Probabilistic speed-density relationship for pedestrians based on data driven space and time representation

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WORKSHOP ON PEDESTRIAN MODELS 2014

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Introduction

Objective
Mathematical framework providing the detailed characterization of the pedestrian flow

Motivation

- Heterogeneity
- Complex interactions
- Multidirectional flows
Data

Data collection

• Surveys and counting
• Pedestrian tracking

Pedestrian studies

• Field data
  
  (Fruin, 1971; Navin and Wheeler 1969; Lam et al. 2003; Rastogi et al. 2013)

• Controlled experiments
  
  (Daamen and Hoogendoorn 2003; Seyfried et al., 2010; Kretz et al., 2006; Wong et al., 2010)
Data

Visiosafe technology

- Spin-off of EPFL
- Gare de Lausanne
- Anonymous sensor based pedestrian tracking
  - Thermal sensors
  - Range sensors
- Vision processing outcome
  \[(t, x(t), y(t), \text{pedestrian}_{id})\]

Ga re de Lausanne

Pedestrian underpass West

- The busiest walking area in the station
- Area $\approx 685 \text{m}^2$
- The maximum occupation $\approx 250$ pedestrians
- Area covered by 32 sensors
Fundamental flow indicators

- Density \((k)\)
- Speed \((v)\)
- Flow \((q)\)
- Fundamental diagram

\[ q = v \cdot k \]

source: (Daamen et al., 2005)
Fundamental flow indicators

Issues

- Spatio-temporal discretization is arbitrary
  - Results may be highly sensitive
  - Loss of heterogeneity
- Pedestrian flow is multidirectional
  (Lam et al. 2003; Wong et al., 2010)

Pedestrian-oriented flow characterization

- Detailed pedestrian tracking input
- Data driven space and time discretization
Density indicator

Pedestrian flow
- Number of pedestrians per unit of space at a given time

Spatial discretization
- Discretization units are too small - many remain empty
- Discretization units are too large - loss of information
Spatial discretization

Voronoi tessellations

- \( p_1, p_2, \ldots, p_N \) is a finite set of points
- Voronoi space decomposition assigns a region to each point

\[
V(p_i) = \{ p \mid \| p - p_i \| \leq \| p - p_j \|, i \neq j \}
\]

Spatial discretization

Numerical instability
- Small polygons allocated to pedestrians in very dense areas

Delaunay triangulation
- Clustering of critical cells
- $\xi$, threshold distance

$$d(p_i, p_j) < \xi, \forall i, j$$
Spatial discretization

Numerical instability

- Small polygons allocated to pedestrians in very dense areas

Sensitivity analyses

- $\xi = 0.4m$
- $\omega_i$, weight associated to the corresponding space
Spatial discretization

Presence of obstacles

• Assumption: two points can be connected by a straight line
• Voronoi diagram for points and Voronoi diagram areas

\[
d(p_i, O) = \min_{o_j} \{ \|p_i - o_j\| \mid o_j \in O \}
\]
Density indicator

Definition

- Set of points: pedestrians
  \[ p_i = (x_i, y_i, t_i) \]
- Pedestrian-oriented density indicator
  \[ k_i = \frac{\omega_i}{|V(p_i)|} \]

Voronoi density map
Speed indicator

Pedestrian flow

- Instantaneous speed - rate of change of position of a pedestrian with respect to time and at a particular point.

Time discretization

- Discretization interval is too small - noisy observations
- Discretization interval is too large - lower precision
Time discretization

<table>
<thead>
<tr>
<th>Moment</th>
<th>$\Delta t=0.1s$</th>
<th>$\Delta t=0.2s$</th>
<th>$\Delta t=0.3s$</th>
<th>$\Delta t=0.4s$</th>
<th>$\Delta t=0.5s$</th>
<th>$\Delta t=0.6s$</th>
<th>$\Delta t=0.7s$</th>
<th>$\Delta t=0.8s$</th>
<th>$\Delta t=0.9s$</th>
<th>$\Delta t=1s$</th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>1.1161</td>
<td>1.1158</td>
<td>1.1156</td>
<td>1.1155</td>
<td>1.1153</td>
<td>1.1152</td>
<td>1.1150</td>
<td>1.1149</td>
<td>1.1148</td>
<td>1.1147</td>
</tr>
<tr>
<td>2</td>
<td>0.4175</td>
<td>0.3296</td>
<td>0.2956</td>
<td>0.2747</td>
<td>0.2591</td>
<td>0.2465</td>
<td>0.2358</td>
<td>0.2263</td>
<td>0.2179</td>
<td>0.2104</td>
</tr>
<tr>
<td>3</td>
<td>5.7853</td>
<td>2.5957</td>
<td>1.7703</td>
<td>1.4310</td>
<td>1.2544</td>
<td>1.1476</td>
<td>1.0740</td>
<td>1.0188</td>
<td>0.9744</td>
<td>0.9363</td>
</tr>
</tbody>
</table>

- Kruskal-Wallis test ($H=4.61$, df=9, $p=0.87$)
  The moments represent the same population at 95% confidence level
Speed indicator

Definition

• Space-time representation
  \[ p_i = (x_i, y_i, t_i) \]

• Pedestrian-oriented speed indicator
  \[ v_i = \frac{\|p_i(t+\Delta t) - p_i(t-\Delta t)\|}{2\Delta t}, \quad \Delta t = 1s \]
Empirical speed-density relationship

Speed-density profiles

February 11.-15., 2013.: morning peak hour
Probabilistic approach

Kumaraswamy distribution

- Defined on the bounded region $[l, u]$
- Two non-negative shape parameters $\alpha$ and $\beta$
- The simple closed form of pdf $f(x)$ and cdf $F(x)$

$$f(x) = \frac{\alpha \cdot \beta \cdot (x-l)^{\alpha-1} \cdot ((u-l)^{\alpha} - (x-l)^{\alpha})^{\beta-1}}{(u-l)^{\alpha \cdot \beta}}$$

$$F(x) = 1 - (1 - \left(\frac{x-l}{u-l}\right)^{\alpha})^{\beta}$$

Probabilistic approach

Speed-density relationship

\[ V \sim f(\alpha(k), \beta(k), l(k), u(k)) \]

- \( f \) - Kumaraswamy pdf
- \( V \) - speed
- \( k \) - density level
- \( \alpha, \beta \) - shape parameters
- \( u, l \) - boundary parameters
Probabilistic approach

Specification of speed-density relationship

\[ V \sim f(\alpha(k), \beta(k), l(k), u(k)) \]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Specification#1</th>
<th>Specification#2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha(k) )</td>
<td>( a_\alpha k^3 + b_\alpha k^2 + c_\alpha k + d_\alpha )</td>
<td>( a_\alpha k^3 + b_\alpha k^2 + c_\alpha k + d_\alpha )</td>
</tr>
<tr>
<td>( \beta(k) )</td>
<td>( a_\beta \exp(b_\beta k) )</td>
<td>( a_\beta \exp(b_\beta k) )</td>
</tr>
<tr>
<td>( u(k) )</td>
<td>( a_u \exp(b_u k) )</td>
<td>( a_u k^3 + b_u k^2 + c_u k + d_u )</td>
</tr>
<tr>
<td>( l(k) )</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
Probabilistic approach

Maximum likelihood estimation

\[
\log L = \sum_{i=1}^{n} \log(\alpha(k_i)) + \sum_{i=1}^{n} \log(\beta(k_i)) + \sum_{i=1}^{n} (\alpha(k_i) - 1) \log(v_i - l(k_i)) + \sum_{i=1}^{n} (\beta(k_i) - 1) \log((u(k_i) - l(k_i))^\alpha(k_i) - (v_i - l(k_i))^\alpha(k_i)) - \sum_{i=1}^{n} \alpha(k_i) \beta(k_i) \log(u(k_i) - l(k_i))
\]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Specification #1</th>
<th>Specification #2</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a_\alpha)</td>
<td>-0.0076</td>
<td>0.0498</td>
</tr>
<tr>
<td>(b_\alpha)</td>
<td>0.0961</td>
<td>-0.2823</td>
</tr>
<tr>
<td>(c_\alpha)</td>
<td>-0.3781</td>
<td>-0.0207</td>
</tr>
<tr>
<td>(d_\alpha)</td>
<td>2.2185</td>
<td>2.0089</td>
</tr>
<tr>
<td>(a_\beta)</td>
<td>44.8191</td>
<td>45.362</td>
</tr>
<tr>
<td>(b_\beta)</td>
<td>-0.1057</td>
<td>-0.5945</td>
</tr>
<tr>
<td>(a_u)</td>
<td>7</td>
<td>0.0002</td>
</tr>
<tr>
<td>(b_u)</td>
<td>0</td>
<td>-0.0002</td>
</tr>
<tr>
<td>(c_u)</td>
<td>0.0010</td>
<td>8.0017</td>
</tr>
</tbody>
</table>

\(<log \ L> = -891880 \quad -932990.<\/div>
Probabilistic approach

Speed-density relationship

\[ V \sim f(\alpha(k), \beta(k), l(k), u(k)) \]

\[ \alpha(k) = a_\alpha k^3 + b_\alpha k^2 + c_\alpha k + d_\alpha \]
\[ \beta(k) = a_\beta \exp(b_\beta k) \]
\[ u(k) = 7 \]
\[ l(k) = 0 \]

\[ a_\alpha = -0.0076, \, b_\alpha = 0.0961, \, c_\alpha = -0.3781, \, d_\alpha = 2.2185 \]
\[ a_\beta = 44.8191, \, b_\beta = -0.1057 \]
Probabilistic approach

Validation

- Moments of empirical and predicted discrete joint distributions
- Kruskal-Wallis test ($H=0.33$, df=1, $p=0.5637$)

The model and data represent the same population at 95% confidence level

<table>
<thead>
<tr>
<th>Moments</th>
<th>Data</th>
<th>Model prediction</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.9333</td>
<td>0.9856</td>
</tr>
<tr>
<td>2</td>
<td>0.1845</td>
<td>0.2376</td>
</tr>
<tr>
<td>3</td>
<td>0.0426</td>
<td>0.0648</td>
</tr>
<tr>
<td>4</td>
<td>0.1521</td>
<td>0.1769</td>
</tr>
</tbody>
</table>
Conclusion

- Pedestrian-oriented flow characterization
- Data-driven space and time discretization
- Probabilistic methodology to describe observed heterogeneity
- Model estimation and validation based on pedestrian tracking input
- Case study: Gare de Lausanne
Future directions

- The framework is insufficient to explain the multidirectional nature of pedestrian flows
- Solution investigated: a stream-based approach
- Final objective: integration of the stream-based concept with the developed probabilistic framework
Thank you
References


References


Rastogi, R., Chandra, S. et al. (2013). Pedestrian flow characteristics for different pedestrian facilities and situations.
