Route choice models: bringing behavioral aspects into shortest path

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Outline

Introduction

The problem

- Given an origin o and a destination d,
- given a mode of transportation
- how does a traveler select a path to travel from o to d?

Introduction

Motivation

- Each route choice contributes to congestion
- We want to predict it
- We want to mitigate it

Network representation

Nodes

- Intersections, bus stops, airports, train stations, parkings
- Centroids: subset of nodes, potential origins and destinations

Links

- Connecting two nodes
- Oriented edges
- Associated with attributes: length, travel time, cost, level of comfort, capacity, etc.

Modes of transportation

- Single mode
- or multi-modal

Shortest paths

Definition

- Given a network
- given a cost associated with each link
- given an origin o and a destination d,
- what is the path with the minimum total cost from o to d.

Assumptions

- Path attributes are link-additive
- Link attributes are summarized into a generalized cost
- Link cost can be negative, but no cycle with negative cost

Shortest path

Algorithms

- Bellman (1957)–Ford (1956)
- Dijkstra (1959)
- A* (Hart et al. (1968))
- Hub labeling (Abraham et al. (2011), specialized for road networks)
- and many variants

Main properties

- No enumeration of path
- Efficient implementations

Assignment

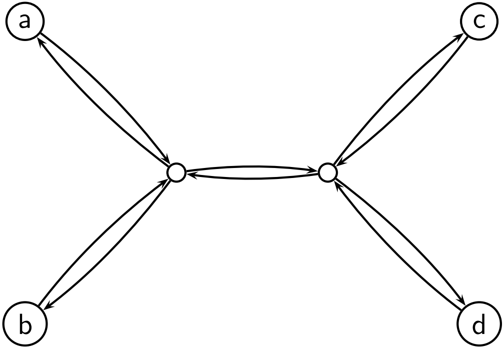
Motivation

- We are interested in congestion.
- Suppose that we know how travelers select their route.
- How do we measure the impact on the traffic through the network?

Problem definition

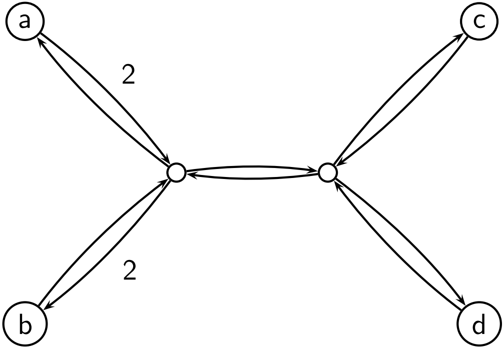
- For each origin o and destination d, we know the number of travelers performing the trip during the period of interest: q_{od}.
- We know the route choice model.
- What is the flow on each link of the network?

Assignment: example



	a	b	c	d
a	0	2	3	7
b	4	0	2	3
c	4	4	0	1
d	1	1	2	0

Assignment: example



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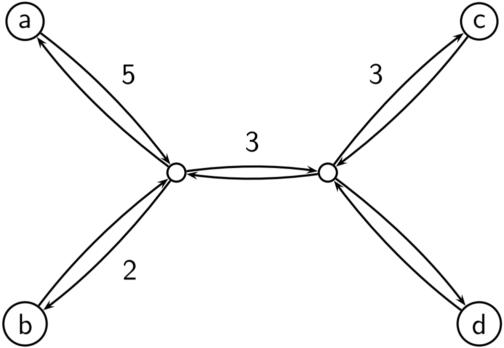

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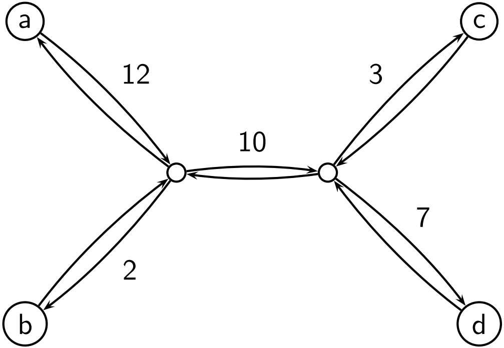

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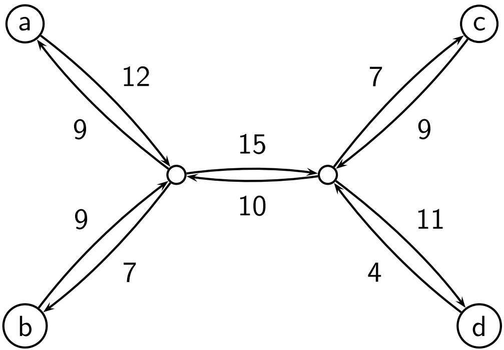
Assignment: example



	a	b	c	d
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Model

Assignment matrix

- Vector of OD flows: $q \in \mathbb{R}^{m \times 1}$
- Vector of link flows: $x \in \mathbb{R}^{n \times 1}$
- Total number of paths: p (potentially extremely large)
- Path-link incidence matrix: $P \in \mathbb{R}^{n \times p}$

$$P_{\ell k} = 1 \text{ if link } \ell \text{ belongs to path } k\text{, 0 otherwise}$$

- Route choice matrix: $R \in \mathbb{R}^{p \times m}$

$$R_{kj} \text{ proportion of OD flow } j \text{ using path } k$$

- Assignment map:

$$x = PRq$$

- Assignment matrix: $A = PR \in \mathbb{R}^{n \times m}$

All or nothing assignment

Assumptions

- Travel time is given for each link
- Every traveler takes the shortest path from o to d

Consequences

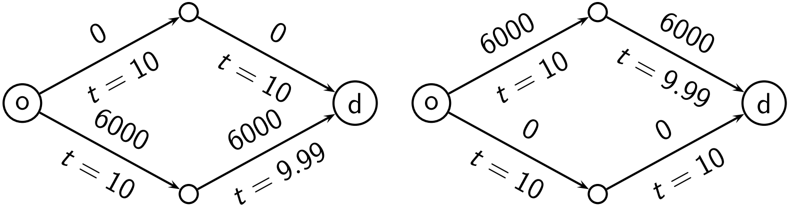
- Each column of R contains exactly one 1 and all zeros
- Assignment matrix can be built directly without enumerating the paths

All or nothing assignment

Limitation: non robust

Minor variations of the data may generate significantly different output

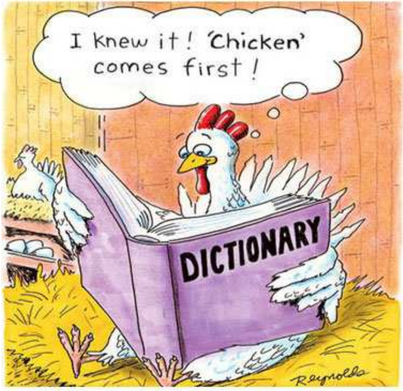
Assignment of 6000 units of flow



All or nothing assignment

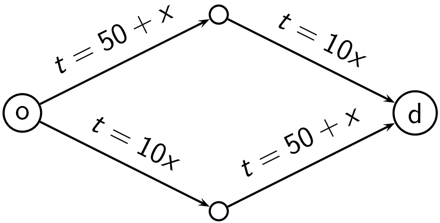
Limitation: ignores congestion

- Travel time increases with flow
- Flow depends on route choice
- Route choice depends on travel time



Accounting for congestion

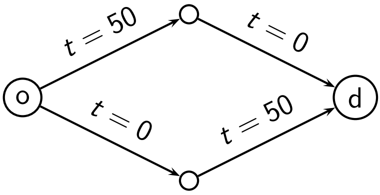
Example



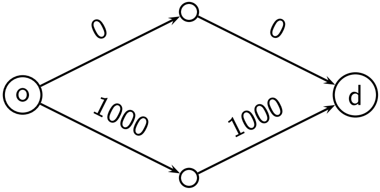
x: 1000 units of flow

Accounting for congestion

Empty network

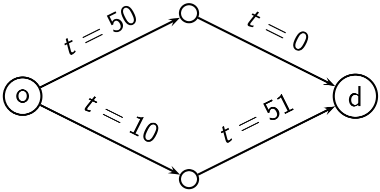


Load 1000 units of flows

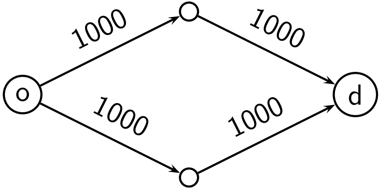


Accounting for congestion

Network with 1000 units

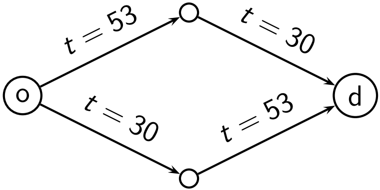


Load another 1000 units of flows

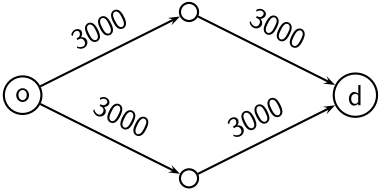


Accounting for congestion

Network with 6000 units



Result: Nash equilibrium



Nash equilibrium

Definition

The network is in *Nash equilibrium* or *user equilibrium* if no traveler can improve her travel time by unilaterally changing routes.

Property

For each OD pair, the travel time on all used paths are equal, and lower or equal to the travel time on any unused path

1994

Nash equilibrium

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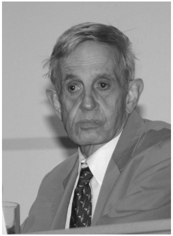
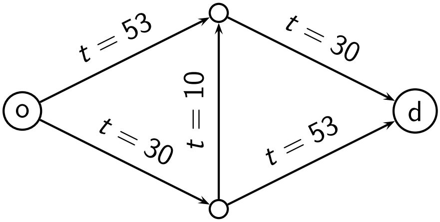


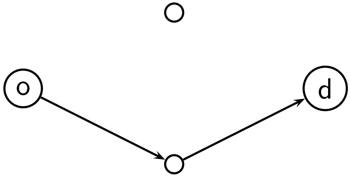
1994

Back to the example

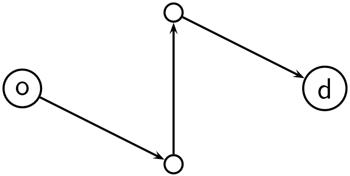
Construct a new link: $t = 10 + x$



Before: 83 min.

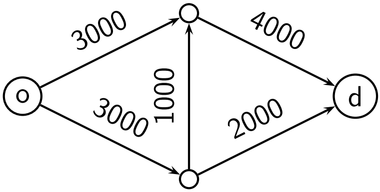


After: 70 min.

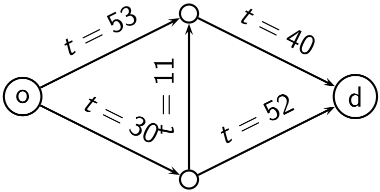


Back to the example

Flows

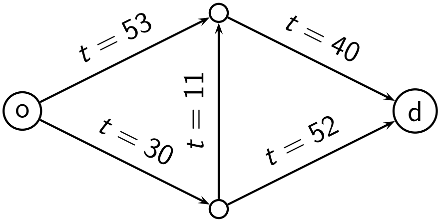


Travel times

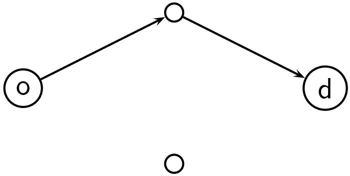


Back to the example

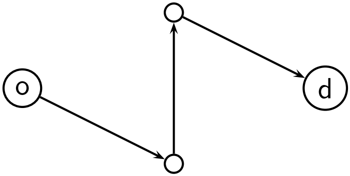
Travel times



Before: 93 min.

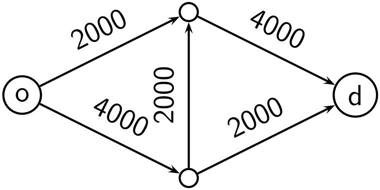


After: 81 min.

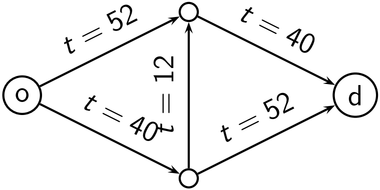


Back to the example

Flows

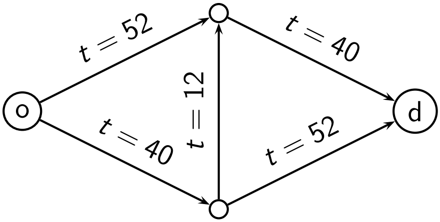


Travel times: Nash equilibrium

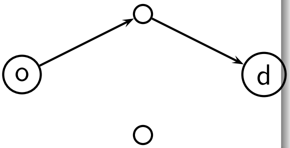


Back to the example

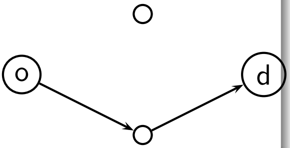
Travel times: Nash equilibrium



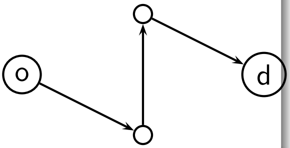
Path 1: $t = 92$



Path 2: $t = 92$

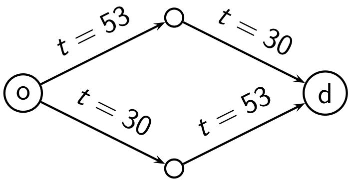


Path 3: $t = 92$

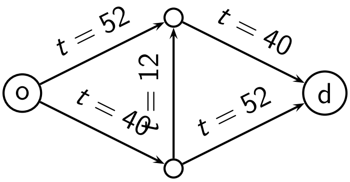


Braess paradox

Before: $t = 83$



After: $t = 92$



- Increasing the capacity of the network deteriorates its overall performance
- If travelers coordinate (coalition), they can be better off
- If not, they pay the "price of anarchy"
- Braess (1968)

Solution algorithm

Beckmann transformation (Beckmann et al. (1956))

- Equivalent nonlinear optimization problem
- Traffic equilibrium conditions = optimality conditions of the optimization problem

Frank-Wolfe algorithm (Frank and Wolfe (1956))

- Shortest paths
- Convex combinations

Outline

Behavioral models

Traffic equilibrium

- Inherit the non robustness of all or nothing assignment
- Everybody has exactly the same behavior

Behavioral models

Choice models

- Account for the heterogeneity of behavior
- Theoretical foundations: utility theory
- Operational models used in transportation, marketing, etc.

Choice models

Theoretical foundations

- Random utility theory
- Choice set: $\mathcal{C}_n$
- Logit model:

$$P(i|\mathcal{C}_n) = \frac{e^{V_{in}}}{\sum_{j \in \mathcal{C}_n} e^{V_{jn}}}$$

2000

Route choice models

Advantages

- Link-additivity not necessary
- Traveler specific attributes
- Utility can be estimated from real data

Drawbacks

- Enumeration of paths
- Structural correlation among alternatives

Sampling of alternatives: McFadden (1978)

Sampling

Consider $D \subset \mathcal{C}_n$ sampled

$$\begin{aligned}
\text{Prob}(D, i) &= \text{Prob}(D|i)P(i|\mathcal{C}_n) \\
&= P_n(i|D)\,\text{Prob}(D) \\
&= P_n(i|D) \sum_{k \in D} \text{Prob}(D|k)P(k|\mathcal{C}_n)
\end{aligned}$$

Therefore,

$$P_n(i|D) = \frac{\text{Prob}(D|i)P(i|\mathcal{C}_n)}{\sum_{j \in D} \text{Prob}(D|j)P(j|\mathcal{C}_n)}$$

Sampling of alternatives: McFadden (1978)

Model based on sample of alternatives

$$P_n(i|D) = \frac{\text{Prob}(D|i)P(i|\mathcal{C}_n)}{\sum_{k\in D}\text{Prob}(D|k)P(k|\mathcal{C}_n)}$$

Logit model

$$P(i|\mathcal{C}_n) = \frac{e^{V_{in}}}{\sum_{j\in\mathcal{C}_n} e^{V_{jn}}}$$

Sampling with logit

$$P_n(i|D) = \frac{\text{Prob}(D|i)e^{V_{in}}}{\sum_{k\in D}\text{Prob}(D|k)e^{V_{kn}}} \frac{\sum_{j\in\mathcal{C}_n} e^{V_{jn}}}{\sum_{j\in\mathcal{C}_n} e^{V_{jn}}}$$

Sampling of alternatives: McFadden (1978)

Sampling with logit

$$
\begin{aligned}
P_n(i|D) &= \frac{\text{Prob}(D|i)e^{V_{in}}}{\sum_{k\in D}\text{Prob}(D|k)e^{V_{kn}}} \frac{\sum_{j\in\mathcal{C}_n} e^{V_{jn}}}{\sum_{j\in\mathcal{C}_n} e^{V_{jn}}} \\
&= \frac{\text{Prob}(D|i)e^{V_{in}}}{\sum_{k\in D}\text{Prob}(D|k)e^{V_{kn}}} \\
&= \frac{e^{V_{in}+\ln\text{Prob}(D|i)}}{\sum_{k\in D} e^{V_{kn}+\ln\text{Prob}(D|k)}}
\end{aligned}
$$

Sampling of alternatives: McFadden (1978)

Comments

- Choice probability can be approximated using a sample of alternatives
- Terms involving $\mathcal{C}_n$ cancel out with logit
- Condition: $\text{Prob}(D|k) \neq 0$, for each $k \in D$
- Generalized to more complex models by Bierlaire et al. (2008)

Sampling of paths: challenges

Importance sampling: prefer shorter paths

Path p is sampled with probability

$$\pi_p = \frac{e^{-\lambda L_p}}{\sum_{q \in \mathcal{C}_n} e^{-\lambda L_q}}$$

Calculate correction Prob($D|k$)

Frejinger et al. (2009)

Draw D from $\mathcal{C}_n$

Flötteröd and Bierlaire (2013)

Metropolis-Hastings

Principles

- Let $b_j = \exp(-\lambda L_j)$, $j \in \mathcal{C}_n$
- Let $B = \sum_{j\in\mathcal{C}_n} b_j$. B cannot be computed.
- We want to simulate a r.v. with pmf $\pi_j = b_j/B$.
- Consider a Markov process on $\mathcal{C}_n$ with transition probability Q.
- Define another Markov process with the same states in the following way:
  - Assume the process is in state i, that is $X_t = i$,
  - Simulate the (candidate) next state j according to Q.
  - Define

$$X_{t+1} = \begin{cases} j & \text{with probability } \alpha_{ij} \\ i & \text{with probability } 1-\alpha_{ij} \end{cases}$$

Metropolis-Hastings

Accept-reject probability

Derived from the theory of Markov processes:

$$\alpha_{ij} = \min\left(\frac{b_j B Q_{ji}}{b_i B Q_{ij}}, 1\right) = \min\left(\frac{b_j Q_{ji}}{b_i Q_{ij}}, 1\right)$$

Does not involve B.

In practice: define a Markov process Q

- Q is generating a sequence of paths
- Too little variability: slow convergence
- Too much variability: random search
- Transition probabilities Q_{ij} and Q_{ji} must be calculated.

Markov process Q

State $i = (\Gamma, a, b, c)$

- a path Γ
- three node indices $a < b < c$ within that path
- Node indices are important to compute Q_{ij} and Q_{ji}

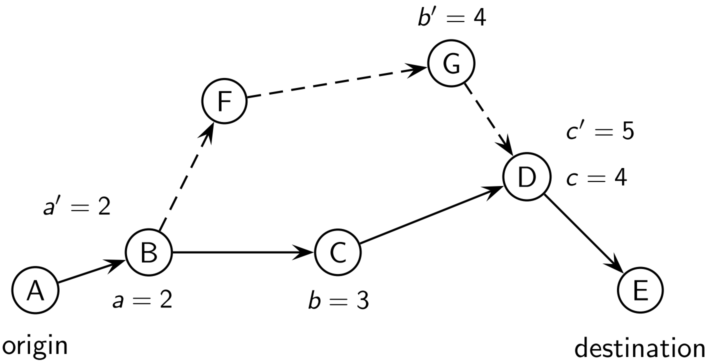
First type of transition: shuffle

Re-sample (uniformly) $a < b < c$ within path Γ

Second type of transition: splice

- sample a node v "near" the path segment $\Gamma(a, c)$
- connect $\Gamma(a)$ to v
- connect v to $\Gamma(c)$
- let new b point at v, update c

Markov process Q



Case study: Tel Aviv

Large network

- 17118 links
- 7879 nodes
- Movie...

Summary

Route choice behavior

- Shortest paths: efficient algorithm, limited realism
- Accounting for congestion: traffic equilibrium
- Accounting for behavioral heterogeneity: random utility models

Sampling of path

- Allows to approximate choice probability
- Importance sampling
- Metropolis-Hastings algorithms

Future work

Making the models more complex

Lai and Bierlaire (2014)

Making the models simpler

Kazagli and Bierlaire (2014)

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