Divergence-free phase contrast MRI

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***Abstract*—In this work, we extend a separate magnitude and phase regularization framework for Phase Contrast MRI by incorporating the divergence-free condition.**

Introduction 3D phase-contrast (PC) MRI is a powerful tool to assess hemodynamic parameters. However, this method is hampered by long acquisition times and residual phase errors due to system imperfections. The latter can be adressed by incorporating physical priors, such as the approximate incompressibility of blood [1]. Using compressed sensing (CS) for scan acceleration, regularizers are often designed for magnitude reconstruction [2], and therefore, may not be robust for phase encoding. In [3] it was demonstrated that improved phase accuracy can be achieved by separate magnitude and phase regularization. In this work, we extend this framework for PC MRI by incorporating the divergence-free and smoothness condition of the velocity flow field.

Theory In a 4-point PC experiment, the velocities at position $\boldsymbol{r}$ along three orthogonal ($i = 1, 2, 3$) directions are given by $v_i(\boldsymbol{r}) = (\phi_i(\boldsymbol{r}) - \phi_0(\boldsymbol{r}))/k_{v,i}$. ϕ_i and ϕ_0 denotes the phase of the velocity encoded $\rho_i(\boldsymbol{r})$, and reference image $\rho_0(\boldsymbol{r})$, respectively. $k_{v,i}$ is the first moment of the applied bipolar gradient along i. Using incoherent undersampling and collecting all four images into $\boldsymbol{\rho} \in \mathbb{C}^{4n}$ and the acquired k-space samples into $\boldsymbol{d} \in \mathbb{C}^{4m}$, the separate magnitude-phase reconstruction problem is initialized with the solution minimizing the following convex baseline cost function:

$$\Psi_1(\boldsymbol{\rho}) = \frac{1}{2}\|\boldsymbol{d} - (\boldsymbol{I}_4 \otimes \boldsymbol{E})\boldsymbol{\rho}\|_2^2 + \lambda_1\|(\boldsymbol{I}_4 \otimes \boldsymbol{B})\boldsymbol{\rho}\|_1 + \lambda_2\|(\boldsymbol{H} \otimes \boldsymbol{I}_n)\boldsymbol{\rho}\|_1, \quad (1)$$

with encoding matrix $\boldsymbol{E}$ relating the reconstructed images to the acquired k-space trajectory, $\boldsymbol{B}$ an operator implementing several sparsifying transforms and $\boldsymbol{H}$ the last 3 rows of the Hadamard matrix, producing sparse complex difference images with signal concentrated in the vessels. The image is then decomposed into its magnitude $m_j \in \mathbb{R}$, and phase component $\theta_j = e^{i\varphi_j} \in \mathbb{C}$, s.t. $\boldsymbol{\rho} = \boldsymbol{m} \circ \boldsymbol{\theta}$. Both components are reconstructed by minimizing the cost function,

$$\Psi_2(\boldsymbol{m}, \boldsymbol{\theta}) = \frac{1}{2}\|\boldsymbol{d} - (\boldsymbol{I}_4 \otimes \boldsymbol{E})(\boldsymbol{m} \circ \boldsymbol{\theta})\|_2^2 + \lambda_1\|(\boldsymbol{I}_4 \otimes \boldsymbol{B})\boldsymbol{m}\|_1 + \lambda_2 \sum_{i=1}^{3} \|\boldsymbol{m}_i - \boldsymbol{m}_0\|_1 + \mathcal{R}(\boldsymbol{\varphi}), \quad (2)$$

with separate phase regularization

$$\mathcal{R}(\boldsymbol{\varphi}) = \lambda_3 \Big\| \boldsymbol{\omega} \circ \Big(\sum_{i=1}^{3} \nabla_i (\boldsymbol{\varphi}_i - \boldsymbol{\varphi}_0) \Big) \Big\|_2^2 + \lambda_4 \sum_{i=1}^{3} \|\boldsymbol{C}(\boldsymbol{\varphi}_i - \boldsymbol{\varphi}_0)\|_2^2,$$

where $\boldsymbol{\omega} \in \{0, 1\}^n$ is a masking vector, ∇_i the gradient operator along i and $\boldsymbol{C}$ the 3D finite difference matrix penalizing divergence and enforcing smoothness of the vector field, respectively.

Following [4], Eq. 1 and Eq. 2 are reformulated as constrained optimization problems using variable splitting, where parts of the objective are decoupled by introducing equality constraints. These constraints are incorporated by adding augmented Lagrangian terms with additional split variables and Lagrange multipliers and finally an unconstrained optimization problem is solved. The optimization consists of iteratively updating split variables, unknown variables and Lagrange multipliers until some convergence criterion is met.

Methods Simulated PC MRI data with phase wraps was generated using a computational fluid dynamic vector field in a U-bend. Gaussian noise was added (SNR = 30) and the resulting reference images were projected on 8-fold undersampled radial trajectories in 3D k-space. Reconstruction was then performed on the reduced data by subsequently minimizing Eq. 1 and Eq. 2.

Results Figure 1 shows the noisy reference velocity fields in y and x direction respectively. Next to them, estimated velocity and divergence profiles are shown for the noiseless reference, the baseline and the proposed method. From the results we can see that the proposed method is able to deal with the phase wrapping that can occur in the reference data. The divergence of the vector field in both directions is also descreased.

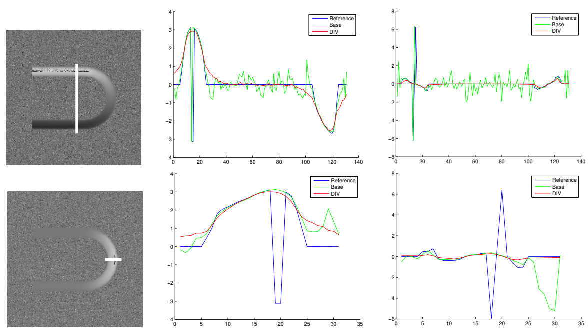

Fig. 1. Top: Noisy ref. velocity in y direction, Velocity profiles, Divergence profiles. Bottom: Noisy ref. velocity in x direction, Velocity profiles, Divergence profiles.

Discussion In this work, an extension of the separate magnitude and phase regularization on PC MRI has been developed and evaluated. Results show that the proposed method overcomes phase wrapping problems that can occur in the reference data and simultaneously miminizes the divergence of the resulting velocity vector field.

References

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