$\mathcal{K}$	set of all locations
$\mathcal{L}$	set of all pixels
$\mathcal{Z}$	set of all possible depth values
X_k	binary occupancy variable for a location k
Z_i	observed depth value variable at pixel i
z^{∞}	special value for when no depth is observed
M_{ki}	segmentation mask at pixel i for location k
$\mathcal{S}_k$	object silhouette for k-th location
$\|\mathcal{S}_k\|$	number of pixels in the k-th silhouette
$\langle\cdot\rangle_p$	expectation wrt a distribution p
$\sigma(x)$	sigmoid function $(1+e^{-x})^{-1}$
ρ_k	approximate posterior $Q(X_k=1)$
π^{∞}	probability of observing z^{∞}
π°	probability of observing an outlier
$\Delta_{l,i}$	$\log\theta_{l,i}(z)$, $l \in \mathcal{K}\cup\{bg\}$
τ_{ki}	$\prod_{l<k, i\in \boldsymbol{M}_l}(1-\rho_l)$

Table 1: Notations

We know that X_k is a Bernoulli variable ($Q(X_k=1)+Q(X_k=0)=1$), and keeping in mind Eq. 7 from the paper:

$$Q(X_k=1)=\frac{\exp(\langle\log P(\boldsymbol{Z},\boldsymbol{X},\boldsymbol{M})|X_k=1\rangle)}{\tilde{Z}_{X_K}}$$

$$Q(X_k=0)=\frac{\exp(\langle\log P(\boldsymbol{Z},\boldsymbol{X},\boldsymbol{H})|X_k=0\rangle)}{\tilde{Z}_{X_K}}$$

which allows us to find the normalizing factor $\tilde{Z}_{X_K}$ and get the following update on $\rho_k = Q(X_k=1)$ (Eq. 8 in the paper):

$$\rho_k = \sigma(\langle\log P(\boldsymbol{Z},\boldsymbol{X},\boldsymbol{M}|X_k=0)\rangle_{Q(\boldsymbol{X}/X_k)} - \langle\log P(\boldsymbol{Z},\boldsymbol{X},\boldsymbol{M}|X_k=1)\rangle_{Q(\boldsymbol{X}/X_k)})$$

Let's take a closer look at the conditional expectation of the joint $\langle\log P(\boldsymbol{Z},\boldsymbol{X},\boldsymbol{M}|X_k=\xi)\rangle_{Q(\boldsymbol{X}/X_k)}, \xi\in\{0,1\}$ (omitting $\boldsymbol{M}$ for clarity):

$$\langle \log P(\boldsymbol{Z}, \boldsymbol{X} | X_k = \xi) \rangle_{Q(\boldsymbol{X}/X_k)} =$$
$$\langle \log P(\boldsymbol{Z}|\boldsymbol{X}) P(\boldsymbol{X}) | X_k = \xi \rangle_{Q(\boldsymbol{X}/X_k)} =$$
$$\textstyle\sum_{i \in \mathcal{S}_k} \langle \log P(Z_i|\boldsymbol{X}) | X_k = \xi \rangle + \sum_{l \in \mathcal{K}} \langle \log P(X_l) | X_k = \xi \rangle$$

where we used an assumption of conditional independence between pixels, and also assumption of independence of occupancies a-priori.

We now need to evaluate the expectation of the observation likelihood $\log P(Z_i|\boldsymbol{X})$ conditioned on $X_k = \xi$ for $\xi \in \{0, 1\}$. Under our generative model, each pixel is either generated by one of the silhouettes, or by the background. If it was generated by some silhouette $\mathcal{S}_l$, then, first, all the silhouettes that are closer to the camera should be absent (which happens with probability $\tau_{l-1,i}$), and, second, the silhouette itself should be present (which happens independently with probability ρ_l). Otherwise, all the silhouettes are absent (probability $\tau_{|\mathcal{K}|,i}$), and pixel was generated by the background. Now, let's write down the expected log-likelihood for observing some value z at pixel $i \in \mathcal{L}$ (omitting segmentation masks for clarity):

$$\langle \log P(Z_i = z|\boldsymbol{X}) \rangle_{Q(\boldsymbol{X})} = \sum_{l \in \mathcal{K}} \tau_{l-1,i} \rho_l \log \theta_{li}(z) + \tau_{|\mathcal{K}|,i} \log \theta_{bg,i}(z)$$

When conditioned on $X_k = 1$ the expectation will be:

$$\sum_{l<k} \tau_{l-1,i} \rho_l \log \theta_{li}(z) + \tau_{k-1,i} \log \theta_{ki}(z)$$

And conditioned when on $X_k = 0$:

$$\sum_{l<k} \tau_{l-1,i} \rho_l \log \theta_{li}(z) + \frac{\sum_{l>k} \tau_{l-1,i} \rho_l \log \theta_{li}(z) + \tau_{|\mathcal{K}|,i} \log \theta_{bg,i}(z)}{(1 - \rho_k)}$$

Now, one can evaluate $\langle \log P(\boldsymbol{Z}, \boldsymbol{X} | X_k = \xi) \rangle_{Q(\boldsymbol{X}/X_k)}$ (expectations for the priors is trivial, see e.g. Fleuret'09). If one substitutes these into Eq. 8, one will get Eq. 10:

$$\rho_k = \sigma(\log \tfrac{1-\epsilon}{\epsilon} -$$
$$\textstyle\sum_{i \in \boldsymbol{M}_k} \tau_{k-1,i} \log \theta_{k,i} -$$
$$\textstyle\sum_{i \in \boldsymbol{M}_k} \frac{1}{1-\rho_k} (\sum_{l>k, i \in \boldsymbol{M}_l} \tau_{l-1,i} \rho_l \log \theta_{l,i} + \tau_{|\mathcal{K}|,i} \log \theta_{bg,i}))$$