# Forecasting Uncertainty in Electricity Demand


Tri Kurniawan Wijaya tri-kurniawan.wijaya@epfl.ch

Mathieu Sinn mathsinn@ie.ibm.com

Bei Chen beichen2@ie.ibm.com

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Our general model for electricity demand:

$$Y_t = \mu(x_t) + \sigma(x_t)\epsilon_t$$

$y_t$ = the demand at time $t$
$x_t$ = vector of covariates
$\epsilon_t$ = the error at time $t$ (zero mean, unit variance)

$$\mathbb{E}[Y_t] = \mu(x_t)$$
$$\mathrm{Var}(Y_t) = \sigma^2(x_t)$$

In practice, $\mu(\cdot)$ (conditional mean) and $\sigma^2(\cdot)$ (conditional variance) are unknown!

Estimate $\mu(\cdot)$ and $\sigma^2(\cdot)$ from empirical data
No bootstrapping, no ensemble

## 1 Estimating the conditional mean $\mu(\cdot)$

- Use Generalized Additive Model (GAM):

$$F(\mu(x_t)) = \sum_{i=1}^{I} f_i(x_t)$$

$F$: link function, e.g., identity or logarithm

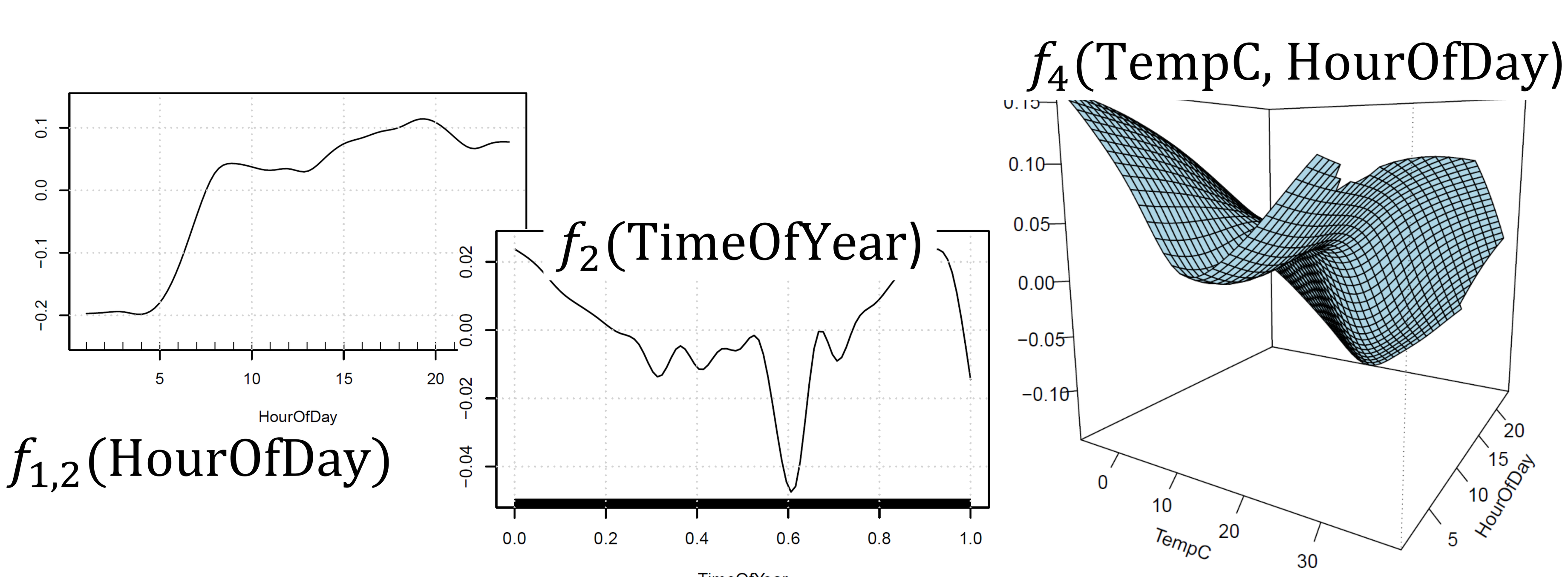
$f_i$: transfer function

$$f_i(x_t) = \mathbf{1}_{(x_t \in A_i)}\, \beta_i^T\, b_i(x_t)$$

$b_i$: basis functions

- Fit a GAM to $y_t$ to get an estimate $\hat{\mu}(\cdot)$ of $\mu(\cdot)$

$$\log(\mathbb{E}[Y_t]) = \beta_0 + \beta_1 \texttt{tmrwDayType}_t + \sum_{j=1}^{8} \mathbf{1}_{(\texttt{DayType}_t = j)} f_{1,j}(\texttt{HourOfDay}_t) + f_2(\texttt{TimeOfYear}_t) + f_3(\texttt{TempC}_t, \texttt{Humidity}_t) + f_4(\texttt{TempC}_t, \texttt{HourOfDay}_t) + f_5(\texttt{LagNSameDTLoad}_t) + \sum_{j=1}^{8} \mathbf{1}_{(\texttt{DayType}_t = j)} f_{6,j}(\texttt{LagLoad}_t)$$




- Forecasting the conditional mean $\hat{y}_t = \hat{\mu}(x_t)$

## 2 Estimating the conditional variance $\sigma^2(\cdot)$

- We use another GAM:

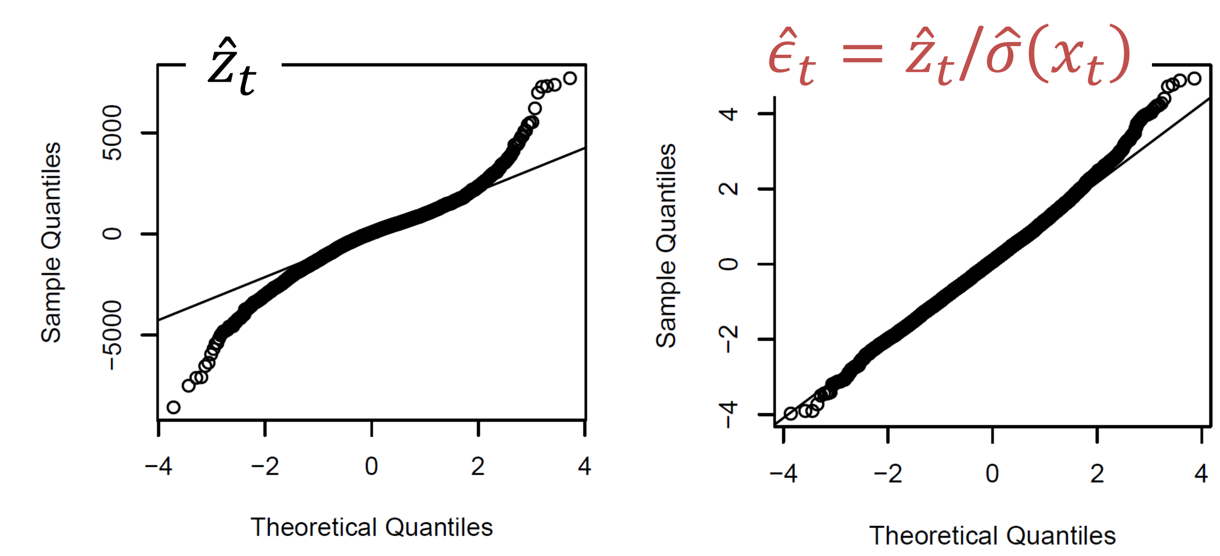
$$G(\sigma^2(x_t)) = \sum_{j=1}^{J} g_i(x_t)$$

- Let $z_t = \sigma(x_t)\epsilon_t$
$\hat{z}_t = \hat{y}_t - y_t$

- Fit a GAM to $\hat{z}_t^2$ to get an estimate $\hat{\sigma}^2(\cdot)$ of $\sigma^2(\cdot)$

$$\log(\mathbb{E}[\hat{z}_t^2]) = \beta_0 + \beta_1 \texttt{DayType}_t + \sum_{j=1}^{5} \mathbf{1}_{(\texttt{DayType}_t = j)} f_{1,j}(\texttt{HourOfDay}_t) + f_2(\texttt{TimeOfYear}_t) + f_3(\texttt{TempC}_t, \texttt{Humidity}_t) + f_4(\texttt{TempC}_t, \texttt{HourOfDay}_t)$$

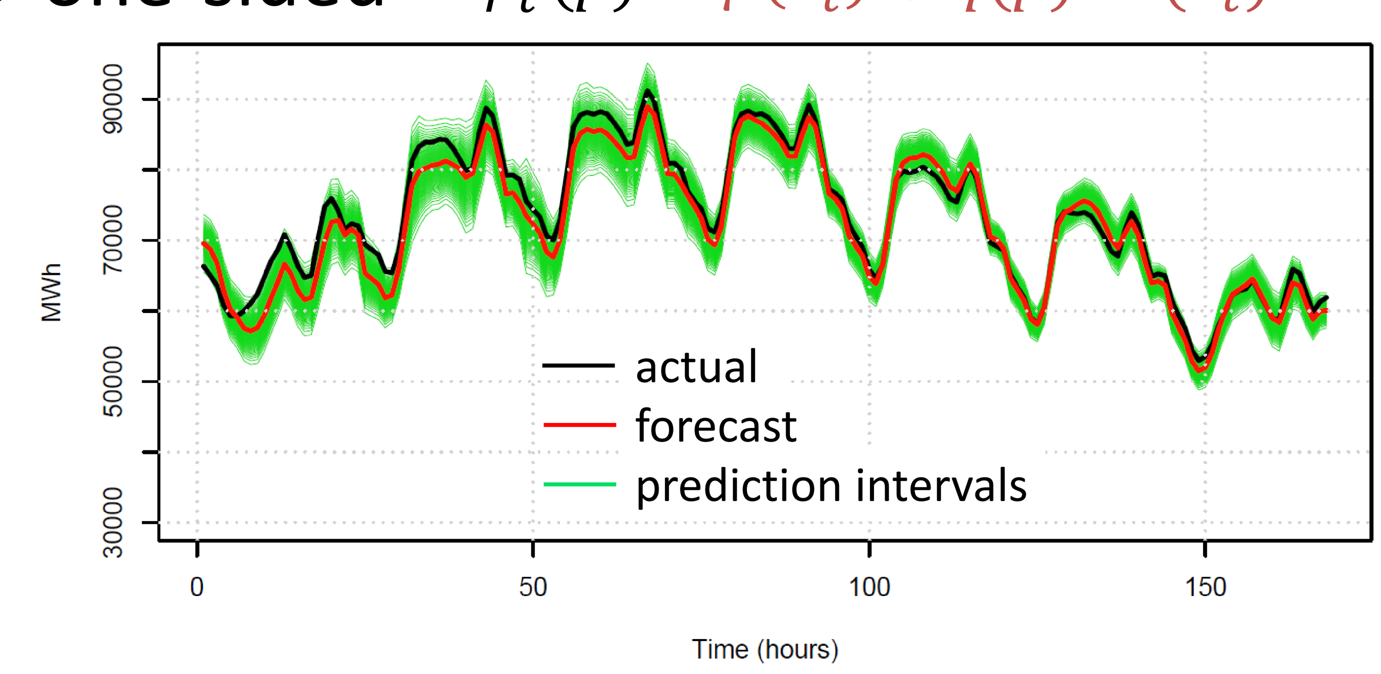
Normal Q-Q Plot




## 3 Computing prediction intervals

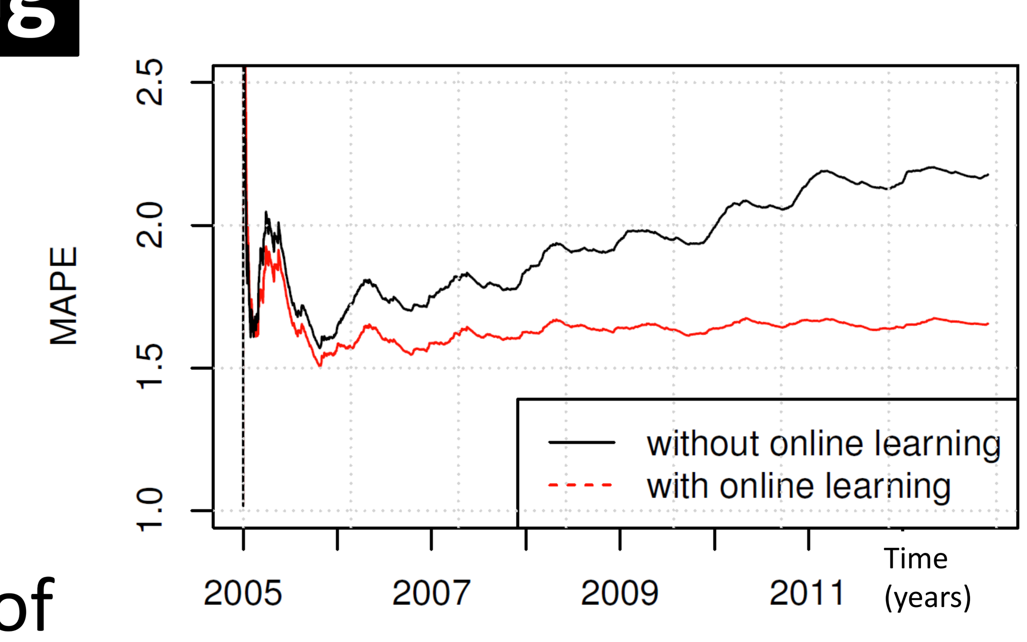
$q(\cdot)$ = the quantile function of the standard normal dist.

- two-sided $\phi_t^2(p) = \hat{\mu}(x_t) \pm \left| q\left(\frac{1-p}{2}\right) \right| \cdot \hat{\sigma}(x_t)$
- one-sided $\phi_t^1(p) = \hat{\mu}(x_t) + q(p) \cdot \hat{\sigma}(x_t)$




## 4 Online learning

- Adaptive learning for smoothing functions (Ba et al., NIPS, 2012)




- Adaptive construction of prediction intervals

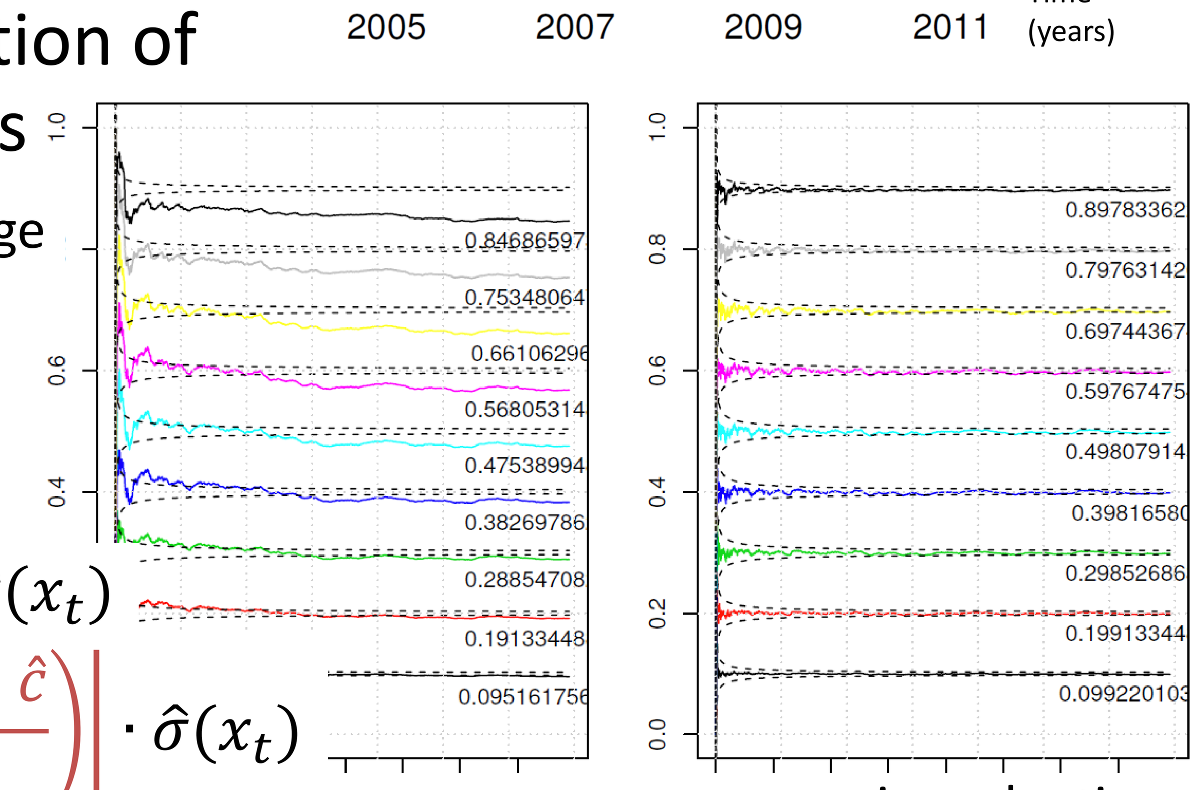
$c$ = current empirical coverage

$$\alpha c + (1-\alpha)\hat{c} = p$$

$$\hat{c} = \frac{p - \alpha c}{1 - \alpha}$$

$$\phi_t^1(p) = \hat{\mu}(x_t) + q(\hat{c}) \cdot \hat{\sigma}(x_t)$$

$$\phi_t^2(p) = \hat{\mu}(x_t) \pm \left| q\left(\frac{1-\hat{c}}{2}\right) \right| \cdot \hat{\sigma}(x_t)$$




The dataset is available at `https://github.com/tritritri/uncertainty`