Advancing plasma turbulence understanding through a rigorous Verification and Validation procedure: a practical example

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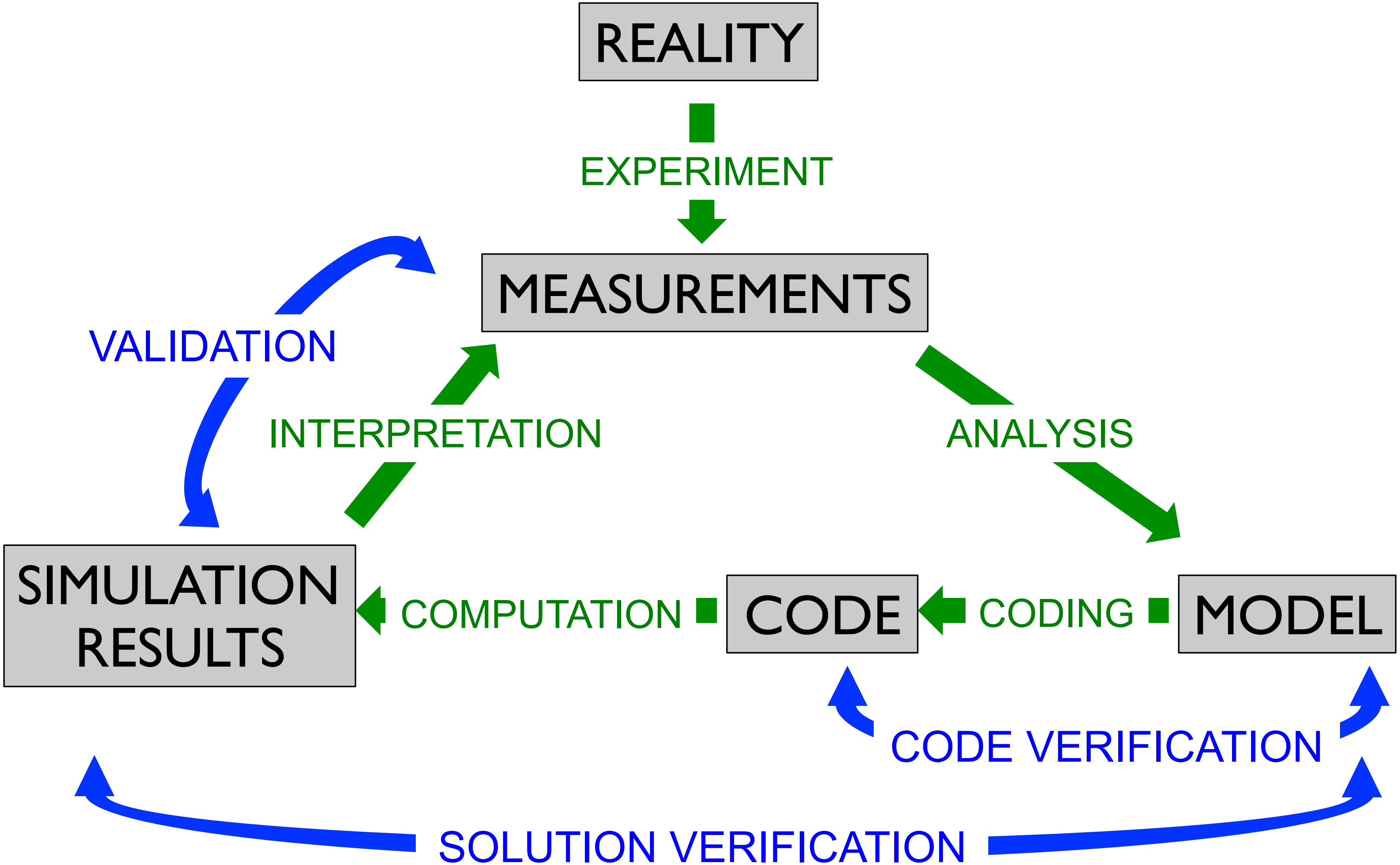
What does "Verification & Validation" (V&V) mean?

What V&V methodology did we use?

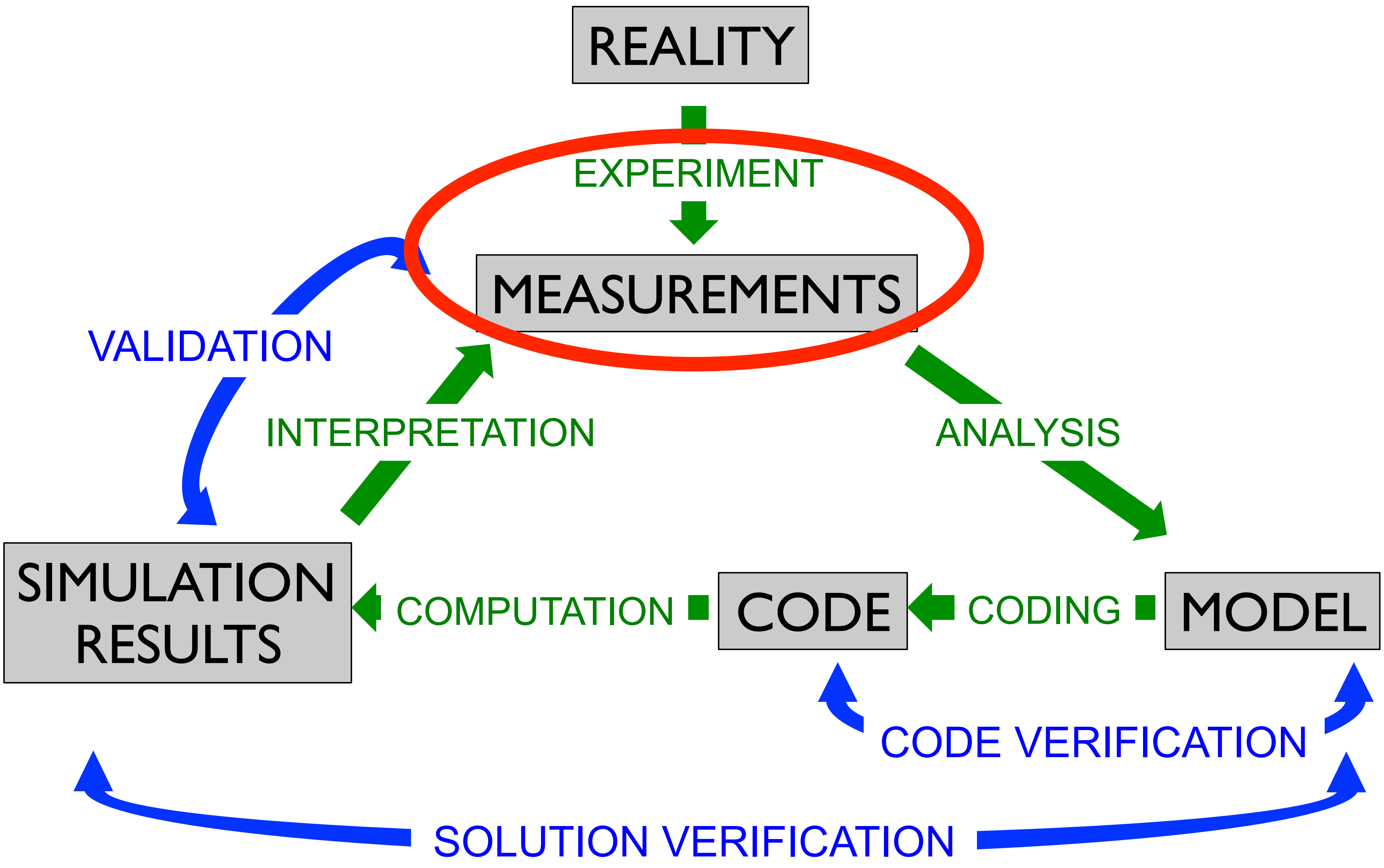
A practical example: GBS code and TORPEX experiment

What have we learned?

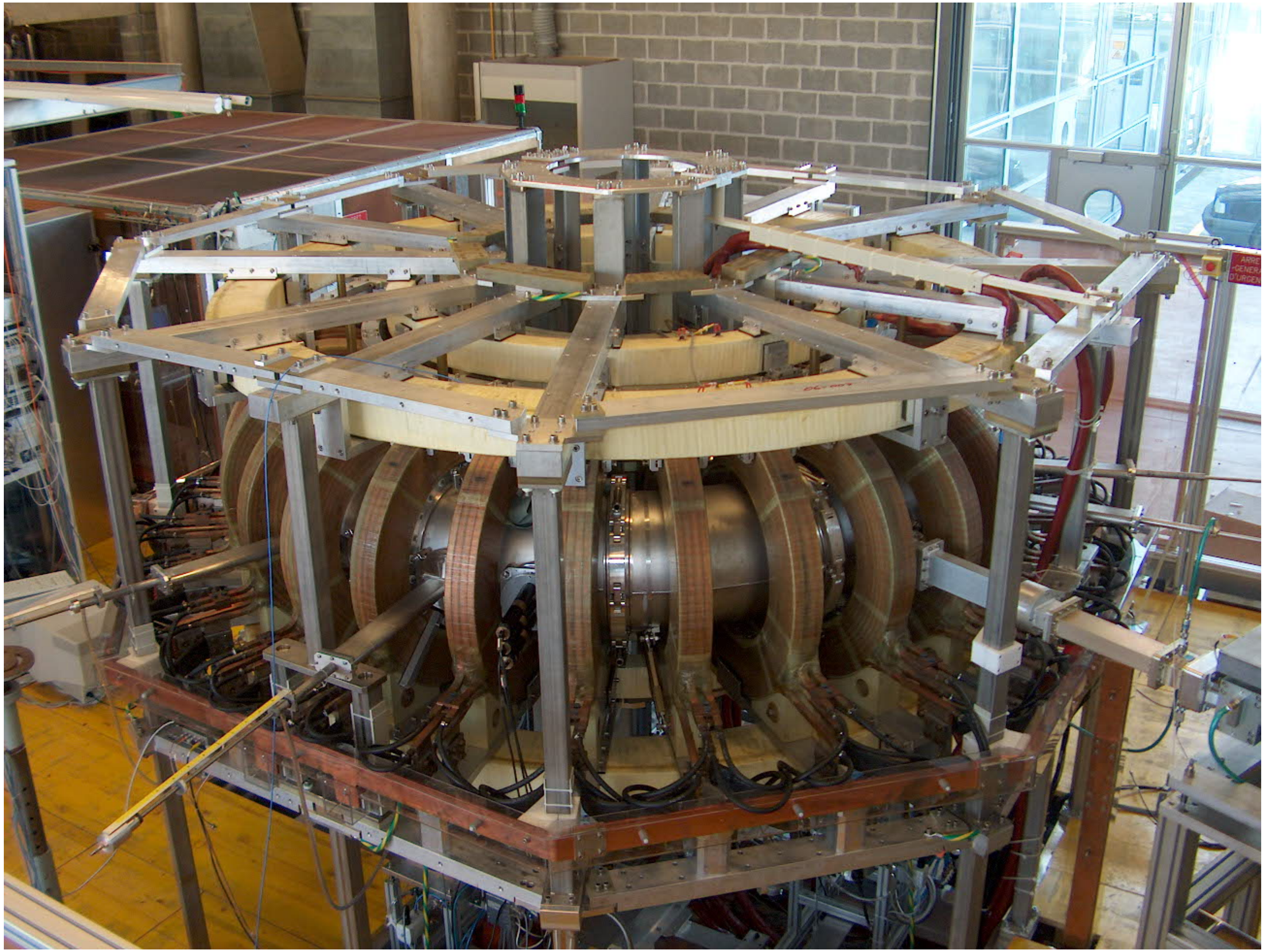
Verification & Validation



Verification & Validation

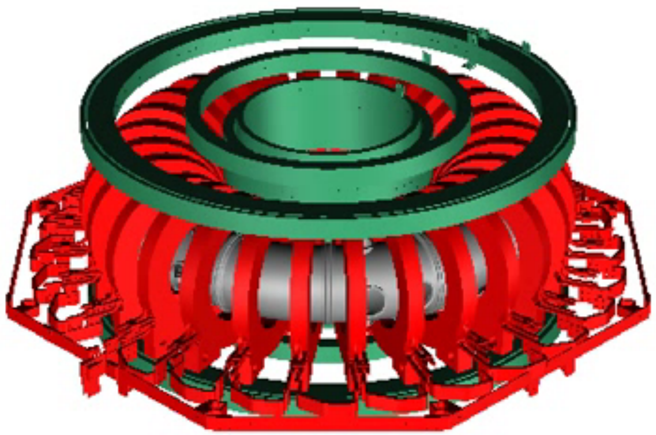


The TORPEX device



Fasoli et al., PoP 2006; PPCF 2010

The TORPEX device



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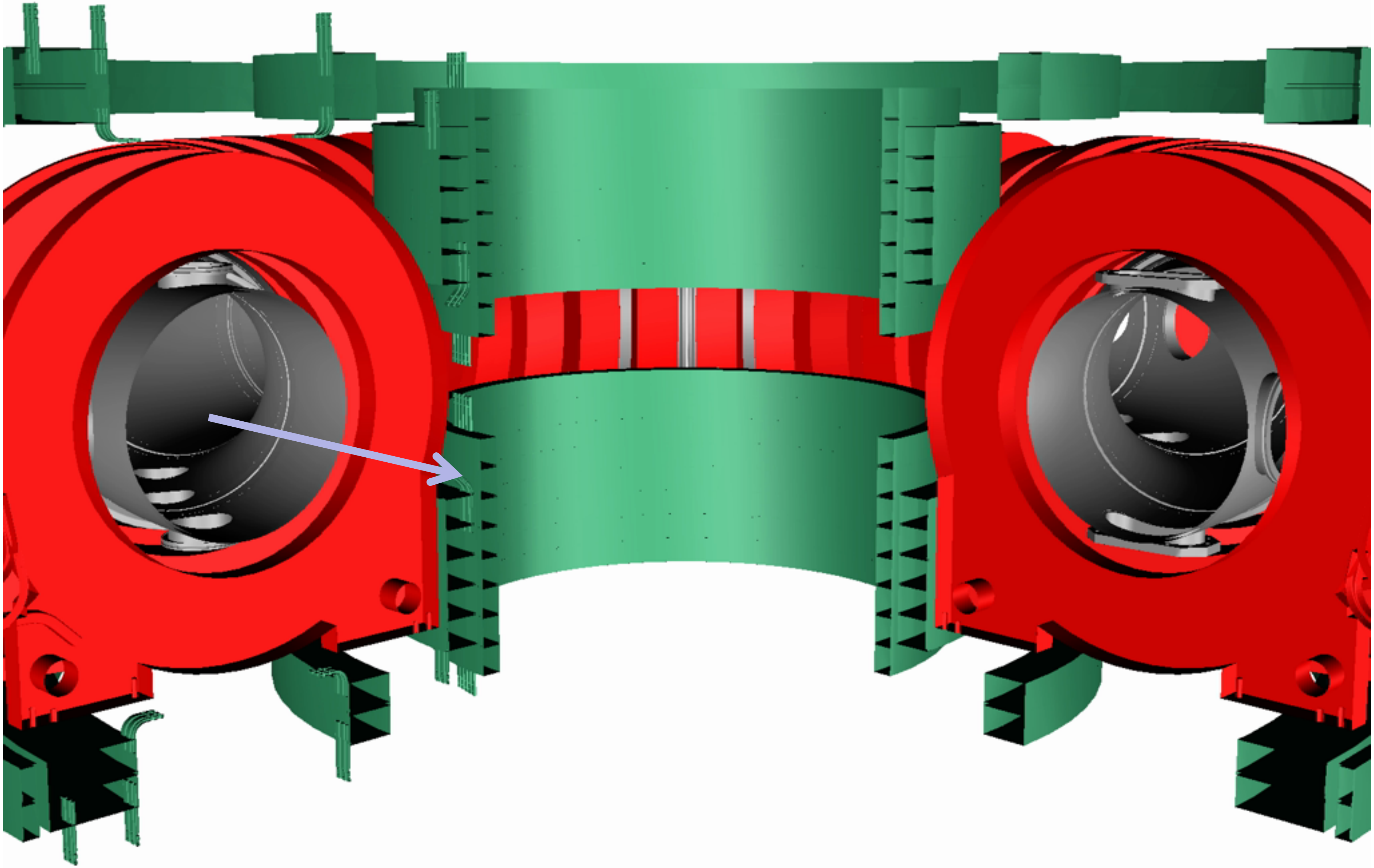

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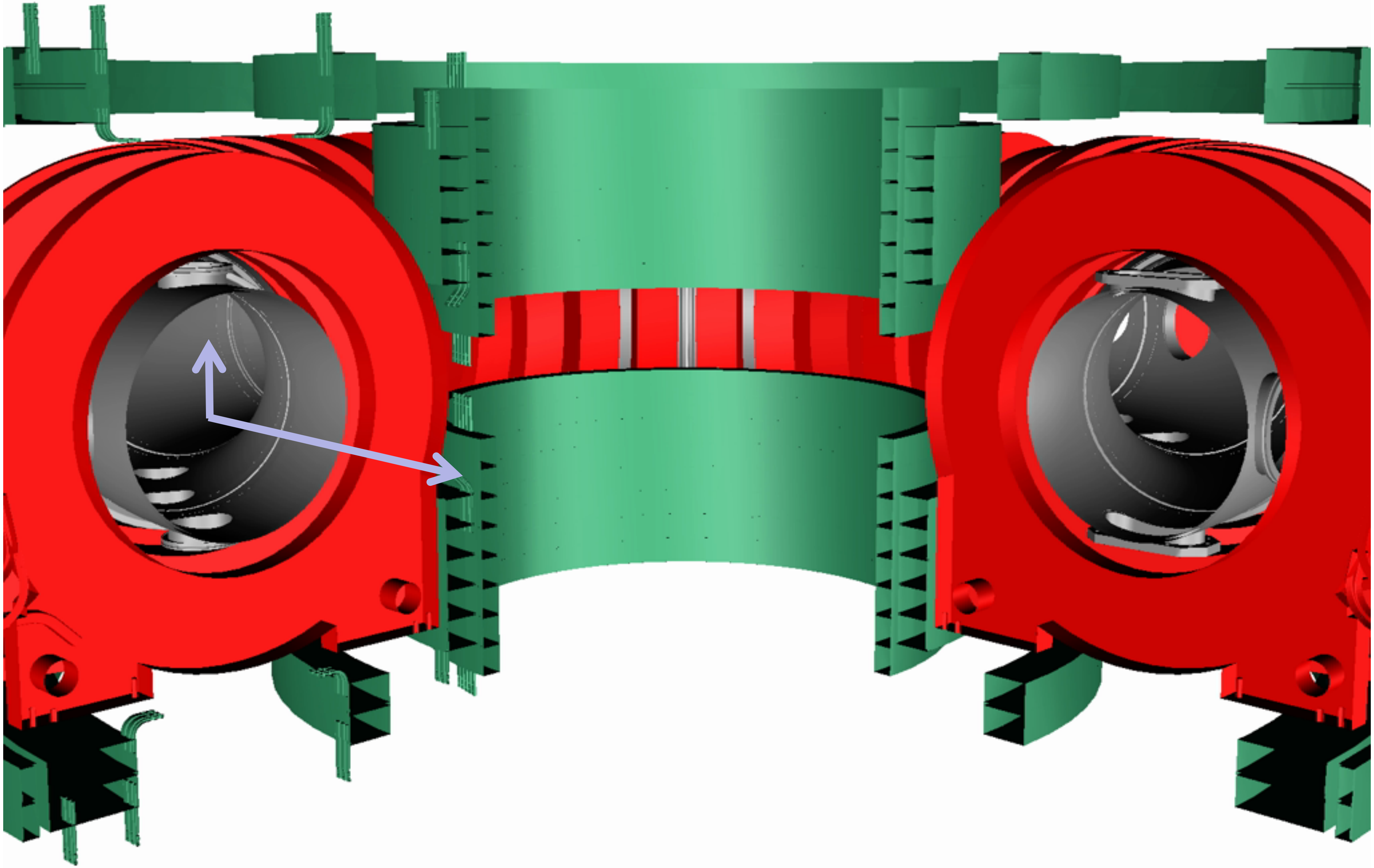

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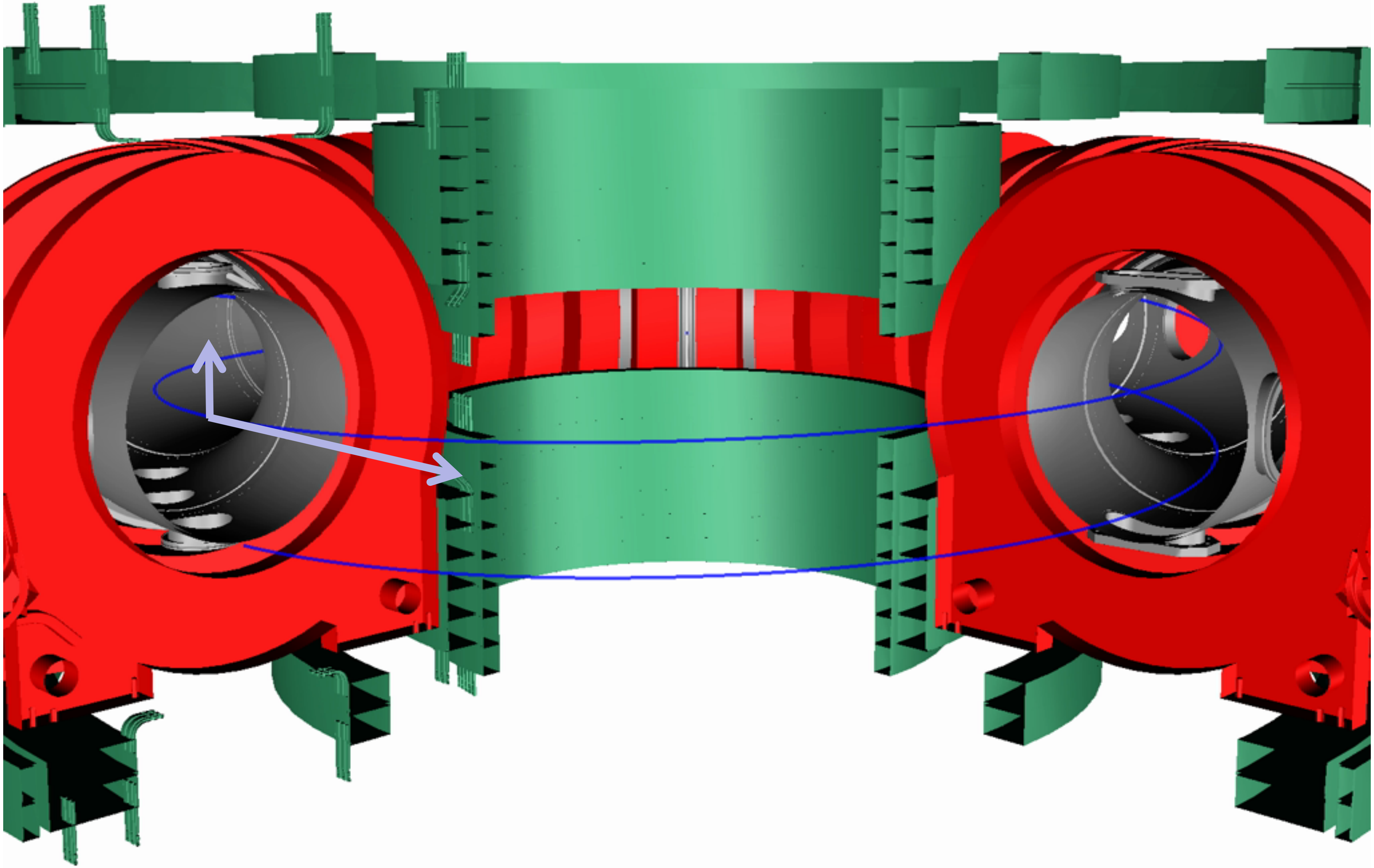
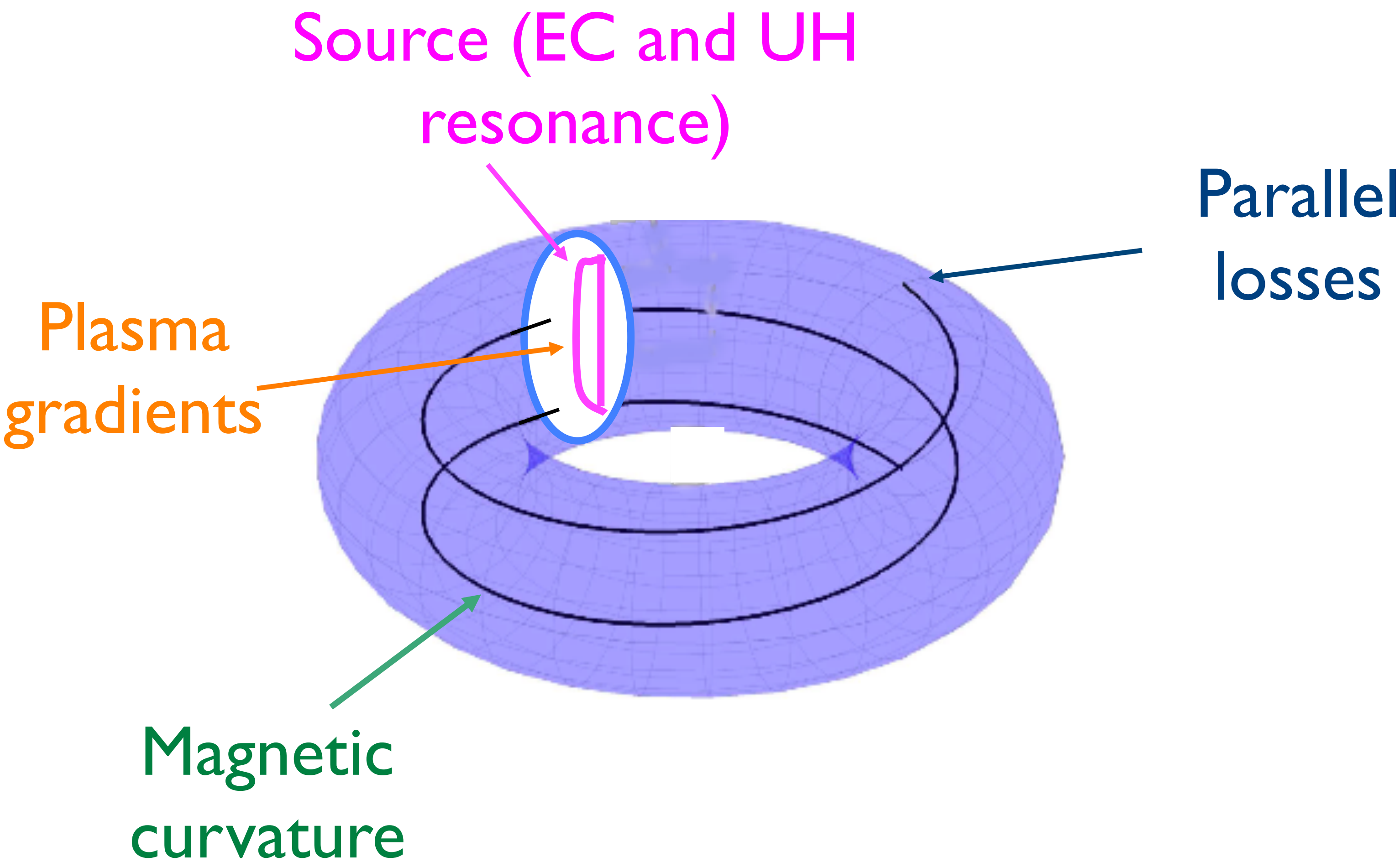
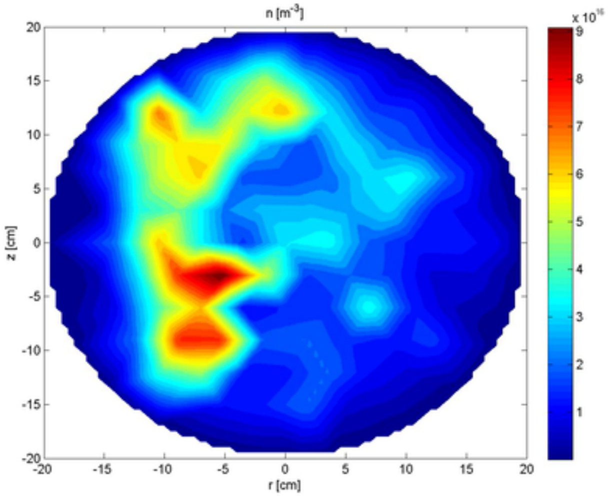


Key elements of the TORPEX device

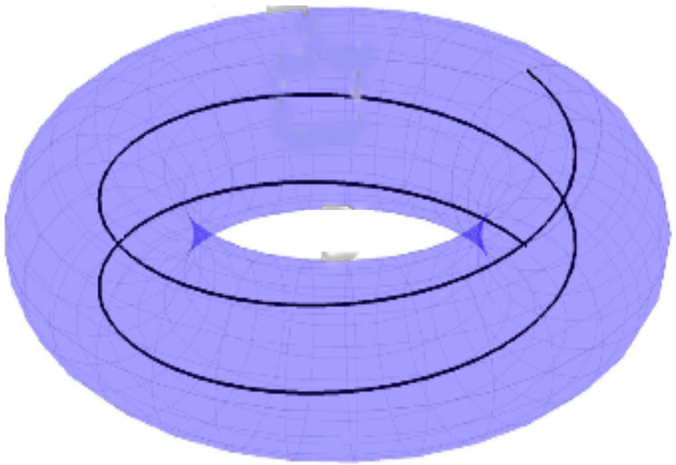


TORPEX: an ideal verification & validation testbed



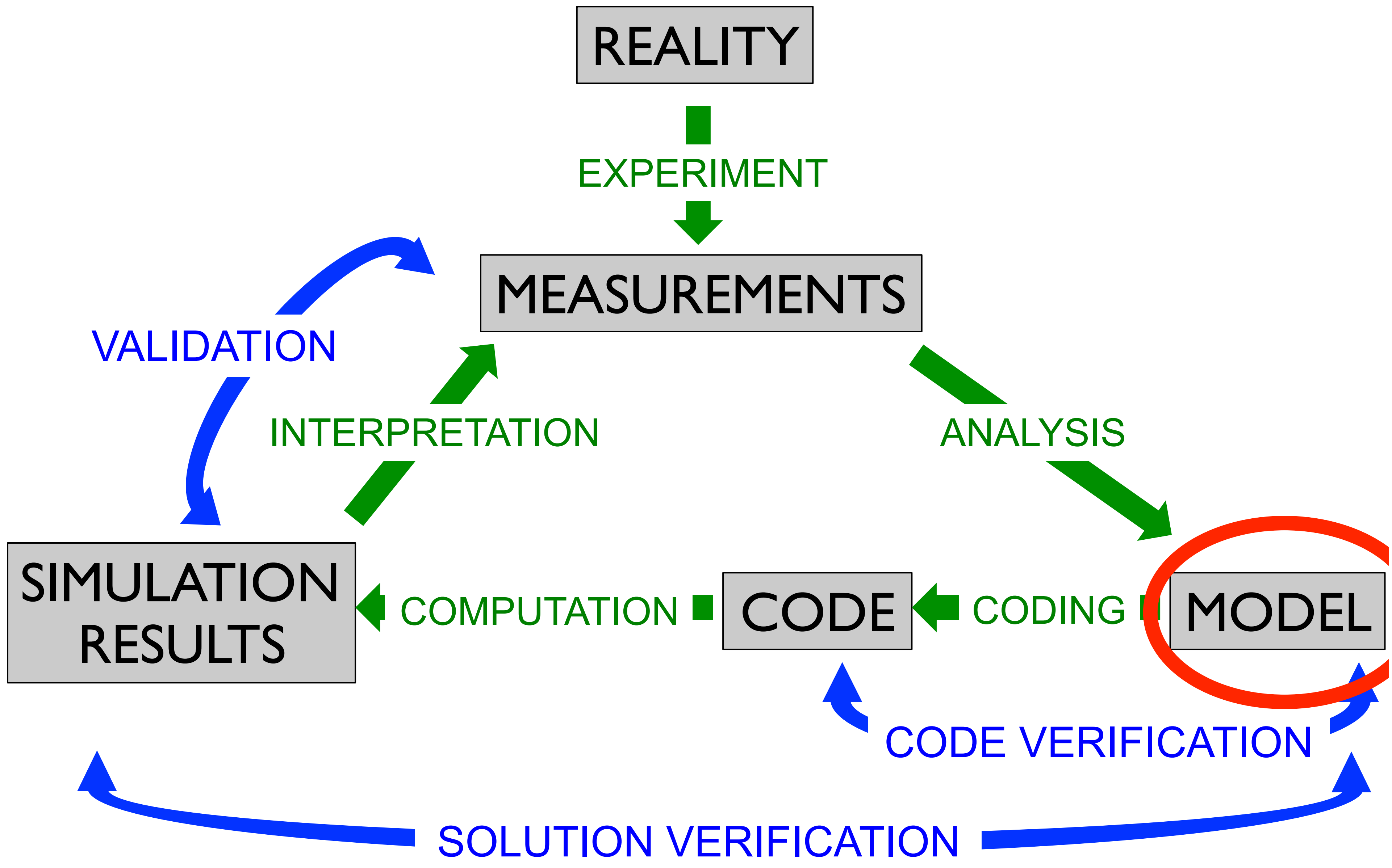
- Complete set of diagnostics, full plasma imaging possible

Example: N=2



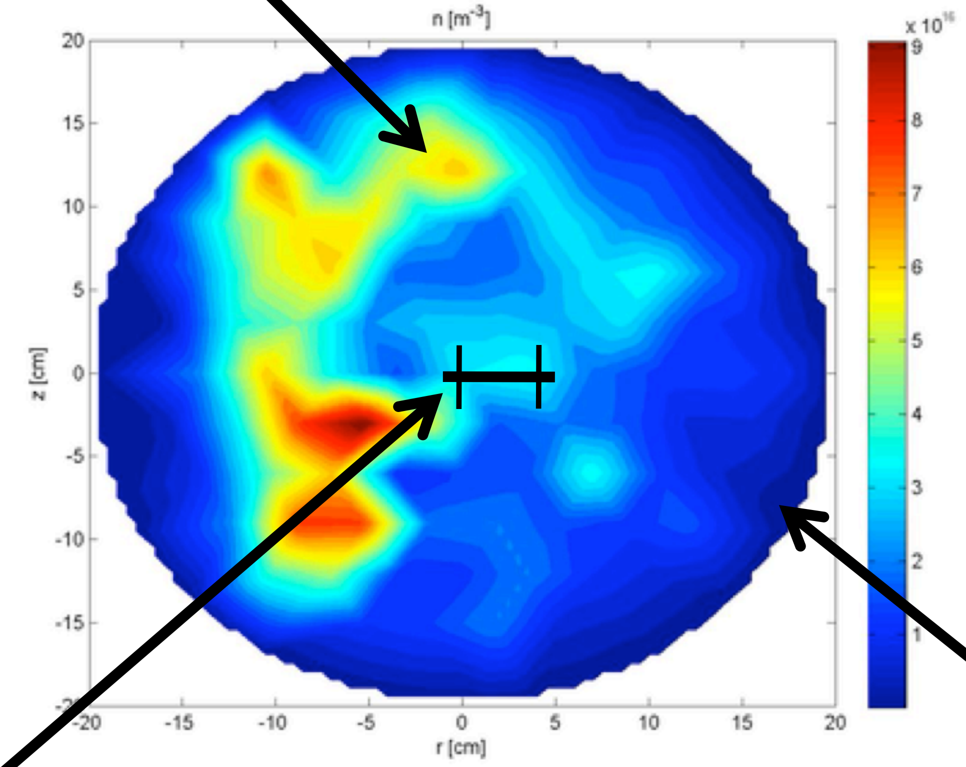
- Parameter scan, N – number of field line turns

Verification & Validation



Properties of TORPEX turbulence

$n_{fluc} \sim n_{eq}$



Collisional

$L_{eq} \sim L_{fluc}$

$L >> \rho_i$

The model

Collisional Plasma → **Braginskii model** → ($\rho_i << L$, $\omega << \Omega_{ci}$, $\beta << 1$) → **Electrostatic Drift-reduced Braginskii equations**

ExB Convection; Magnetic curvature; Parallel dynamics; Source

$$\frac{\partial n}{\partial t} + [\phi, n] = \hat{C}(nT_e) - n\hat{C}(\phi) - \nabla_{\|}(nV_{\|e}) + S$$

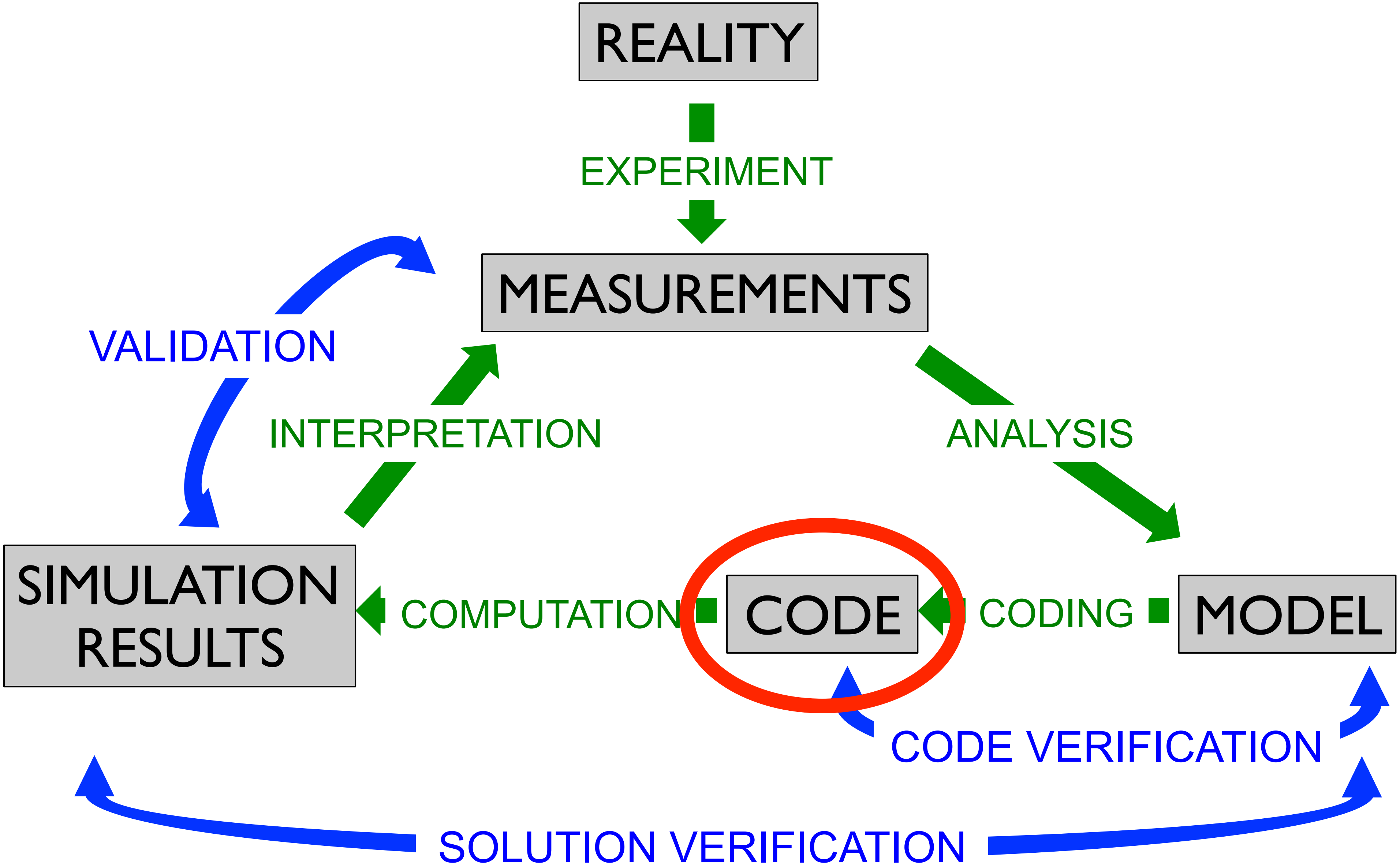
T_e, Ω (vorticity) → similar equations

$V_{\|e}$, $V_{\|i}$ → parallel momentum balance

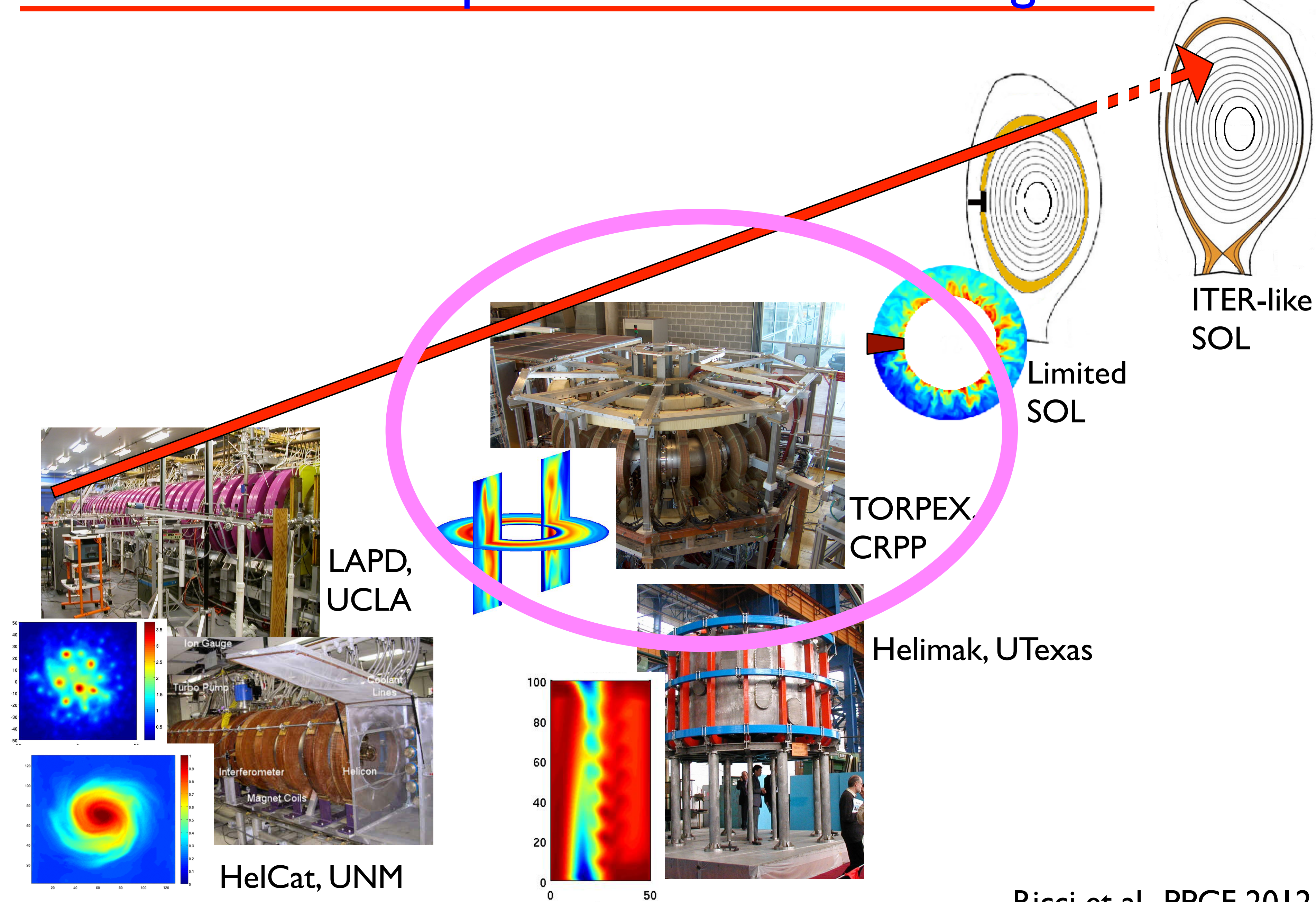
$\nabla_{\perp}^2 \phi = \Omega$

Quasi steady state – balance between:
plasma source, perpendicular transport, and parallel losses

Verification & Validation

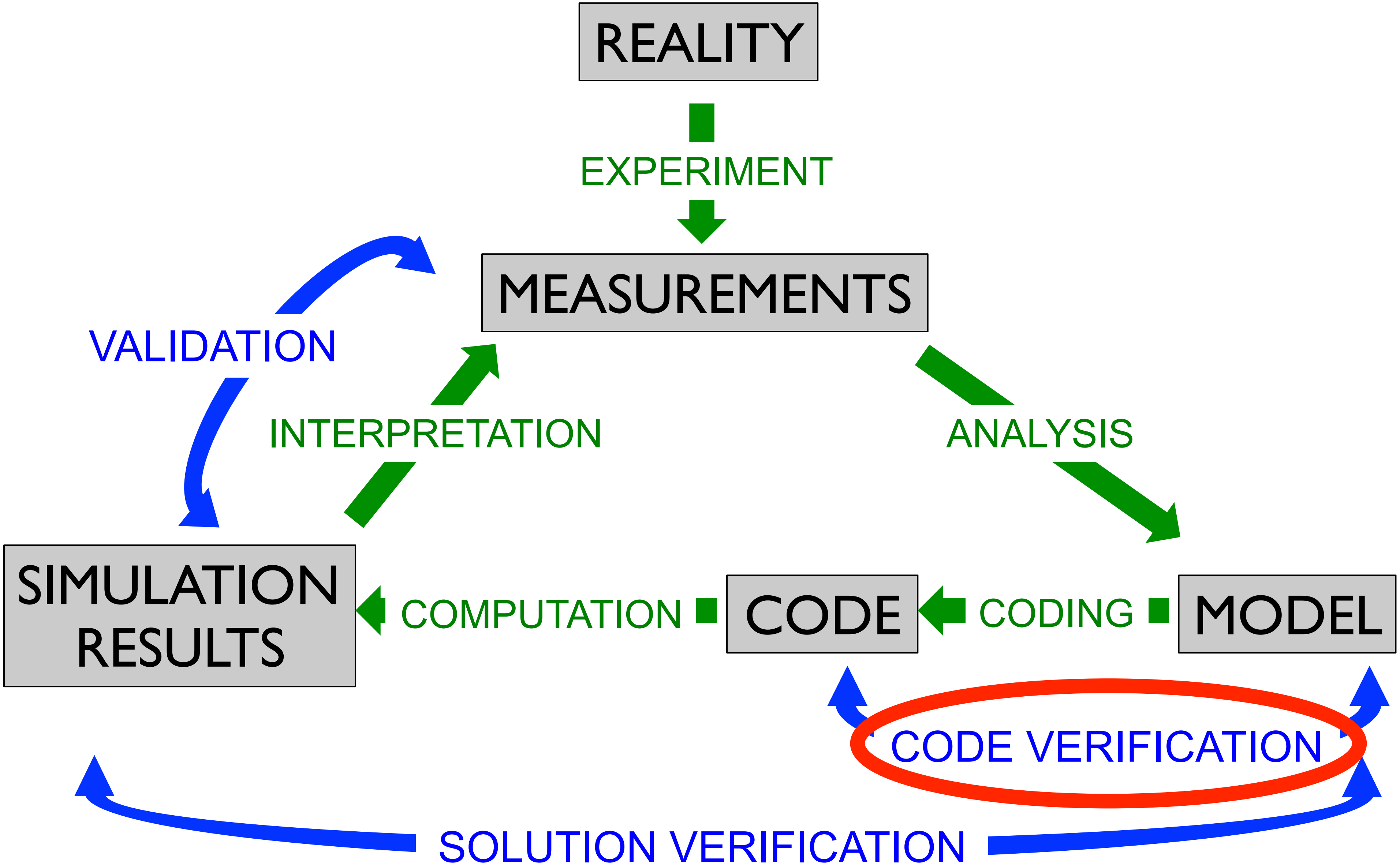


GBS: simulation of plasma turbulence in edge conditions



Ricci et al., PPCF 2012

Verification & Validation



Code verification, the techniques

1) Simple tests
2) Code-to-code comparisons (benchmarking)
3) Discretization error quantification
4) Convergence tests
5) Order-of-accuracy tests

NOT RIGOROUS (1–2)

RIGOROUS, requires analytical solution (3–5)

Only verification ensuring convergence and correct numerical implementation

Riva et al., PoP 2014

Order-of-accuracy tests, method of manufactured solution

Our model: $A(f) = 0, \quad f$ unknown

We solve $A_n(f_n) = 0,$ but $\epsilon_n = f_n - f = ?$

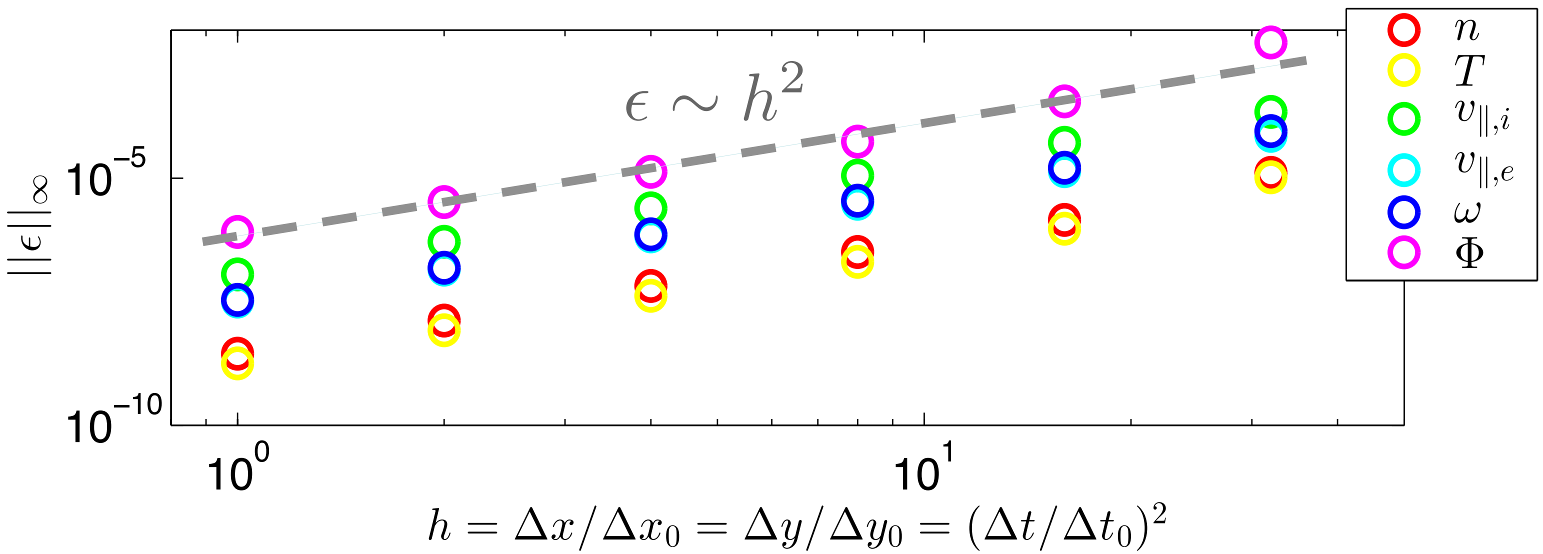
Method of manufactured solution:

1) we choose $g,$ then $S = A(g)$

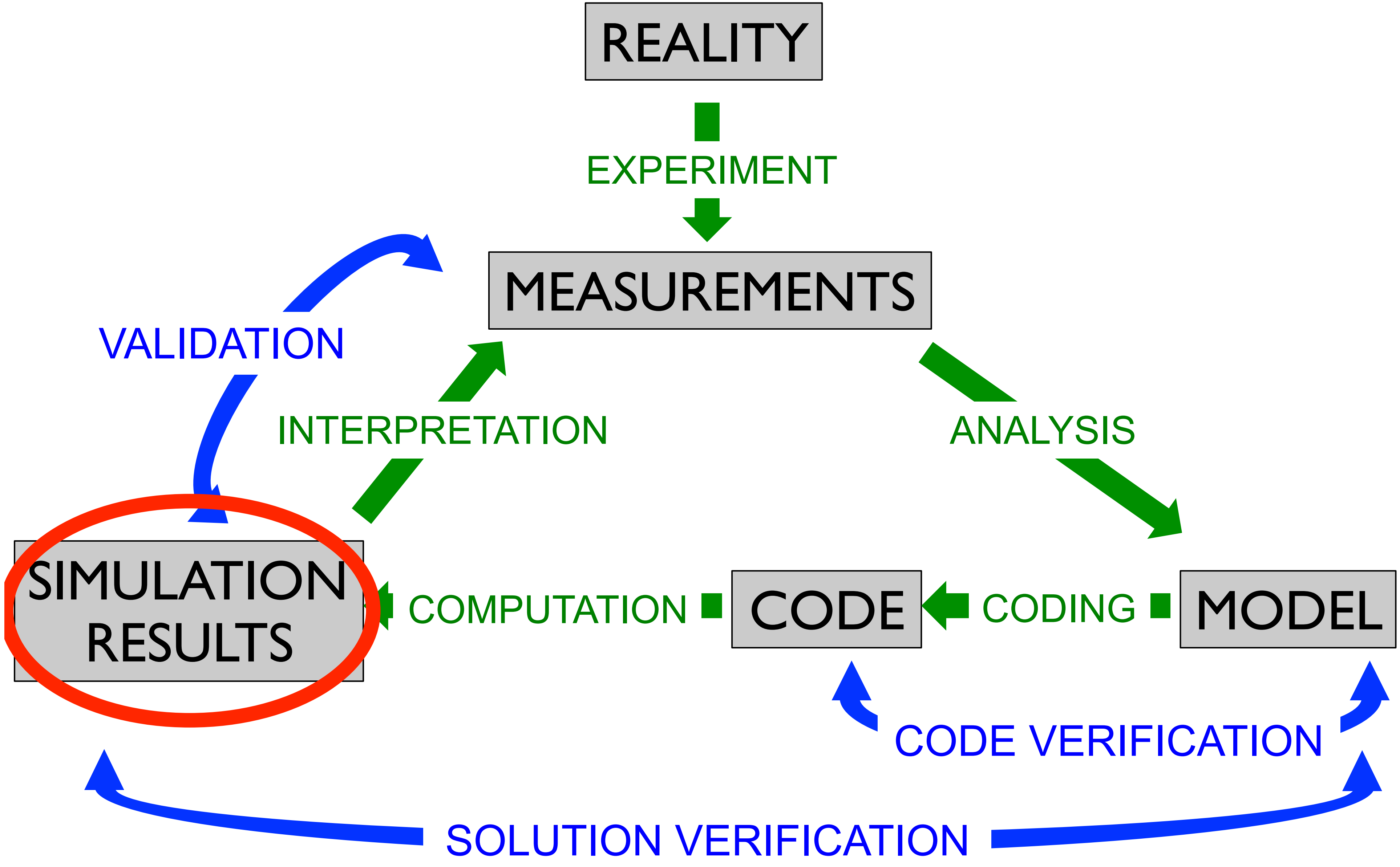
2) we solve: $A_n(g_n) - S = 0$

$\epsilon_n = g_n - g$

For GBS:

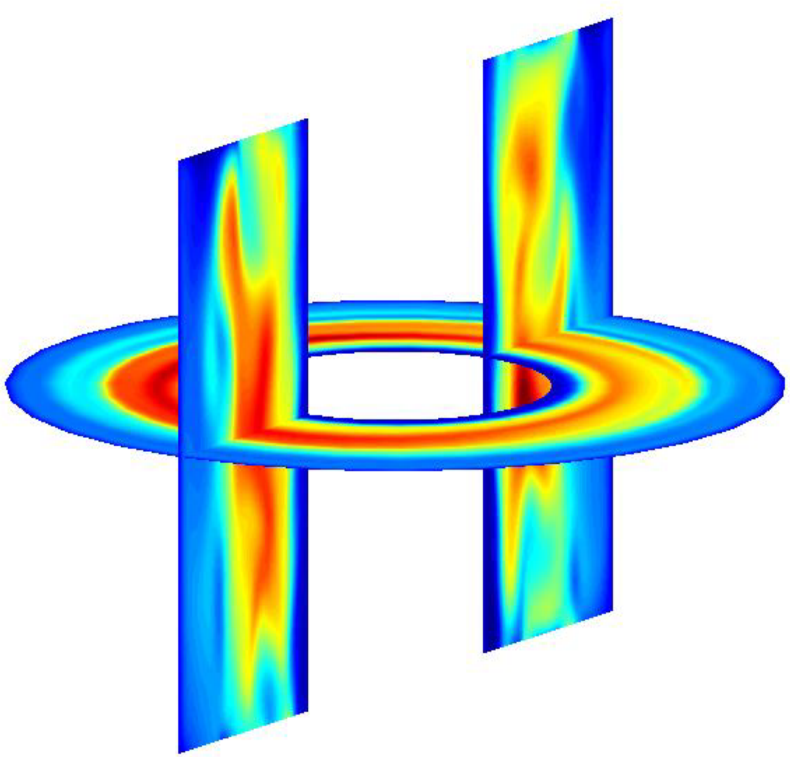


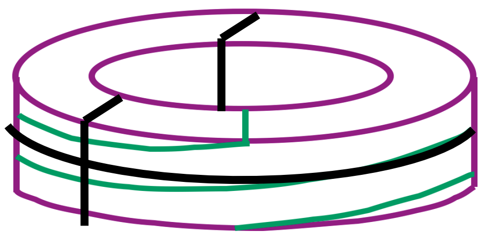
Verification & Validation



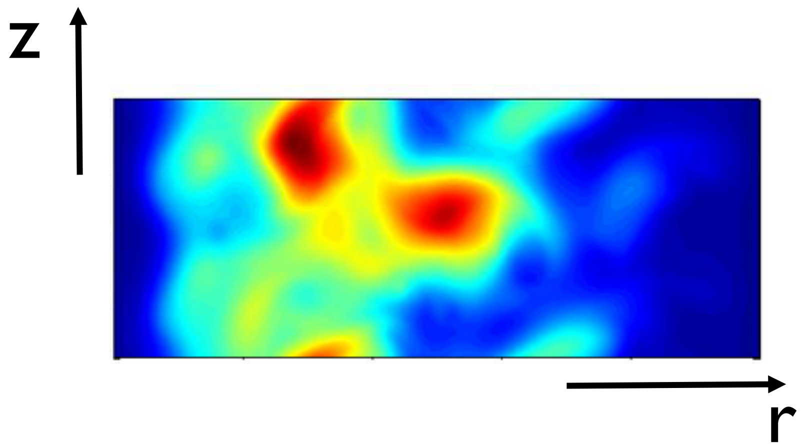
3D and 2D GBS simulations

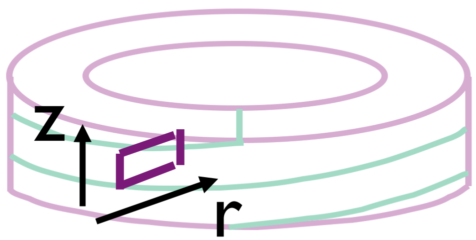
Fully 3D version



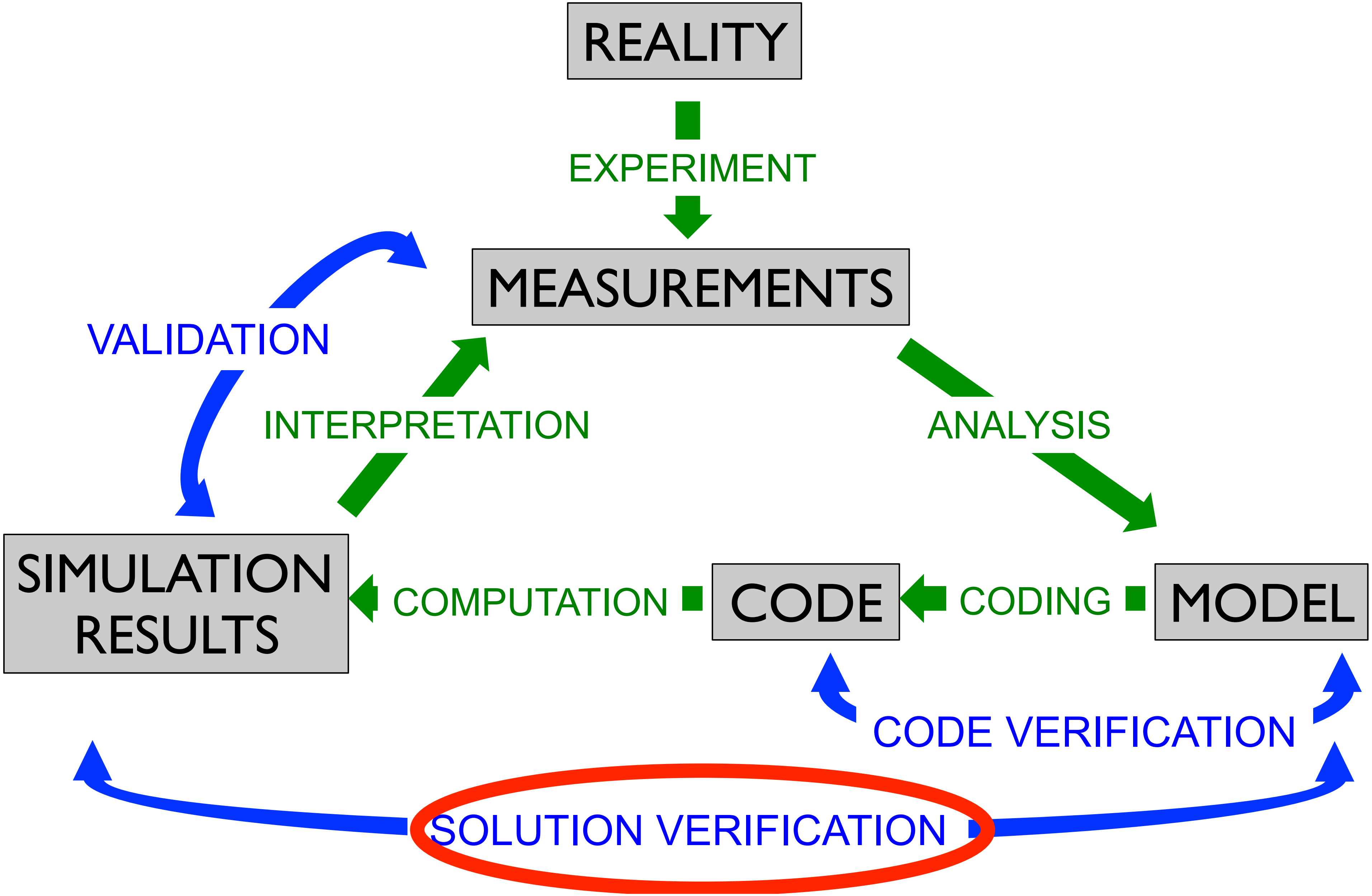


2D version ($k_{||}$=0 hypothesis)



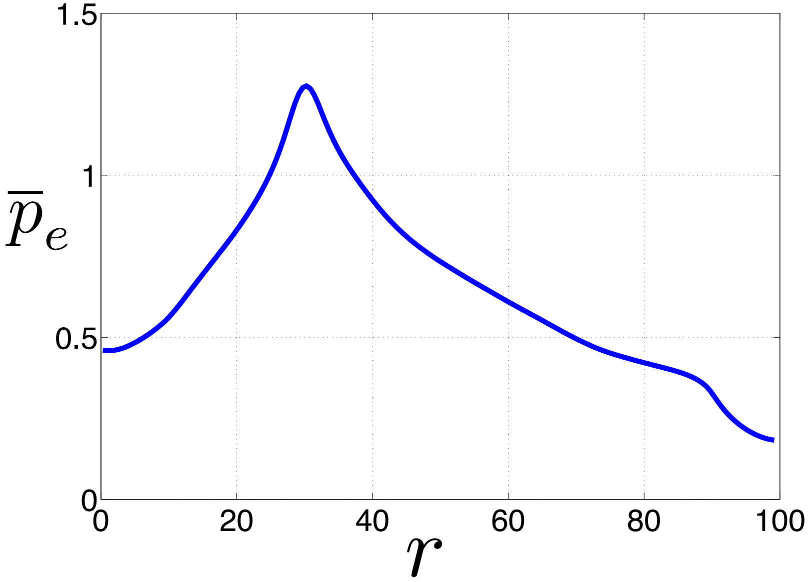


Verification & Validation



Solution verification, numerical error estimate

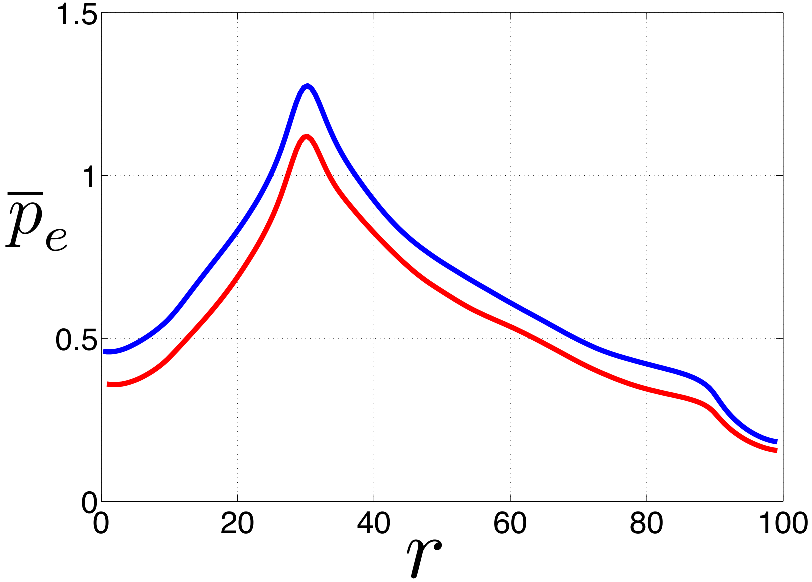
1. Calculate f on standard grid, f_s



Riva et al., PoP 2014

Solution verification, numerical error estimate

1. Calculate f on standard grid, f_s
2. Calculate f on a grid coarsened by α, f_c



Riva et al., PoP 2014

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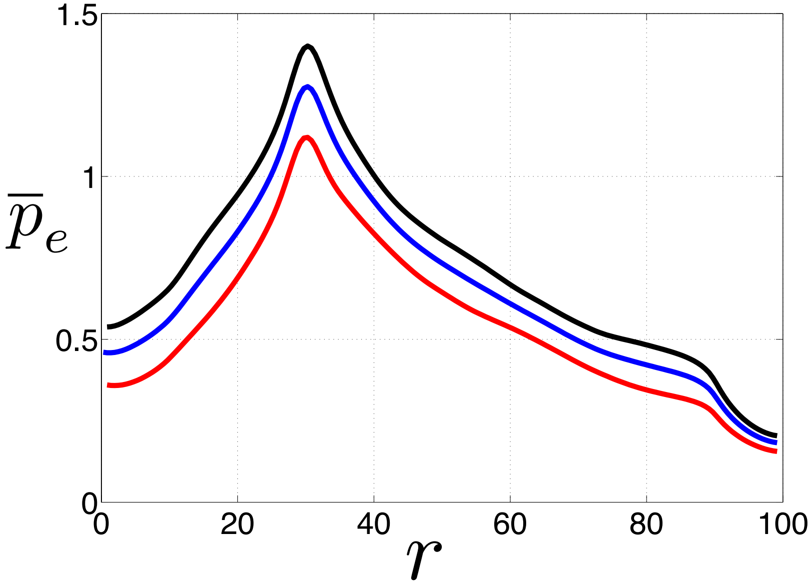
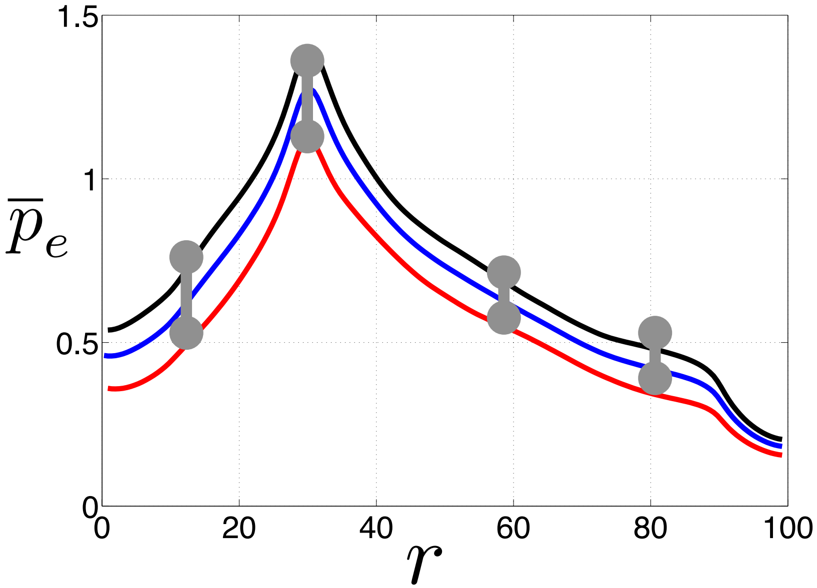


1. Calculate f on standard grid, f_s
2. Calculate f on a grid coarsened by α, f_c
3. Compute Richardson extrapolation

$$\overline{f} = f_s + (f_s - f_c)/(\alpha^p - 1)$$

Riva et al., PoP 2014

Solution verification, numerical error estimate



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4. Compute

$$\epsilon = |(f_s - f_c)/(\alpha^p - 1)|$$

Riva et al., PoP 2014

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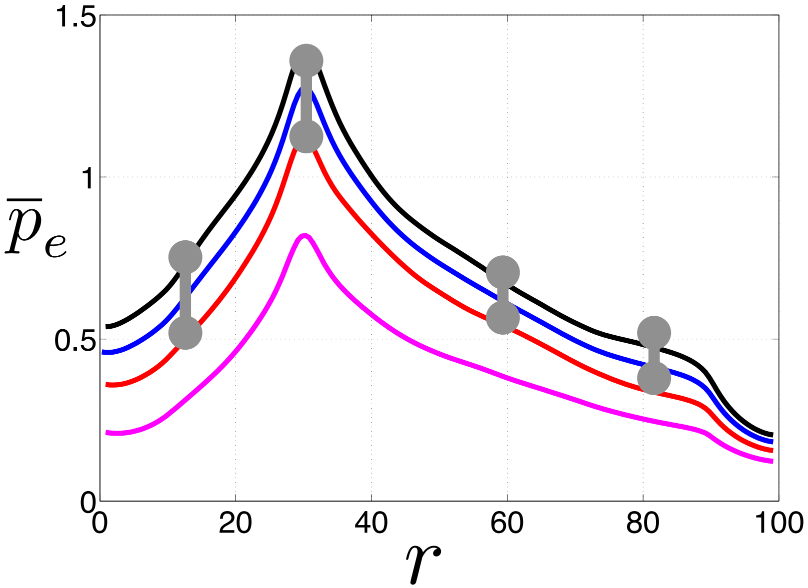

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4. Compute

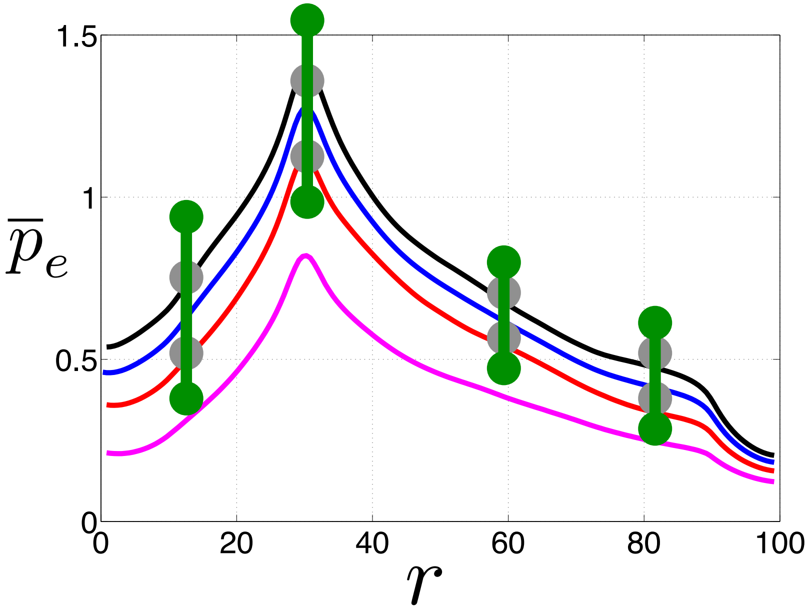
$$\epsilon = |(f_s - f_c)/(\alpha^p - 1)|$$

5. Calculate f on a grid even coarser, by α^2, f_{cc} , and evaluate

$$\hat{p} = \frac{\ln[(f_{cc} - f_c)/(f_c - f_s)]}{\ln(\alpha)}$$

Riva et al., PoP 2014

Solution verification, numerical error estimate



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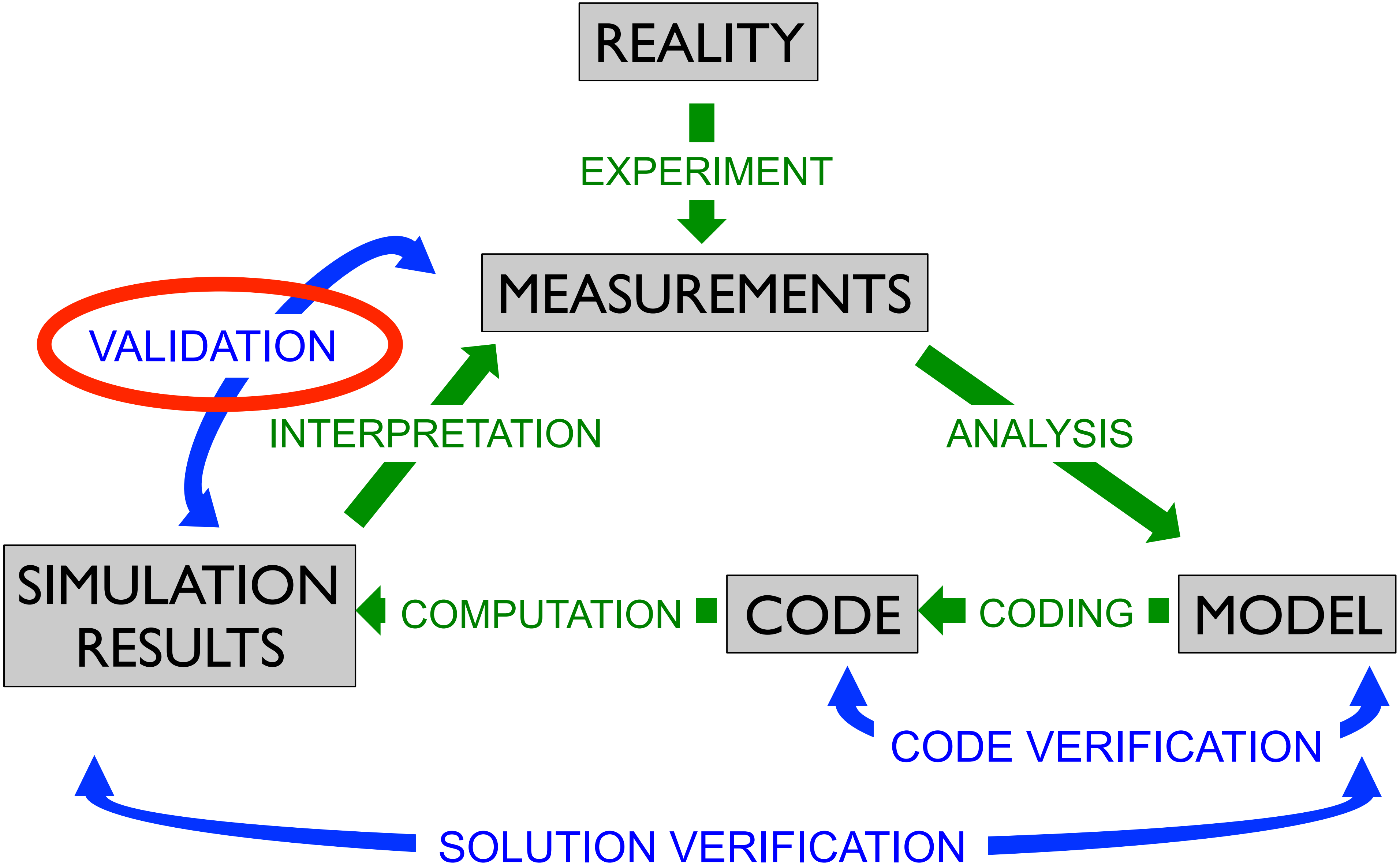
$$\hat{p} = \frac{\ln[(f_{cc} - f_c)/(f_c - f_s)]}{\ln(\alpha)}$$

6. Compute the GCI error estimate

$$\mathrm{GCI} = \frac{F_s\,|f_s - f_c|}{(\alpha^{\tilde{p}} - 1)\,|f_s|}$$

Riva et al., PoP 2014

Verification & Validation



Validation goals

- Make progress in physics understanding

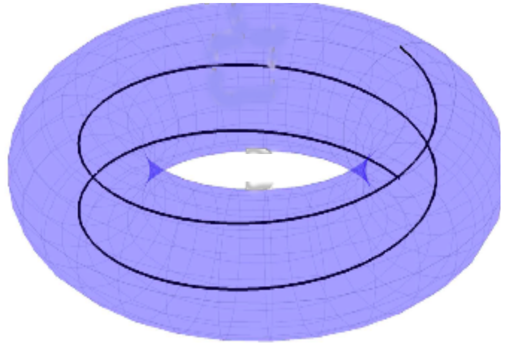
- Compare experiments and simulations to assess physics of the model

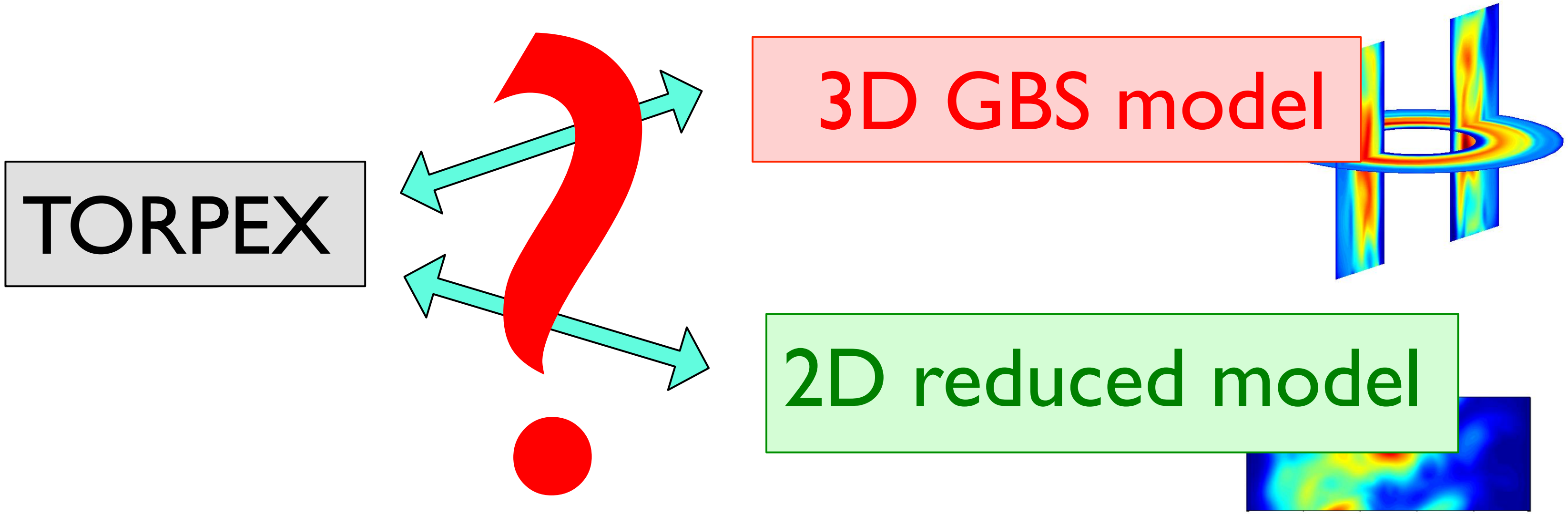
- Consider different models and parameter scans to guide us to key physics

- Avoid fortuitous agreement

- Rigorous tool, but easy to use

Our project, paradigm of turbulence code validation





For the 2 codes, what is the agreement of experiment and simulations as a function of N?

Are 3D effects important?

Our physics progress: role of 3D in TORPEX physics?

The validation methodology

[Based on ideas of Terry *et al.*, PoP 2008; Greenwald, PoP 2010]

What quantities can we use for validation? The more, the better…

- Definition & evaluation of the validation observables

What are the uncertainties affecting measured and simulation data?

- Uncertainty analysis

For one observable, within its uncertainties, what is the level of agreement?

- Level of agreement for an individual observable

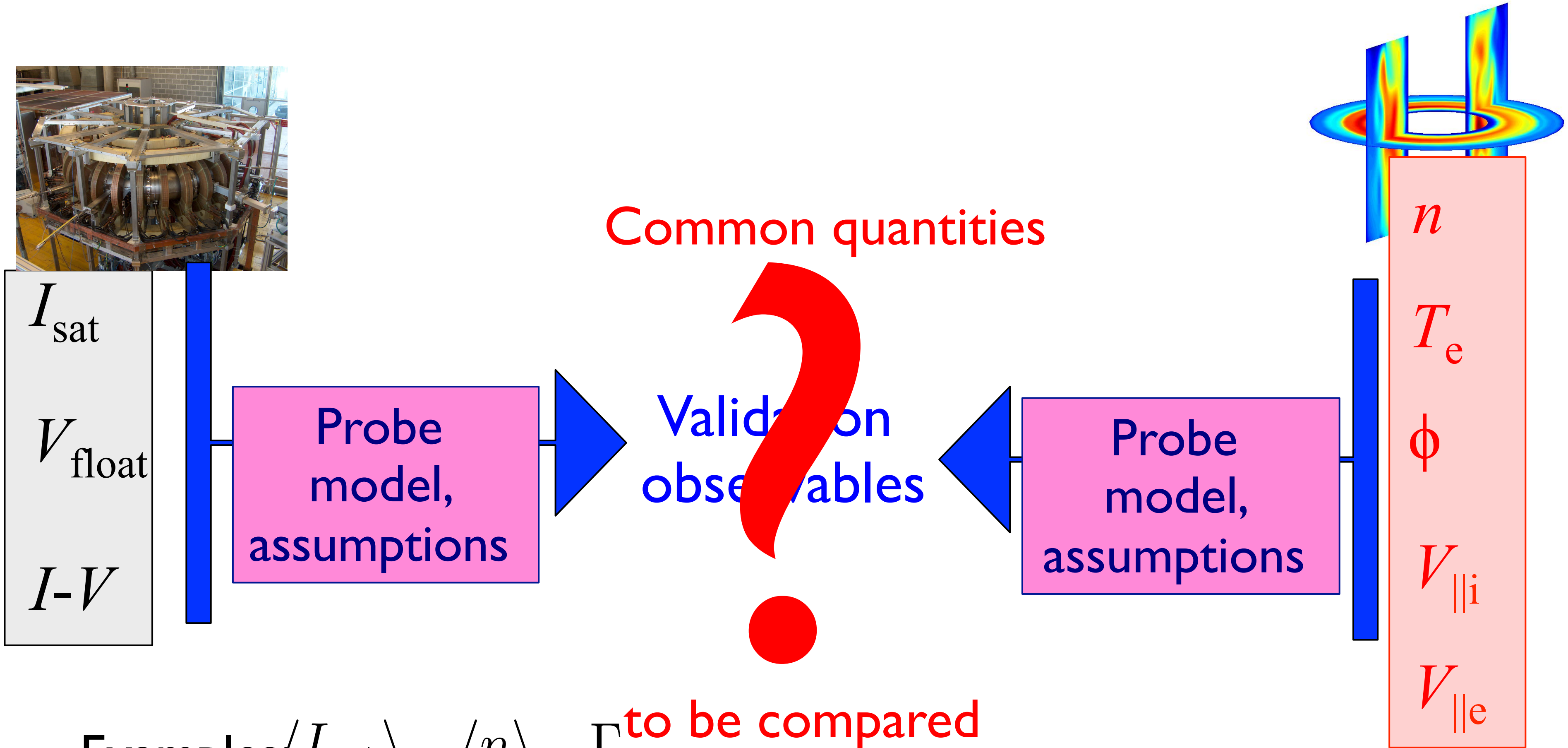
How directly can an observable be extracted from simulation and experimental data? How worthy is it, i.e. what should be its weight in a composite metric?

- The observable hierarchy

How to evaluate the global agreement and how to interpret it

- Composite metric

Definition of the validation observables



- Examples: $\langle I_{\rm sat} \rangle_t\,,\ \langle n \rangle_t\,,\ \Gamma, \ldots$

- A validation observable should not be a function of the others

- 11 observables for our validation:

$$\langle n(r) \rangle_t\,,\ \langle T_e(r) \rangle_t\,,\ \langle I_{\rm sat}(r) \rangle_t\,,\ \delta I_{\rm sat}/I_{\rm sat},\ k_v,\ {\rm PDF}(I_{\rm sat}),\ \ldots$$

Uncertainty analysis

Experiment

$$\Delta x^2 = \Delta x^2_{\mathrm{fit}} + \Delta x^2_{\mathrm{prb}} + \Delta x^2_{\mathrm{rep}} + \Delta x^2_{\mathrm{fin}}$$

- $\Delta x^2_{\mathrm{fit}}$: I-V Fitting
- $\Delta x^2_{\mathrm{prb}}$: Probe properties, measurement uncertainties
- $\Delta x^2_{\mathrm{rep}}$: Plasma reproducibility
- $\Delta x^2_{\mathrm{fin}}$: Finite statistics

Simulation

$$\Delta y^2 = \Delta y^2_{\mathrm{num}} + \Delta y^2_{\mathrm{inp}} + \Delta y^2_{\mathrm{fin}}$$

- $\Delta y^2_{\mathrm{num}}$: Numerics
- $\Delta y^2_{\mathrm{inp}}$: Input parameters - scan in resistivity and boundary conditions
- $\Delta y^2_{\mathrm{fin}}$: Finite statistics

Agreement with respect to an individual observable

Distance:

Experimental measurements

Simulation results

$$d = \sqrt{\frac{1}{G}\sum_{i=1}^{G}\frac{(x_i - y_i)^2}{\Delta x_i^2 + \Delta y_i^2}}$$

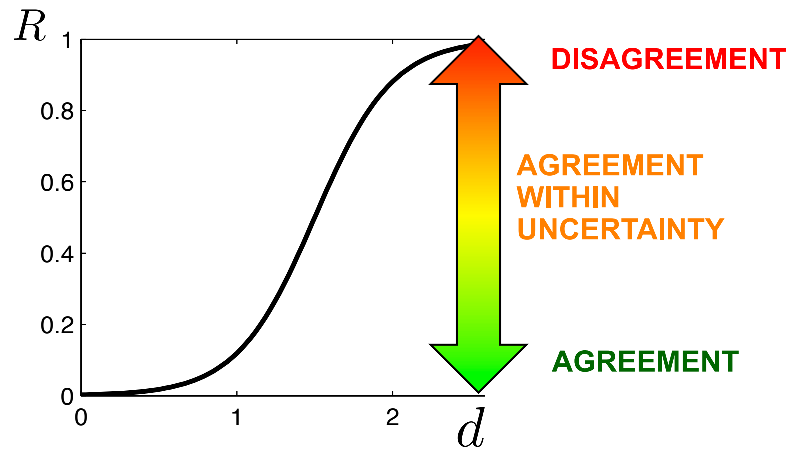
Average over all points

Normalization to uncertainties

Level of agreement:

$$R = \frac{\tanh[(d - d_0)/\lambda] + 1}{2}$$

$$d_0 = 1.5$$
$$\lambda = 0.5$$



Observable hierarchy

Not all the observables are equally worthy…

The hierarchy assesses the assumptions used for their deduction

h^{exp} : # of assumptions to get the observable from experimental data

h^{sim} : same for simulation results

→ $$h = h^{exp} + h^{sim}$$

Examples: - $\langle n \rangle_t$: $h^{\mathrm{exp}} = 1$, $h^{\mathrm{sim}} = 0$, $h = 1$

- $\Gamma_{I_{\mathrm{sat}}}$: $h^{\mathrm{exp}} = 2$, $h^{\mathrm{sim}} = 1$, $h = 3$

Composite metric

Level of agreement

$$R_j = \frac{\tanh[(d_j - d_0)/\lambda] + 1}{2}$$

Hierarchy level

$$H_j = 1/(h_j + 1)$$

Sensitivity

$$S_j = \exp\left(- \frac{\Sigma_i \Delta x_{j,i} + \Sigma_i \Delta y_{j,i}}{\Sigma_i |x_{j,i}| + \Sigma_i |y_{j,i}|} \right)$$

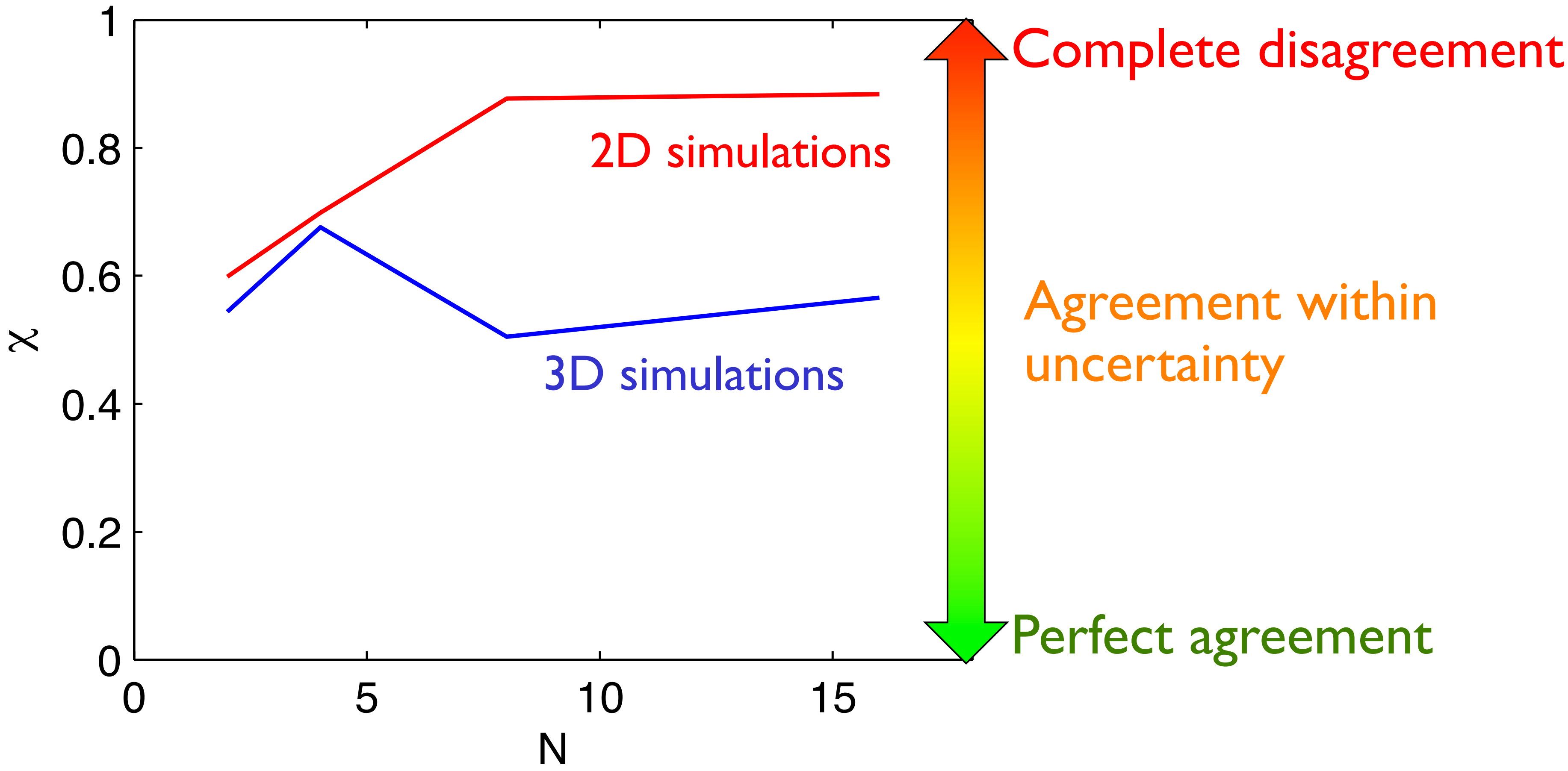
Sum over all the observables

$$\chi = \frac{\Sigma_j R_j H_j S_j}{\Sigma_j H_j S_j}$$

Normalization:

- $\chi = 0$: perfect agreement
- $\chi = 0.5$: agreement within uncertainty
- $\chi = 1$: total disagreement

The validation results



Ricci et al., PoP 2009, PoP 2011

Why 2D and 3D work equally well at low N and 2D fails at high N?
What can we learn on the TORPEX physics?

Flute instabilities - ideal interchange mode

$k_{||} = 0$ ➡

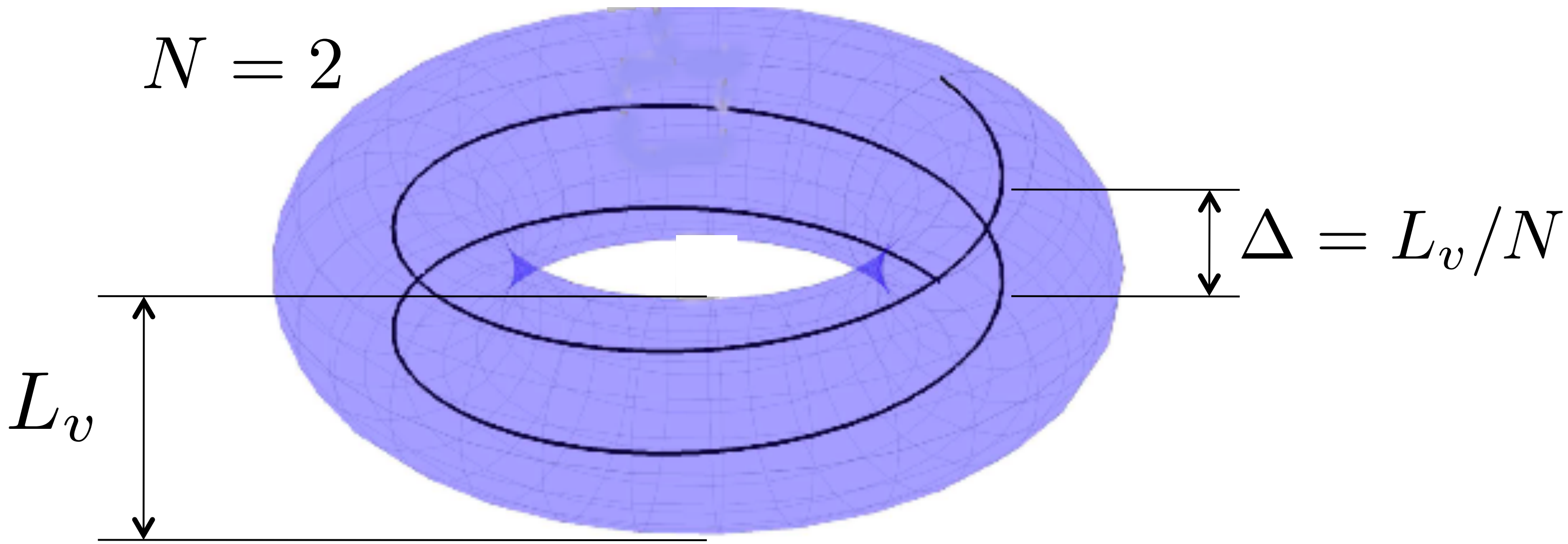
n + T_e eqs. ⟶ $$\frac{\partial p_e}{\partial t} = \frac{c}{B}[\phi, p_e]$$

Vorticity eq. ⟶ $$\frac{\partial \nabla_\perp^2 \phi}{\partial t} = \frac{2B}{cm_i R n}\frac{\partial p_e}{\partial y}$$

➡ $$\gamma = \gamma_I \qquad \gamma_I = c_s\sqrt{\frac{2}{L_p R}}$$

Compressibility stabilizes the mode at $k_v \rho_s > 0.3\gamma_I R/c_s$

Anatomy of a $k_{\parallel} = 0$ perturbation



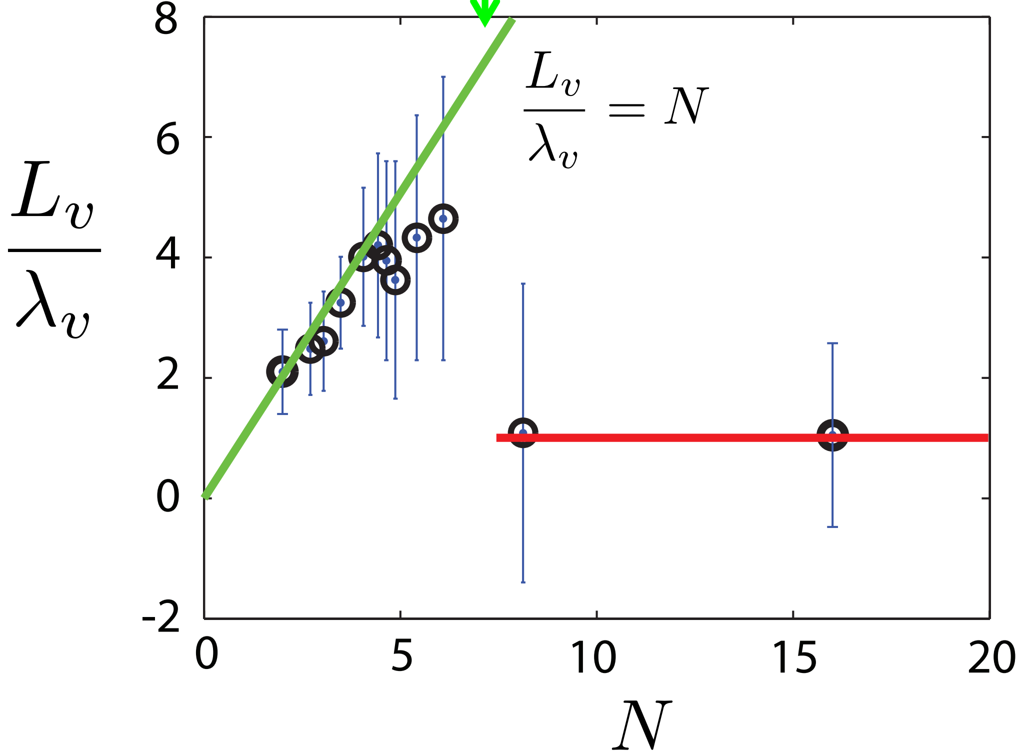
λ_v : longest possible vertical wavelength of a perturbation

If $k_{\parallel} = 0$ then $\lambda_v = \Delta = \dfrac{L_v}{N}$

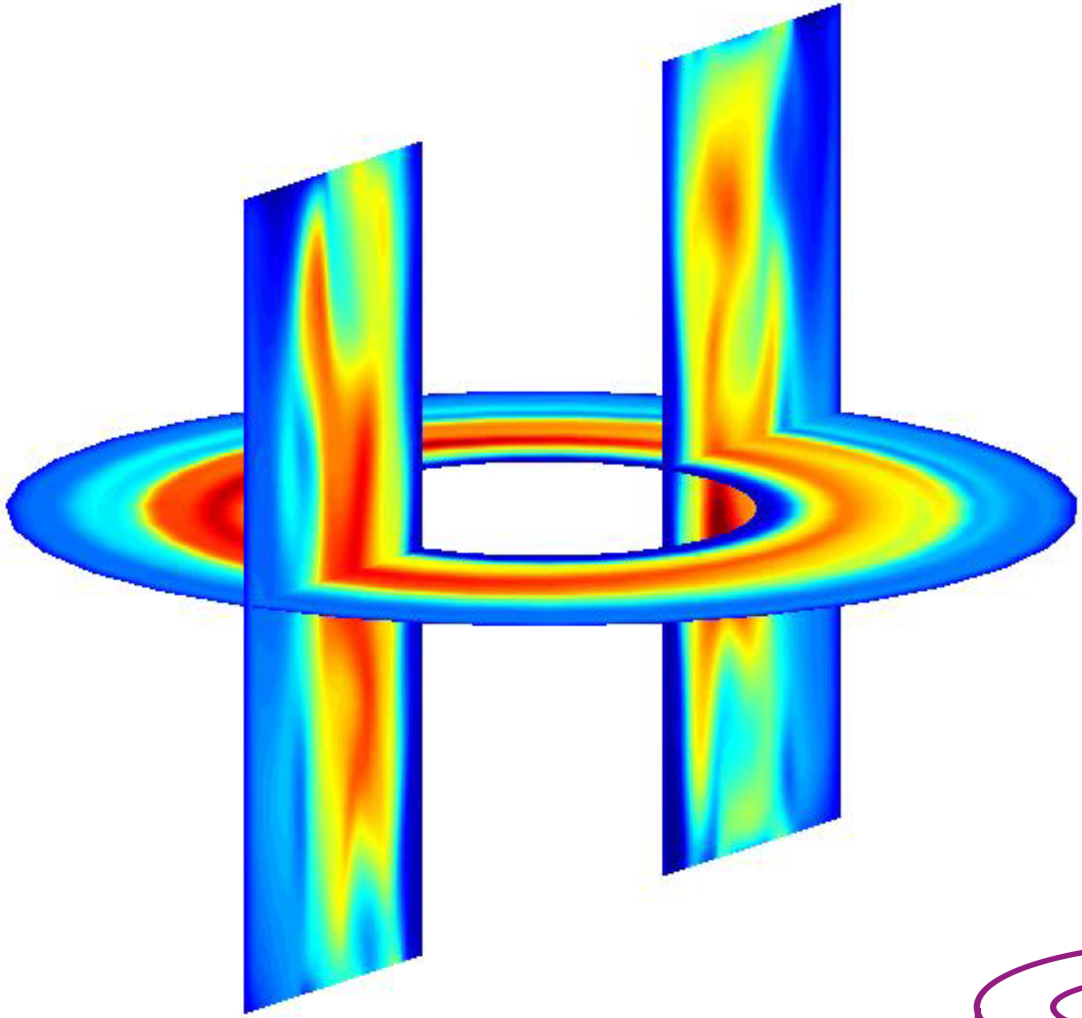
TORPEX shows $k_{\|} = 0$ turbulence at low N

$$k_{\|} = 0 \quad (\lambda_v = L_v / N)$$

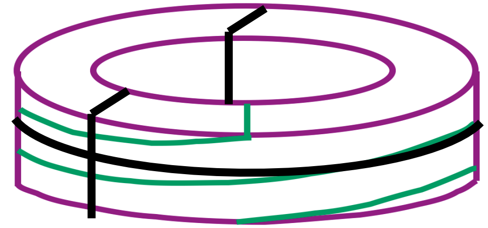
Ideal interchange regime



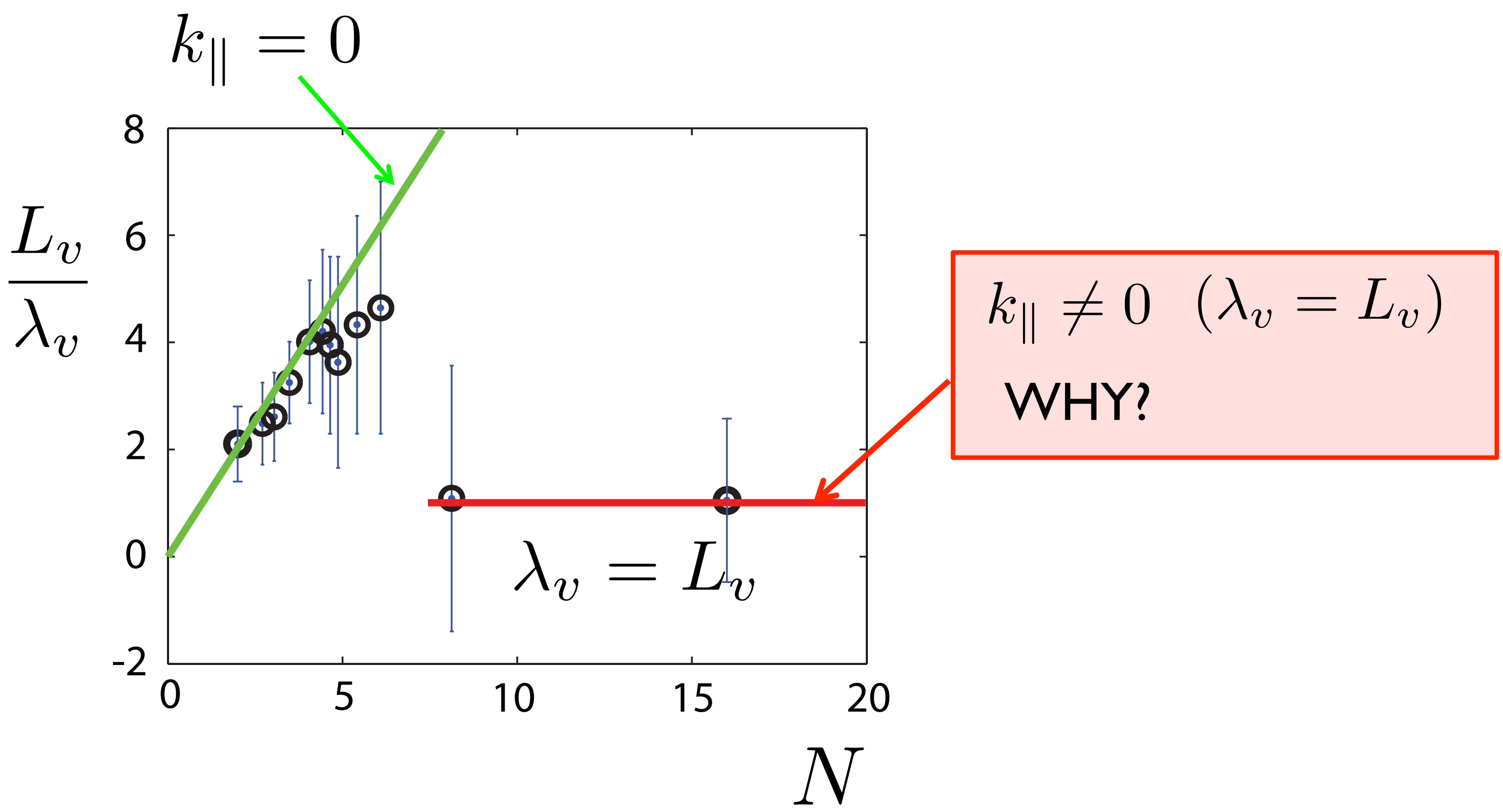
For N~1-6, ideal $k_{\|} = 0$ interchange modes dominant



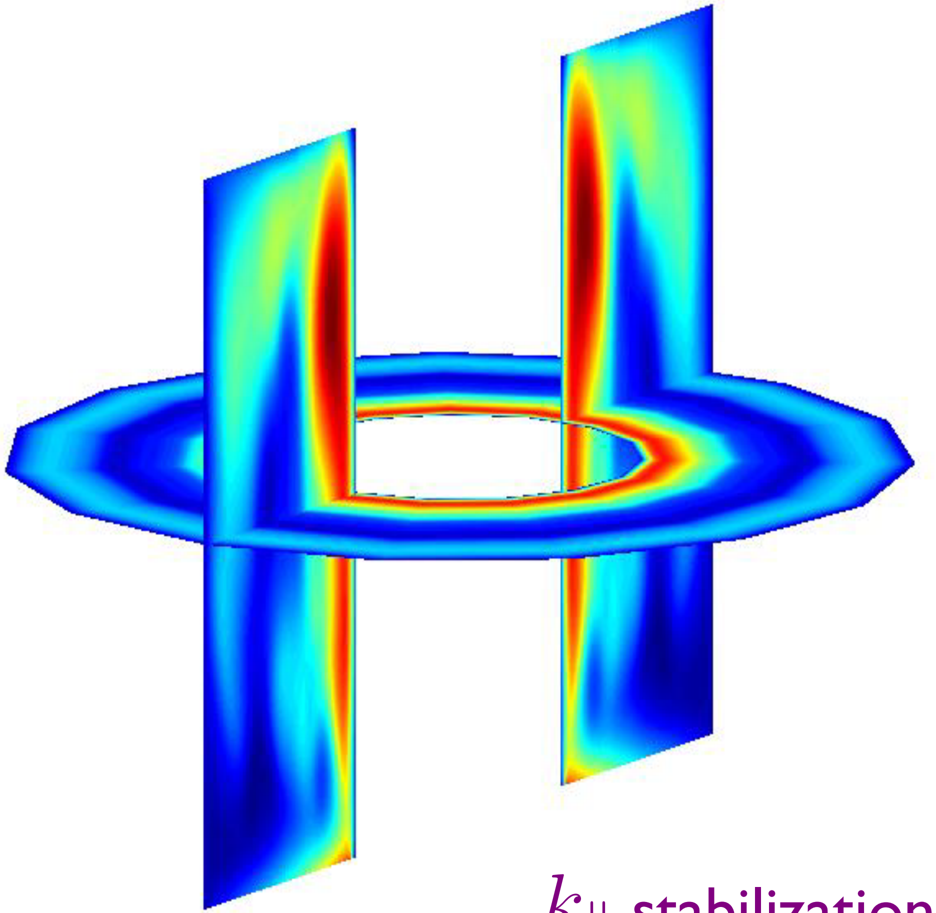
N=2



Turbulence changes character at N>7



At high N>7, Resistive Interchange Mode turbulence



Toroidally symmetric

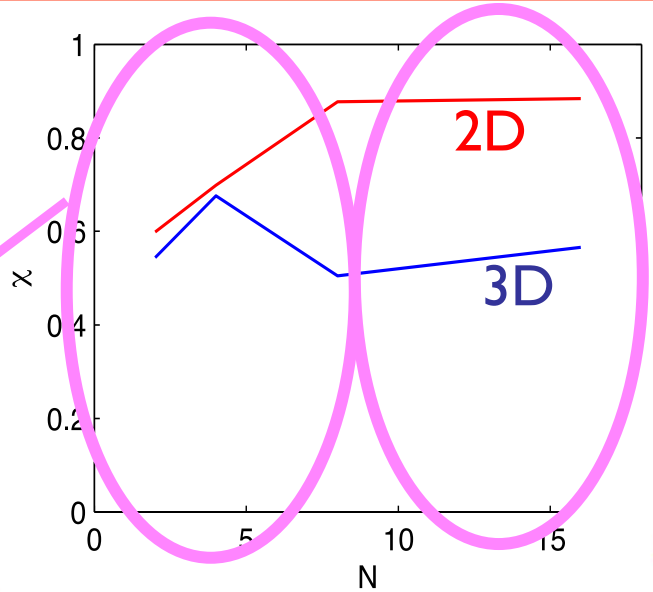
$\lambda_v \sim L_v$

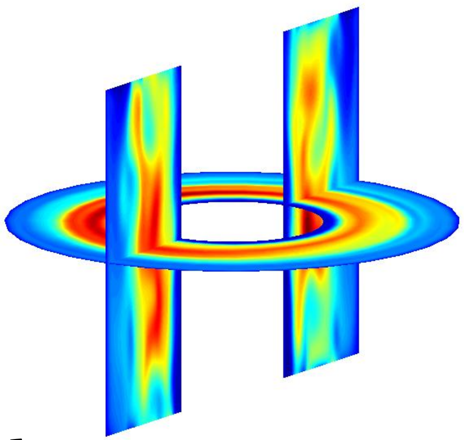
$k_{\|}$ stabilization, requires high N and $\eta_{\|} \neq 0$

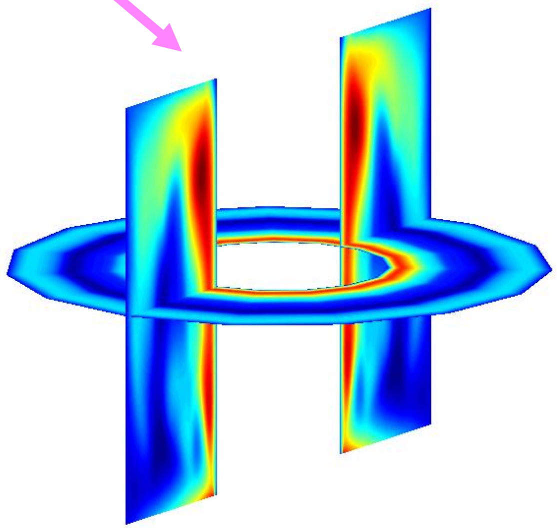
Introducing $k_{\|} \neq 0$ modes

$$\gamma^2 = \gamma_I^2 - \gamma \frac{4\pi V_A^2 k_{\|}^2}{\eta_{\|} c^2 k_y^2}, \quad \gamma_I = c_s \sqrt{\frac{2}{R L_p}}$$

Interpretation of the validation results







$$k_{\|} = 0$$

- Ideal interchange turbulence
- 2D model appropriate

$$k_{\|} \neq 0$$

- Compressibility stabilizes ideal interchange
- Resistive interchange turbulence
- 2D model not appropriate

Ricci & Rogers, PRL 2010

Where can a Verification & Validation exercise help?

1. Make sure that the code works correctly, and asses the numerical error

   The correct implementation of GBS rigorously shown, the discretization error estimate for the quantity of interest estimated

2. Compare codes

   2D and 3D simulations agree with experimental measurements similarly at low N.

   Global 3D simulations are needed to describe the plasma dynamics at high N.

3. Let the physics emerge

   Two turbulent regimes: ideal interchange mode at low N and non-flute modes at high N.

   Parameter scans have a crucial role