S-N-P FATIGUE CURVES USING MAXIMUM LIKELIHOOD

Method for fatigue resistance curves with application to straight and welded rebars

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INTRODUCTION

When checking the fatigue life of steel bridges with concrete deck slabs, both steel details and embedded reinforcing steel bars (rebars) must be considered. In this paper a method for estimation of fatigue resistance curves is presented with application to straight and welded rebars. Rebar fatigue resistance is traditionally presented in the form of *S-N-P* curves which relate the applied stress range, *S*, to the *p*-quantile of fatigue life, *N*. These *S-N-P* curves are obtained from rebar fatigue tests at constant stress amplitudes. Test results for hot rolled (HR), cold worked (CW) and quenched and self-tempered (QST) rebars can be found in [1-7]. Both HR and CW rebars show a ductile cross section consisting of pearlite-ferrite microstructure and low or medium Carbon content [8-9]; however, HR and CW rebars have been mainly replaced by QST rebars since the 1970's. QST rebars are produced from a specific thermal treatment called Thermex or Tempcore [10-11] which results in a different microstructure at the surface and in the core. Typically the surface is a hard martensite, whereas the core consists of pearlite-ferrite. Fatigue datasets for different rebar connections such as lapping, coupling or welding as well as for corroded rebars can be found from tests performed in the 1970's.

In the EN standards, characteristic *S-N* curves are created by fitting a linear regression to the experimental failure data points and translating the linear regression mean curve to the *p*-lower hyperbolic prediction bound (typically $p=0.05$), at 1 million cycles [12]. This approach has several limitations: 1) run-out test results are neglected; 2) a constant amplitude fatigue limit (CAFL) (stress below which tested bars experience no fatigue damage) is arbitrarily chosen to begin at 1 million cycles for straight rebars and at 10 million cycles for welded rebars [13]; and 3) *S-N* curves are based on fatigue data scatter in the finite-life region (*N* less than 1 million cycles) resulting in less accuracy in the high cycle fatigue (HCF) region (*N* over 1 million cycles). The statistical method recommended by the EN standards is currently used in the standards for concrete structures [14]. Analysing the fatigue data using more statistically robust approaches may overcome some of these issues.

In this paper a Maximum Likelihood (ML) method-based approach is used together with Monte-Carlo Simulations (MCS) to estimate *S-N-P* curves of straight and welded rebars. Run-out test results are considered and particular attention is given to the position of the CAFL. The influence of rebar size and rebar type is studied. Comparisons between ML-based *S-N-P* curves and EN-based characteristic S-N curves are made. The approach proposed in this study is presented with application to straight and welded rebars but it has generic applicability for estimating fatigue *S-N-P* curves of fatigue sensitive details of steel bridges like welded and bolted connections.

1 STATISTICAL EVALUATION OF S-N-P CURVES

This section presents the statistical method for estimation of characteristic *S-N* curves recommended by the EN standards [12], and presents the formulation for the ML-based approach.

1.1 Statistical evaluation of S-N-P curves based on EN background documentation

A linear statistical model is used to define the relationship between the logarithm of the number of cycles to failure, $Y=ln(N)$, and the logarithm of the nominal stress range, $X=ln(S)$:

$$Y = \beta_0 + \beta_1 \cdot X + \varepsilon(0, \sigma) \quad (1)$$

In *Eq. 1*, β_0 and β_1 are respectively the intercept and the slope of the *S-N* curve in the log-log plane. It is assumed that the model error ε can be modeled with a normal random variable, with an expected value equal to zero and standard deviation equal to σ. The model $E(Y)=\beta_0+\beta 1 \cdot X$, which represents the mean value of *ln(N)* for an assigned stress range, is fitted to the experimental dataset $(y_1, x_1)\ldots(y_n, x_n)$ using the least square method (LSM). Only failure points are considered.
According to the LSM, the unbiased, normally distributed estimators of model parameters are:

$$\hat{\beta}_1 = \sum_{i=1}^{n} \frac{(x_i - \bar{x}) \cdot (y_i - \bar{y})}{(x_i - \bar{x})^2} = \frac{S_{xy}}{S_{xx}} \quad (2)$$

$$\hat{\beta}_0 = \bar{y} - \beta_1 . \bar{x} \quad (3)$$

Since $\hat{\beta}_0$ and $\hat{\beta}_1$ are normally distributed in repeated sampling, it follows that $\hat{Y} = \hat{\beta}_0 + \hat{\beta}_1 \cdot X$ is also normally distributed. In order to obtain the characteristic S-N curve, a 100(α)% lower hyperbolic prediction bound can be determined around the mean regression line, using the following expression:

$$\hat{\beta}_0 + \hat{\beta}_1 \cdot x^* + t_{\alpha,n-2} \cdot StD \cdot \sqrt{1 + \frac{1}{n} + \frac{(x^* - \bar{x})^2}{S_{xx}}} \quad (4)$$

In *Eq. 4*, *StD* is the sample standard deviation, $t_{\alpha,n-2}$ is the a-quantile of the Student's T distribution with n-2 degrees of freedom and x* is the natural logarithm of the reference stress range. Characteristic S-N curves are determined by translating the mean regression line to the corresponding point of the 5% lower hyperbolic prediction bound, at 1 million cycles. The constant amplitude fatigue limit (CAFL) is arbitrarily chosen to begin at 1 million cycles for straight bars and at 10 million cycles for welded bars [13].

1.2 Statistical evaluation of S-N-P curves based on maximum likelihood approach

As previously mentioned, this EN standard approach is limited because: 1) run-out test results are neglected (loss of information) 2) the CAFL position is arbitrarily chosen; and 3) prediction bounds of linear regression curves are based on fatigue data scatter in the finite-life region resulting in less accuracy in the HCF region. To overcome these limitations, Pascual et al. [15] proposed a 5-parameter random fatigue limit (RFL) model that fit a nonlinear S-N curve having a random CAFL, to a complete fatigue dataset using ML estimation. In [15], characteristic S-N curves were determined by finding lower α-confidence bounds of *p*-quantile S-N curves (typically α=75% and *p*=0.05). This ML model proposed by Pascual is still affected by two limitations: 1) the choice of the α-confidence level for the lower bound of the *p*-quantiles is arbitrary; and 2) RFL-based S-N curves are nonlinear and are not easily comparable to the current standard linear *S-N* curves from the EN method.
This study proposes a bi-linear random fatigue limit (BLRFL) model that fits a bi-linear median *S-N* curve to a complete fatigue dataset, using again ML estimation. *S-N-P* curves are computed using Monte-Carlo Simulations (MCS), whereby the arbitrary choice of the α-confidence level for the lower bound of the *p*-quantiles is not required.
The dependence between fatigue life and stress range is modeled as follows:

$$Y = \frac{\beta_0 + \beta_1 \cdot X}{H(X - V)} + \varepsilon(0, \exp(\sigma_Y)) \quad (5)$$

Where *H(·)* is the unit step function and *V* is the natural logarithm of CAFL. *Y* and *V* are assumed to be normal distributed random variables:

$$Y = \mathrm{Normal}(\mu_Y, \exp(\sigma_Y)) \quad (6)$$

$$V = \mathrm{Normal}(\mu_V, \exp(\sigma_V)) \quad (7)$$

The location parameter of the *Y* distribution is:

$$\mu_Y = \frac{(\beta_0 + \beta_1 \cdot X)}{H(X - V)} \tag{8}$$

The conditional probability density function of $Y|V$ is:

$$f_{Y|V} = \frac{1}{\sigma_Y} \phi_{Y|V}(x, y, v; \beta_0, \beta_1, \sigma_Y) \tag{9}$$

The marginal probability density function of Y is:

$$f_Y = \int_{-\bowtie}^{x} \frac{1}{\sigma_Y . \sigma_V} \phi_{Y|V}(x, y, v; \beta_0, \beta_1, \sigma_Y) \cdot \phi_V(v; \mu_V, \sigma_V) \cdot dv = f_Y(x, y; \underline{\vartheta}) \tag{10}$$

Similarly the marginal cumulative distribution function of Y is:

$$F_Y = \int_{-\bowtie}^{x} \frac{1}{\sigma_V} \Phi_{Y|V}(x, y, v; \beta_0, \beta_1, \sigma_Y) \cdot \phi_V(v; \mu_V, \sigma_V) \cdot dv = F_Y(x, y; \underline{\vartheta}) \tag{11}$$

Where the model parameter vector is indicated as:

$$\underline{\vartheta} = (\beta_0, \beta_1, \sigma_Y, \mu_V, \sigma_V) \tag{12}$$

The sample likelihood is:

$$L(\underline{\vartheta}) = \prod_{i=1}^{N_{tot}} \left[f_Y(x_i, y_i; \underline{\vartheta}) \right]^{\delta_i} \cdot \left[1 - F_Y(x_i, y_i; \underline{\vartheta}) \right]^{1-\delta_i} \tag{13}$$

where $\delta_i = 1$ for the failure points and $\delta_i = 0$ for run-out points. The negative sample log-likelihood is:

$$-\ln\left(L(\underline{\vartheta})\right) = -\left(\sum_{i=1}^{N_{fail}} \ln(f_Y(x_i, y_i; \underline{\vartheta})) + \sum_{i=1}^{N_{runouts}} \ln(1 - F_Y(x_i, y_i; \underline{\vartheta})) \right) \tag{14}$$

The maximum likelihood estimate of the parameters vector $\underline{\vartheta}$ is the vector that minimizes the negative sample log-likelihood.

$$E(\underline{\vartheta}) = [E(\beta_0), E(\beta_1), E(\sigma_Y), \mathrm{E}(\mu_V), E(\sigma_V)] \tag{15}$$

The inverse of the Fisher information matrix is the asymptotic covariance matrix $\underline{\underline{C}}$ and gives information on the uncertainty of the stochastic model. Once the vector $E(\underline{\vartheta})$ and the covariance matrix $\underline{\underline{C}}$ have been computed, following MCS approach is used to estimate *S-N-P* curves:

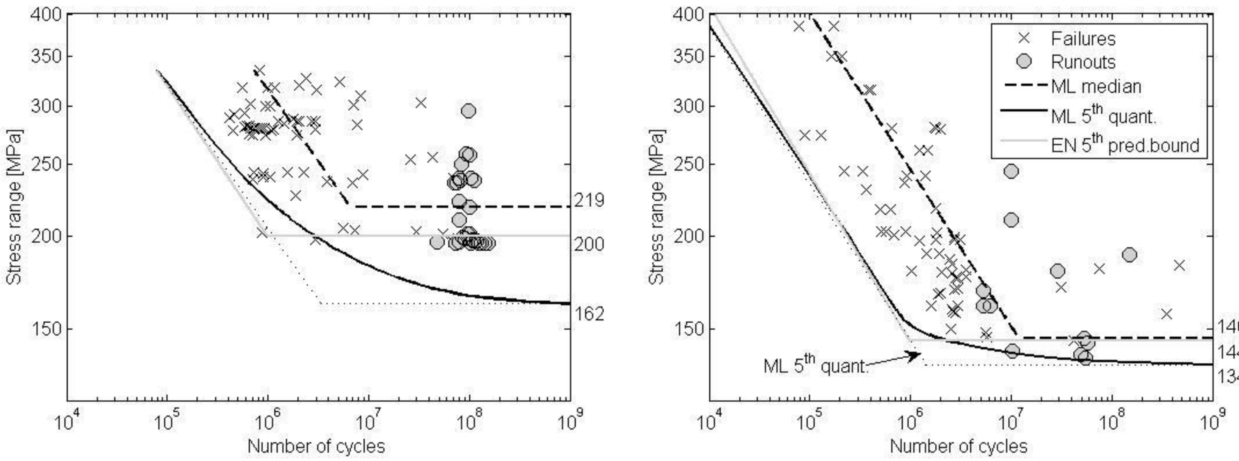
- A stress range S is selected
- 10^6 values of Y are sampled using $E(\underline{\vartheta})$ and $\underline{\underline{C}}$ information
- P_f is computed for each value of the sample
- the p-quantile of the fatigue life, N that gives $E(P_f)= p$, for the selected stress range
- the process is repeated

2 RESULTS OF STATISTICAL ANALYSIS

Five different experimental datasets were analysed using both the EN- and ML-based approaches. The five different data sets represent:

- HRCW straight rebars with diameter, *d*, smaller than 20 mm [1-2]
- HRCW straight rebars with *d* greater than 20 mm [1-3][7]
- QST straight rebars with *d* smaller than 20 mm [4][anonymous industrial dataset]
- QST straight rebars with *d* greater than 20 mm [4-5]
- HR butt welded rebars (60-degree single-v weld joint) [6]

Figs. 1 to *3 (b)* show the ML-based median S-N curves, the ML-based 5th quantile S-N curves and EN-based characteristic curves for the five considered datasets. ML-based 5th quantile nonlinear *S-N* curves were linearized (dotted lines) for direct comparison with EN-based characteristic S-N curves, using the following approach: 1) a horizontal line at $CAFL_{5\%}$ is traced; 2) a straight line with slope equal to the slope of the median line and starting at the lower abscissa point of the nonlinear curve is intersected with the $CAFL_{5\%}$ horizontal line; 3) the intersection point is the knee point. For all considered datasets, the ML-based 5^{th} quantile S-N curves and EN-based characteristic curves are almost identical while $N<10^6$ cycles.



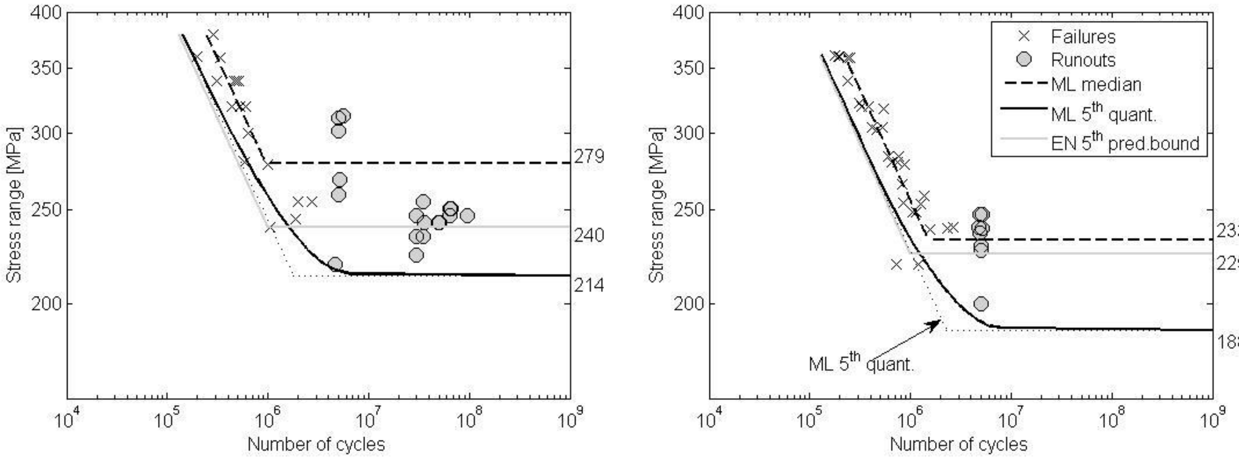
(a) (b)

Fig. 1. S-N curves for HRCW straight rebars; (a) diameter≤20mm; (b) diameter>20 mm

Figs. 1 and *2* show that for HRCW and QST straight rebars ML-approach gives considerably lower estimation of the CAFL with respect to the EN-based approach: ML-approach gives estimates of the knee point between 1.4 and 3.4 million cycles (see *Table 1*).

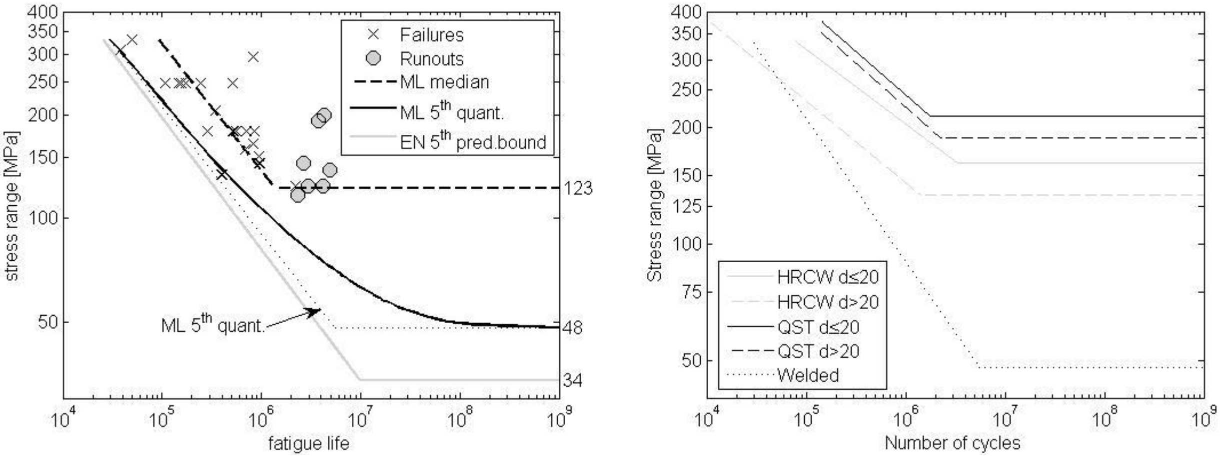
Fig. 3 (a) shows that for HR welded rebars the EN-based approach gives an over conservative estimate of the CAFL of the characteristic curve with respect to the ML-based approach: ML-approach gives estimate of the knee point at 5.5 million cycles (see *Table 1*).

For HRCW and QST straight rebars the fatigue resistance increases by decreasing the diameter of the section; for HRCW straight rebars the CAFL of the ML-based 5^{th} quantile S-N curve decreases from 162 MPa (d≤20 mm) to 134 MPa (d>20 mm) while the fatigue resistance of the ML-based 5^{th} quantile curve at 10^6 cycles decreases from 224 MPa (d≤20 mm) to 152 MPa (d>20 mm). For QST straight rebars, the CAFL of the ML-based 5^{th} quantile S-N curve decreases from 214 MPa (d≤20 mm) to 188 MPa (d>20 mm) while the fatigue resistance of the ML-based 5^{th} quantile curve at 10^6 cycles decreases from 234 MPa (d≤20 mm) to 107 MPa (d>20 mm) (see *Table 1*).



(a) (b)

Fig. 2. S-N curves for QST straight rebars; (a) diameter≤20 mm; (b) diameter>20 mm



(a) (b)

Fig. 3. (a) S-N curves for HR welded rebars (b) ML 5th quant. linearized S-N curves for all experimental datasets

Table 1. Summary of characteristic values of ML-based linearized S-N curves

Type of rebars	Slope	$CAFL_{50\%}$	$CAFL_{5\%}$	knee point	$S(N=10^6)_{5\%}$
HRCW d≤20 mm	-5.17	219 MPa	162 MPa (200 MPa)[1]	$3.4{\cdot}10^6$	224 MPa
HRCW d>20 mm	-4.72	146 MPa	134 MPa (144 MPa)	$1.4{\cdot}10^6$	152 MPa
QST d≤20 mm	-4.46	279 MPa	214 MPa (240 MPa)	$1.8{\cdot}10^6$	258 MPa
QST d>20 mm	-4.39	233 MPa	188 MPa (225 MPa)	$2.3{\cdot}10^6$	234 MPa
Welded	-2.71	123 MPa	48 MPa (34 MPa)	$5.5{\cdot}10^6$	107 MPa

ML-based 5th quantile S-N curves were plotted in *Fig. 3* for the five considered datasets: both for d≤20 mm and for d>20 mm QST straight rebars show higher fatigue resistance with respect to HRCW straight rebars. HR welded rebars have the by far lowest fatigue resistance both in terms of CAFL and of stress range at 10^6 cycles.

Figs. 2 and *3 (a)* show that QST straight rebars and HR welded rebars have a small deviation of 5th quantile curve from the median curve in the finite life region and higher deviation in HCF region. *Fig. 1 (a)* shows that HRCW (d>20 mm) straight rebars have high deviation both in finite life region and in HCF region; HRCW (d>20 mm) straight rebars have high deviation in finite life region and a smaller deviation in HCF region.

3 DISCUSSION OF STATISTICAL ANALYSIS RESULTS

Comparison of the ML-based S-N curves and the EN-based characteristic curves in the HCF region for HRCW and QST straight rebars, indicates that the arbitrary assumption of having the CAFL at 10^6 cycles is unsafe since ML estimates of the S-N curve knee point lie between 1.4 and 3.4 million cycles. On the contrary, comparison of ML-based S-N curves and EN-based characteristic curves in the HCF region for HR welded rebars, indicate that the arbitrary assumption of having the CAFL at 10^7 cycles seems is over conservative since the ML estimate of knee point of the S-N curve lies at 5.5 million cycles.

ML-based linearized 5th quantile S-N curves indicate that fatigue resistance of HRCW and QST straight rebars decreases as the diameter increases. For a given diameter interval, QST straight rebars have higher fatigue resistance with respect to HRCW straight rebars. HR welded rebars have the lowest fatigue resistance within all analysed datasets.

High deviation of the ML-based 5th quantile curve from the median curve was observed in HCF region for QST straight rebars, HRCW (d≤20 mm) straight rebars and HR welded rebars: this is due

[1] In brackets CAFL of the EN-based characteristic curve is given

to the fact that the experimental datasets are highly dispersed in the HCF region. On the contrary a small standard deviation was observed in HCF region for HRCW straight (d>20 mm) rebars, which is probably due to the fact that only run-out points exist at lowest stress ranges in the experimental datasets.

In conclusion the findings of this paper suggest that the limitations included in the current EN recommendations for statistical evaluations of characteristic S-N curves lead to incoherent fatigue resistance estimation in the HCF region for all types of analysed rebars. The ML-approach proposed herein constitutes a powerful tool that can be used to re-define the characteristic S-N curves for straight and welded rebars by taking in account both rebar type and size effect. The estimation of the characteristic S-N curves in the HCF region is directly related to experimental data and the coherence of the estimates can be ameliorated by increasing the significance of the dataset information in the HCF region. Furthermore it has to be noted that the ML-approach is presented with application to straight and welded rebars but it has generic applicability for estimating fatigue *S-N-P* curves of fatigue sensitive details of steel bridges like welded and bolted connections.

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REFERENCES

[1] Tilly G.P., Oct. 1979, Fatigue of Steel Reinforcement Bars in Concrete: a Review, *Fatigue & Fracture of Engineering Materials and Structures*, vol. 2, no. 3, pp. 251–268.

[2] Tilly G.P., Jan. 1984, Fatigue testing and performance of steel reinforcement bars, *Matériaux et Constructions*, vol. 17, no. 97, pp. 43–49.

[3] Rabbat B.G., Corley W.G., 1984, Long-time fatigue properties of high yield reinforcing bars, *Matériaux et Constructions*, vol. 17, no. 97, pp. 35–38.

[4] Zheng H., Abel A., 1999, Fatigue properties of reinforcing steel produced by TEMPCORE process, *Journal of Materials in Civil Engineering*, vol. 11, no. 2, pp. 158–165.

[5] Donnel M.J., Spencer W., Abel A., Fatigue of Tempcore reinforcing bars - the effect of galvanizing, *10th Australasian Conference on the Mechanics of Structures and Materials*,1986, pp. 327–332.

[6] Barone M.R., Cannon J.P., Munse W.H., 1974, “Fatigue behavior of reinforcing steel bars”, *IHR-64*

[7] Hanson J.M., Burton K.T., Hognestad E., 1968, Fatigue Tests of Reinforcing Bars - Effect of Deformation Pattern, *Journal of PCA Research and Development Laboratories*, vol. 10, no. 3, pp. 2–13.

[8] Ray A., Mukerjee D., Sen S.K., Bhattacharya A., Dhua S.K., Prasad M.S., Banerjee N., Popli A.M., Sahu A.K., June 1997, Microstructure and Properties of Thermomechanically Strengthened Reinforcement Bars : A Comparative Assessment of Plain-Carbon and Low-Alloy Steel Grades, *Journal of Materials Engineering and Performance*, vol. 6, pp. 335–343.

[9] MacGregor J.G., Jhamb I.C., Nuttal N., 1971, Fatigue Strength of Hot Rolled Deformed Reinforcing Bars, *ACI Journal*, pp. 169–179.

[10] Rehm G., Russwurm D., 1977, Assessment of Concrete Reinforcing Bars made by Tempcore Process, *Betonwers + Fertigteil-Technik*, vol. 6, pp. 300–307.

[11] Virmani Y.P., Wright W., Nelson R.N., 1991, Fatigue Testing for Thermex Reinforcing Bars, *Public Roads*, vol. 55, no. 3, pp. 72–78.

[12] Eurocode EN 1993 - Part 1 - Background Documentation, European Committee for Standardization, Bruxelles, 1989.

[13] Eurocode 1992: Design of concrete structures - Part 1-1: General rules and rules for buildings.” European Committee for Standardization, Bruxelles, 2005.

[14] fib Model Code for Concrete Structures, Fédération internationale du béton, 2013.

[15] Pascual F.G., Meeker W.Q., 1999, Estimating Fatigue Curves With the Random Fatigue-Limit Model, *Technometrics*.

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KEYWORDS: fatigue, rebars, S-N-P curves, CAFL, maximum likelihood.

ABSTRACT

When checking the fatigue life of steel bridges with concrete deck slabs, both steel details and embedded reinforcing steel bars (rebars) must be considered. In this paper a method for estimation of fatigue resistance curves is presented with application to straight and welded rebars. Rebar fatigue resistance is traditionally presented in the form of *S-N-P* curves which relate the applied stress range, *S*, to the *p*-quantile of fatigue life, *N*. These *S-N-P* curves are obtained from rebar fatigue tests at constant stress amplitudes. In the EN standards, characteristic *S-N* curves are created by fitting a linear regression to the experimental failure data points and translating the linear regression mean curve to the *p*-lower hyperbolic prediction bound (typically p=0.05), at 1 million cycles [1]. This approach has several limitations: 1) run-out test results are neglected; 2) a constant amplitude fatigue limit (CAFL) (stress below which tested bars experience no fatigue damage) is arbitrarily chosen to begin at 1 million cycles for straight rebars and at 10 million cycles for welded rebars [2]; and 3) *S-N* curves are based on fatigue data scatter in the finite-life region (*N* less than 1 million cycles) resulting in less accuracy in the high cycle fatigue (HCF) region (*N* over 1 million cycles). The statistical method recommended by the EN standards is currently used in the standards for concrete structures [3]. Analysing the fatigue data using more statistically robust approaches may overcome some of these issues.

In this paper a Maximum Likelihood (ML) method-based approach is used together with Monte-Carlo Simulations (MCS) to estimate *S-N-P* curves of straight and welded rebars. Run-out test results are considered and particular attention is given to the position of the CAFL. The influence of rebar size and rebar type is studied. Comparisons between ML-based *S-N-P* curves and EN-based characteristic S-N curves are made.

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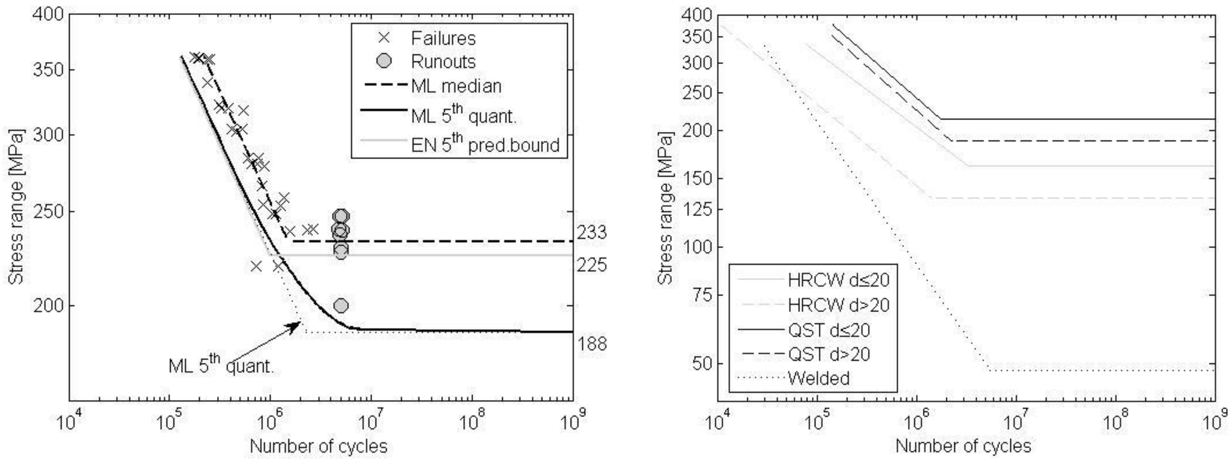
CONCLUSIONS

Comparison of the ML-based S-N curves and the EN-based characteristic curves in the HCF region for hot rolled and cold worked (HRCW) straight rebars and quenched and self-tempered (QST) straight rebars, indicates that the arbitrary assumption of having the CAFL at 10^6 cycles is unsafe since ML estimates of the S-N curve knee point lie between 1.4 and 3.4 million cycles. On the contrary, comparison of ML-based S-N curves and EN-based characteristic curves in the HCF region for HR welded rebars, indicate that the arbitrary assumption of having the CAFL at 10^7 cycles seems is over conservative since the ML estimate of knee point of the S-N curve lies at 5.5 million cycles.

ML-based linearized 5th quantile S-N curves indicate that fatigue resistance of HRCW and QST straight rebars decreases as the diameter increases. For a given diameter interval, QST straight rebars have higher fatigue resistance with respect to HRCW straight rebars. HR welded rebars have

the lowest fatigue resistance within all analysed datasets. High deviation of the ML-based 5th quantile curve from the median curve was observed in HCF region for QST straight rebars, HRCW (d≤20 mm) straight rebars and HR welded rebars: this is due to the fact that the experimental datasets are highly dispersed in the HCF region. On the contrary a small standard deviation was observed in HCF region for HRCW (d>20 mm) straight rebars, which is probably due to the fact that only run-out points exist at lowest stress ranges in the experimental datasets.
In conclusion the findings of this paper suggest that the limitations included in the current EN recommendations for statistical evaluations of characteristic S-N curves lead to incoherent fatigue resistance estimation in the HCF region for all types of analysed rebars. The ML-approach proposed herein constitutes a powerful tool that can be used to re-define the characteristic S-N curves for straight and welded rebars by taking in account both rebar type and size effect. The estimation of the characteristic S-N curves in the HCF region is directly related to experimental data and the coherence of the estimates can be ameliorated by increasing the significance of the dataset information in the HCF region.



(a) (b)

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REFERENCES

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