$$\min \sum_{i\in I}\sum_{t\in T_i} D_i^t \cdot \mathcal{C}_i^t \tag{6}$$

$$\sum_{p\in P_i} x_i^{tp} = 1, \qquad \forall i \in I, \forall t \in T_i, \tag{7}$$

$$\sum_{v\in V^l} y_i^{tplv} = 1, \qquad \forall i \in I, \forall t \in T_i, \forall p \in P_i, \forall l \in L^p, \tag{8}$$

$$\left(d_v^l - d_{v-1}^l\right) = c \cdot z_v^l, \qquad \forall l \in L, \forall v \in V : v > 1, \tag{9}$$

$$\mathcal{C}_i^t \geq 0, \qquad \forall i \in I, \forall t \in T_i, \tag{10}$$

$$d_v^l \geq 0, \qquad \forall l \in L, \forall v \in V^l, \tag{11}$$

$$x_i^{tp} \in (0,1)\,, \qquad \forall i \in I, \forall t \in T_i, \forall p \in P_i, \tag{12}$$

$$y_i^{tplv} \in (0,1)\,, \qquad \forall i \in I, \forall t \in T_i, \forall p \in P_i, \forall l \in L^p, \forall v \in V^l, \tag{13}$$

$$z_v^l \in \mathbb{N}, \qquad \forall l \in L, \forall v \in V^l. \tag{14}$$

The objective function **(6)** is minimizing the passengers' costs. Constraints **(7)** secure that every passenger is using exactly one path to get from his/her origin to his/her destination. Similarly constraints **(8)** make sure that every passenger takes exactly one train on each of the lines in his/her path. Constraints **(9)** model the cyclicity using integer division. When solving the non-cyclic version of the problem, these constraints have to be removed. Constraints **(10)–(14)** set the domains of decision variables.

Pricing Constraints To make the ITTP complete, we need to expand the Routing Model with the pricing constraints. We will add the pricing constraints in blocks of attributes that create the cost of a passenger.

$$s_i^t \geq s_i^{tp} - M \cdot \left(1 - x_i^{tp}\right), \qquad \forall i \in I, \forall t \in T_i, \forall p \in P_i, \tag{15}$$

$$s_i^{tp} \geq f_2 \cdot \left(\left(d_v^{|L|} + h_i^{|L|} + r_i^{p|L|}\right) - t\right) - M \cdot \left(1 - y_i^{tp|L|v}\right), \qquad \forall i \in I, \forall t \in T_i, \forall p \in P_i, \forall v \in V^{|L|}, \tag{16}$$

$$s_i^{tp} \geq f_1 \cdot \left(t - \left(d_v^{|L|} + h_i^{|L|} + r_i^{p|L|}\right)\right) - M \cdot \left(1 - y_i^{tp|L|v}\right), \qquad \forall i \in I, \forall t \in T_i, \forall p \in P_i, \forall v \in V^{|L|}, \tag{17}$$

$$s_i^t \geq 0, \qquad \forall i \in I, \forall t \in T_i, \tag{18}$$

$$s_i^{tp} \geq 0, \qquad \forall i \in I, \forall t \in T_i, \forall p \in P_i. \tag{19}$$

The first block of constraints takes care of the **scheduled delay (SD)**. In our model we have 2 types of scheduled delay: SD for every path (constraints **(19)**) and SD that is linked to the path, which will be the final selected path of a given passenger(s) with a given ideal time (constraints **(18)**). As described in the Section 3, the constraints **(16)** model the earliness of the passengers (Equation 2) and constraints **(17)** model the lateness (Equation 3). Since both of the above constraints are active at the same time, the Equation 4 is implicitly taken care of (greater equal sign). Constraints **(15)** make sure that only one SD is selected – not necessarily the lowest one as it depends on the cost of the whole path, *i.e.* the path with the smallest overall cost will be selected for the given OD pair with a given ideal time.

$$w_i^t \geq w_i^{tp} - M \cdot \left(1 - x_i^{tp}\right), \qquad \forall i \in I, \forall t \in T_i, \forall p \in P_i, \tag{20}$$

$$w_i^{tp} = \sum_{l \in L^p \setminus 1} w_i^{tpl}, \qquad \forall i \in I, \forall t \in T_i, \forall p \in P_i, \tag{21}$$

$$\begin{aligned} w_i^{tpl} \geq \left(\left(d_v^l + h_i^{pl}\right) - \left(d_{v'}^{l'} + h_i^{pl'} + r_i^{pl'} + m\right)\right) \qquad & \forall i \in I, \forall t \in T_i, \forall p \in P_i, \forall l \in L^p : \\ -M \cdot \left(1 - y_i^{tpl'v'}\right) - M \cdot \left(1 - y_i^{tplv}\right), \qquad & l > 1, l' = l - 1, \forall v \in V^l, \forall v' \in V^{l'}, \end{aligned} \tag{22}$$

$$\begin{aligned} w_i^{tpl} \leq \left(\left(d_v^l + h_i^{pl}\right) - \left(d_{v'}^{l'} + h_i^{pl'} + r_i^{pl'} + m\right)\right) \qquad & \forall i \in I, \forall t \in T_i, \forall p \in P_i, \forall l \in L^p : \\ +M \cdot \left(1 - y_i^{tpl'v'}\right) + M \cdot \left(1 - y_i^{tplv}\right), \qquad & l > 1, l' = l - 1, \forall v \in V^l, \forall v' \in V^{l'}, \end{aligned} \tag{23}$$

$$w_i^t \geq 0, \qquad \forall i \in I, \forall t \in T_i, \tag{24}$$

$$w_i^{tp} \geq 0, \qquad \forall i \in I, \forall t \in T_i, \forall p \in P_i, \tag{25}$$

$$w_i^{tpl} \geq 0, \qquad \forall i \in I, \forall t \in T_i, \forall p \in P_i, \forall l \in L^p. \tag{26}$$

The second block of constraints is modeling the **waiting time (WD)**. There are 3 types of waiting time: the final selected waiting time in the best path (constraints **(24)**), the total waiting time of every path (constraints **(25)**) and the waiting time at every transferring point in every path (constraints **(26)**). The constraints **(22)** and **(23)** are complementary constraints that model the waiting time in the transferring points in every path. In other words, these two constraints find the two best connected trains in the two train lines in the passengers' path. Constraints **(21)** add up all the waiting times in one path to estimate the total waiting time in a given path. Constraints **(20)** make sure that only one WT is selected (similarly as constraints **(15)** for SD).

$$\begin{aligned} \mathcal{C}_i^t = q_v \cdot q_w \cdot w_i^t + q_v \cdot a \cdot \sum_{p \in P} x_i^{tp} \cdot (|L^p| - 1) \\ + q_v \cdot \sum_{p \in P} \sum_{l \in L^p} r_i^{pl} \cdot x_i^{tp} + q_v \cdot s_i^t, \qquad \forall i \in I, \forall t \in T_i. \end{aligned} \tag{27}$$

At last, constraints **(27)** combine all the attributes together as in Equation 5 multiplied by the VOT. The complete ITTP model can be seen in Appendix A.

5 Conclusions and Future Work

In this research, we survey the literature on the current planning horizon for the railway passenger service and we identify a gap in the planning horizon – demand based (ideal) timetables. We then define a new way, how to measure the quality of a timetable from the passenger point of view and introduce a definition of such an ideal timetable. We present a formulation of a mixed integer linear problem that can design such timetables. The new Ideal Train Timetabling Problem fits into the current planning horizon of railway passenger service and is in line with the new market structure and the current trend of putting passengers back into consideration, when planning a railway service.

The novel approach not only designs timetables that fit the best the demand, but also creates by itself connection between two trains, when needed. Moreover, the output consists of the routing of the passengers and thus the train occupation can be extracted and be used efficiently, when planning the rolling stock assignment (*i.e.* the Rolling Stock Planning Problem). The ITTP can create both non-cyclic and cyclic timetables.

In the future work, we will focus on efficient solving of the problem. The new definition of a quality of a timetable (the passenger point of view) creates a lot of opportunities for future research: efficient handling of the TOC's fleet, better delay management, robust train timetabling passenger-wise, *etc.*

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A ITTP Model

$$
\begin{aligned}
\min \sum_{i\in I}\sum_{t\in T_i} D_i^t \cdot \mathcal{C}_i^t & \\
\mathcal{C}_i^t = q_v \cdot q_w \cdot w_i^t + q_v \cdot a \cdot \sum_{p\in P} x_i^{tp} \cdot (|L^p| - 1) & \\
+ q_v \cdot \sum_{p\in P}\sum_{l\in L^p} r_i^{pl} \cdot x_i^{tp} + q_v \cdot s_i^t, &\qquad \forall i \in I, \forall t \in T_i, \\
\sum_{p\in P_i} x_i^{tp} = 1, &\qquad \forall i \in I, \forall t \in T_i, \\
\sum_{v\in V^l} y_i^{tplv} = 1, &\qquad \forall i \in I, \forall t \in T_i, \forall p \in P_i, \forall l \in L^p, \\
\left(d_v^l - d_{v-1}^l\right) = c \cdot z_v^l, &\qquad \forall l \in L, \forall v \in V : v > 1, \\
s_i^t \geq s_i^{tp} - M \cdot \left(1 - x_i^{tp}\right), &\qquad \forall i \in I, \forall t \in T_i, \forall p \in P_i, \\
s_i^{tp} \geq f_2 \cdot \left(\left(d_v^{|L|} + h_i^{|L|} + r_i^{p|L|}\right) - t\right) & \\
- M \cdot \left(1 - y_i^{tp|L|v}\right), &\qquad \forall i \in I, \forall t \in T_i, \forall p \in P_i, \forall v \in V^{|L|}, \\
s_i^{tp} \geq f_1 \cdot \left(t - \left(d_v^{|L|} + h_i^{|L|} + r_i^{p|L|}\right)\right) & \\
- M \cdot \left(1 - y_i^{tp|L|v}\right), &\qquad \forall i \in I, \forall t \in T_i, \forall p \in P_i, \forall v \in V^{|L|}, \\
w_i^t \geq w_i^{tp} - M \cdot \left(1 - x_i^{tp}\right), &\qquad \forall i \in I, \forall t \in T_i, \forall p \in P_i, \\
w_i^{tp} = \sum_{l\in L^p\setminus 1} w_i^{tpl}, &\qquad \forall i \in I, \forall t \in T_i, \forall p \in P_i, \\
w_i^{tpl} \geq \left(\left(d_v^l + h_i^{pl}\right) - \left(d_{v'}^{l'} + h_i^{pl'} + r_i^{pl'} + m\right)\right) &\qquad \forall i \in I, \forall t \in T_i, \forall p \in P_i, \forall l \in L^p : \\
- M \cdot \left(1 - y_i^{tpl'v'}\right) - M \cdot \left(1 - y_i^{tplv}\right), &\qquad l > 1, l' = l - 1, \forall v \in V^l, \forall v' \in V^{l'}, \\
w_i^{tpl} \leq \left(\left(d_v^l + h_i^{pl}\right) - \left(d_{v'}^{l'} + h_i^{pl'} + r_i^{pl'} + m\right)\right) &\qquad \forall i \in I, \forall t \in T_i, \forall p \in P_i, \forall l \in L^p : \\
+ M \cdot \left(1 - y_i^{tpl'v'}\right) + M \cdot \left(1 - y_i^{tplv}\right), &\qquad l > 1, l' = l - 1, \forall v \in V^l, \forall v' \in V^{l'},
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{C}_i^t &\geq 0, && \forall i \in I, \forall t \in T_i, \\
d_v^l &\geq 0, && \forall l \in L, \forall v \in V^l, \\
x_i^{tp} &\in (0,1)\,, && \forall i \in I, \forall t \in T_i, \forall p \in P_i, \\
y_i^{tplv} &\in (0,1)\,, && \forall i \in I, \forall t \in T_i, \forall p \in P_i, \\
& && \forall l \in L^p, \forall v \in V^l, \\
z_v^l &\in \mathbb{N}, && \forall l \in L, \forall v \in V^l, \\
s_i^t &\geq 0, && \forall i \in I, \forall t \in T_i, \\
s_i^{tp} &\geq 0, && \forall i \in I, \forall t \in T_i, \forall p \in P_i, \\
w_i^t &\geq 0, && \forall i \in I, \forall t \in T_i, \\
w_i^{tp} &\geq 0, && \forall i \in I, \forall t \in T_i, \forall p \in P_i, \\
w_i^{tpl} &\geq 0, && \forall i \in I, \forall t \in T_i, \forall p \in P_i, \forall l \in L^p.
\end{aligned}
$$