Specification of the cross nested logit model with sampling of alternatives for route choice models

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June 26, 2014
Outline

1. Introduction
2. Sampling of alternatives
3. MEV models
4. Validation on synthetic data
5. Case study with real data
**Motivation**

**Route choice model**
- Given an origin and a destination
- What is the preferred itinerary of a given traveler?

**Main difficulties**
- Very large choice set
- Structural correlation among alternatives
Very large choice set

**Issue**

Number of paths grows exponentially with the number of nodes

**Literature**

- link elimination  Azevedo et al. (1993)
- link penalty  de la Barra et al. (1993)
- labeled paths  Ben-Akiva et al. (1984)
- Sampling  Frejinger et al. (2009)
Introduction

Structural correlation

Issue

Significant physical overlap

Literature

- **C-logit** Cascetta et al. (1996)
- **Path-size** Ben-Akiva and Bierlaire (1999)
- **Link-based cross-nested logit** Prashker and Bekhor (1999)
- **Error components** Ramming (2002); Frejinger and Bierlaire (2007)
In this paper...

Methodology

- Cross Nested logit
- Sampling of alternatives

Builds on...

- McFadden (1978)
- Vovsha and Bekhor (1998)
- Bierlaire et al. (2008)
- Frejinger et al. (2009)
- Guevara and Ben-Akiva (2013)
- Flötteröd and Bierlaire (2013)
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Lai & Bierlaire (EPFL)
CNL and sampling of alternatives
June 26, 2014
Logit model

\[ P(i|C) = \frac{e^{V_i}}{\sum_{j \in C} e^{V_j}} \]

McFadden (1978)

Sampling protocol

- Sample subset \( D \subseteq C \)
- Sampling probability \( q(D|j) \)
- Positive conditioning property

\[ q(D|i) > 0 \implies q(D|j) > 0 \ \forall j \in D. \]
Logit model

\[ P(i|C) \approx P(i|D) = \frac{e^{V_i + \ln q(D|i)}}{\sum_{j \in D} e^{V_j + \ln q(D|j)}} \]

Simple random sampling
- \( q(D|i) = q(D|j) \forall i, j \in C \)
- Correction terms cancel out
- Irrelevant, circuitous paths
- How to draw?

Importance sampling
- In \( q(D|i) \) are confounded with ASC
- In route choice, usually no ASC
- How to draw?
How to draw?

Shortest path-based procedures

- link elimination: deterministic
- link penalty: deterministic
- labeled paths: deterministic
- SP on random costs:
  - some paths have 0 probability to be drawn
  - how to compute the sampling probability?
Metropolis-Hastings algorithm

Features

- Designed to draw from complex distributions
- Does not require the exact pmf/pdf
- Only a quantity proportional to it.
- For instance, to draw a path $p$ with probability

$$\frac{b_p}{\sum_{q \in C} b_q}$$

only $b_p$ are needed.
Metropolis-Hastings algorithm

Methodology

- Design a Markov chain $Q$ visiting the states/paths
- Accept/reject method
- Accept probability depends on
  - target (unnormalized) probabilities
  - transition probabilities of the Markov chain:

$$P(\text{accept}) = \min \left( \frac{b_Q Q_{qp}}{b_p Q_{pq}}, 1 \right)$$
Example

$$b = (20, 8, 3, 1) \quad \pi = \left( \frac{5}{8}, \frac{1}{4}, \frac{3}{32}, \frac{1}{32} \right)$$

$$Q = \begin{pmatrix}
\frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\
\frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\
\frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\
\frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4}
\end{pmatrix}$$

Run MH for 10000 iterations. Collect statistics after 1000

- Accept: [2488, 1532, 801, 283]
- Reject: [0, 952, 1705, 2239]
- Simulated: [0.627, 0.250, 0.095, 0.028]
- Target: [0.625, 0.250, 0.09375, 0.03125]
Sampling of paths

Difficulties

Design $Q$ such that

- Every path can be generated with nonzero probability
- Both $Q_{pq}$ and $Q_{qp}$ are known

Flötteröd and Bierlaire (2013)

- Proof of concept on synthetic data
- Application to Tel Aviv (17K links, 8K nodes)
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5. Case study with real data
MEV models

Generic model

\[ P(i|C) = \frac{\exp(V_i + \ln G_i(C))}{\sum_{j \in C} \exp(V_j + \ln G_j(C))} \]

where \( G_i(C) = G_i(e^{V_1}, \ldots, e^{V_J}) \) is the derivative of the CPGF wrt \( e^{V_i} \).

Cross nested logit

\[ G_i(C) = \sum_{m=1}^{M} \left[ \mu \alpha_{im} e^{V_i(\mu_m - 1)} \left( \sum_{j \in C} \alpha_{jm} e^{\mu_m V_j} \right)^{\mu - \mu_m} \right], \]
**MEV models**

**Generic model**

\[
P(i|C) = \frac{\exp(V_i + \ln G_i(C))}{\sum_{j \in C} \exp(V_j + \ln G_j(C))}
\]

where \( G_i(C) = G_i(e^{V_{1n}}, \ldots, e^{V_j}) \) is the derivative of the CPGF wrt \( e^{V_i} \).

**Cross nested logit**

\[
G_i(C) = \sum_{m=1}^{M} \left[ \mu_{im} e^{V_i(\mu_m - 1)} \left( \sum_{j \in C} \alpha_{jm} e^{\mu_m V_j} \right)^{\frac{\mu - \mu_m}{\mu_m}} \right],
\]
Sampling and MEV

\[ P(i|C) = \frac{\exp(V_i + \ln G_i(C))}{\sum_{j \in C} \exp(V_j + \ln G_j(C))} \]

Sampling correction

- If \( \ln G_j(C) \) is known, same idea as for logit

\[ \Pr(i|D) = \frac{\exp( V_i + \ln G_i(C) + \ln \Pr(D|i))}{\sum_{j \in D} \exp(V_j + \ln G_j(C) + \ln \Pr(D|j))}. \]

- Not confounded with the constants anymore.

Bierlaire et al. (2008)
Correction term

\[ \Pr(D|p) \propto \frac{k_p}{q(p)} \]

where
- \( k_p \) is the number of times path \( p \) has been generated
- \( q(p) \) is the sampling probability of path \( p \)
- \( q(p) \propto b_p \)
Model I

\[
\Pr(i|D) = \frac{\exp(V_i + \ln G_i(C) + \ln \frac{k_i}{b_i})}{\sum_{j \in D} \exp(V_j + \ln G_j(C) + \ln \frac{k_j}{b_j})},
\]
Approximation of \( \ln G_i(C) \)

Guevara and Ben-Akiva (2013)

\[
G_i(C) \approx \hat{G}_i(D, w) = \sum_{m=1}^{M} \left[ \mu \alpha_{im} e^{V_i(\mu_m - 1)} \left( \sum_{j \in D} w_j \alpha_{jm} e^{\mu_m V_j} \right)^{\frac{\mu - \mu_m}{\mu_m}} \right]
\]

where \( w_j \) expansion factor to be defined.
Realized / expected

\[ w_j^G = \frac{k_j}{E[k_j]} = \frac{k_j}{q(j)R} = \frac{k_jB}{b(j)R} \]

where

- \( R \) is the number of draws used to generate \( \mathcal{D} \)
- \( B = \sum_{j \in \mathcal{C}} b(j) \) [Requires enumeration of \( \mathcal{C} \)]

Approximate \( B \)

\[ B = \sum_{j \in \mathcal{C}} b(j) = |\mathcal{C}| \frac{\sum_{i \in \mathcal{C}} b(i)}{|\mathcal{C}|} = |\mathcal{C}| \bar{b}, \]

and

\[ \bar{b} = \frac{\sum_{i \in \mathcal{C}} b(i)}{|\mathcal{C}|} \approx \frac{\sum_{i \in \mathcal{D}} b(i)}{|\mathcal{D}|}. \]
Expansion factors: Guevara and Ben-Akiva (2013)

Approximation

\[ w^G_j = \frac{k_j}{b(j)R|D|} \sum_{i \in D} b(i) \]

which require \(|C|\)

Approximate \(|C|\)

\(N\) random walks in the network

\[ |C| \approx \frac{1}{N} \sum_{i=1}^{N} \frac{1}{\ell(i)}. \]

\(\ell(i)\): likelihood of the path generated by the algorithm during run \(i\)
Expansion factors: Frejinger et al. (2009)

Account for the upper bound

\[ w^F_j = \begin{cases} 
1 & \text{if } b(j)R > B, \\
\frac{B}{b(j)R} & \text{otherwise.}
\end{cases} \]

Same approximation of \( B \)

\[ B \approx \frac{|C|}{|D|} \sum_{i \in D} b(i) \]

Again, requires \(|C|\)
Avoiding $|C|$

- Let $s$ be the path which has been sampled the most in $\mathcal{D}$
- $k_s \geq k_p$, for each $p \in \mathcal{D}$.
- If sample is large enough, $k_s \approx q(s) R$

\[ w_j^G = \frac{k_j}{q(j)R} \approx w_j^L = \frac{k_j}{q(j)R} \frac{q(s) R}{k_s} = \frac{k_j}{b(j)} \frac{b(s)}{k_s} \]

which does not require $B$ or $|C|$. 
Expansion factors

- Guevara and Ben-Akiva (2013)

\[ w_j^G = \frac{k_j}{b(j)R} B \text{ with } B \approx \frac{|C|}{|D|} \sum_{i \in D} b(i) \]

- Frejinger et al. (2009)

\[ w_j^F = \begin{cases} 
1 & \text{if } b(j)R > B, \\
\frac{B}{b(j)R} & \text{otherwise.}
\end{cases} \text{ with } B \approx \frac{|C|}{|D|} \sum_{i \in D} b(i). \]

- Lai and Bierlaire (2014)

\[ w_j^L = \frac{k_j}{b(j)} \frac{b(s)}{k_s} \]
Models to be compared

- Model I: true $G_i$ (impossible in practice)

$$\Pr(i|\mathcal{D}) = \frac{\exp(V_i + \ln G_i(\mathcal{C}) + \ln \frac{k_i}{b(i)})}{\sum_{j \in \mathcal{D}} \exp(V_j + \ln G_j(\mathcal{C}) + \ln \frac{k_j}{b(j)})}$$

- Model II: the proposed model

$$\Pr(i|\mathcal{D}, \mathcal{D}', w) = \frac{\exp(V_i + \ln G_i(\mathcal{D}', w)) + \ln \frac{k_i}{b(i)}}{\sum_{j \in \mathcal{D}} \exp(V_j + \ln G_j(\mathcal{D}', w) + \ln \frac{k_j}{b(j)})}.$$  

- Model III: no expansion factor, no sampling correction (benchmark)

$$\Pr(i|\mathcal{D}, \mathcal{D}') = \frac{\exp(V_i + \ln G_i(\mathcal{D}', 1))}{\sum_{j \in \mathcal{D}} \exp(V_j + \ln G_j(\mathcal{D}', 1))}.$$
Validation on synthetic data

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The network: 170 paths (Frejinger (2008))
The true model: cross-nested logit

Utility

\[ V_i = \beta_L L_i + \beta_{SB} S_{Bi}, \]

“True” parameters

- \( \beta_L = -0.5 \) and \( \beta_{SB} = -0.1 \)
- \( \mu_m = 1.5 \) for each link \( m \)
- \( \alpha_{im} = \ell_m / L_i \)

Data

3000 synthetic choices
Re-estimate the parameters of the true model

**Full choice set**

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Est.</th>
<th>Std err.</th>
<th>t-test (0)</th>
<th>t-test (true)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_L$</td>
<td>-0.501</td>
<td>0.0118</td>
<td>43.1</td>
<td>0.678</td>
</tr>
<tr>
<td>$\beta_{SB}$</td>
<td>-0.0910</td>
<td>0.0240</td>
<td>3.19</td>
<td>0.375</td>
</tr>
<tr>
<td>$\mu_m$</td>
<td>1.49</td>
<td>0.0269</td>
<td>55.2</td>
<td>0.0535</td>
</tr>
</tbody>
</table>
Metropolis-Hastings

\[ b(i) = \exp(-\theta L_i), \quad \theta \geq 0 \]
Validation on synthetic data

Number of generated paths

theta=0.01
theta=0.1
theta=0.3
theta=0.5
theta=1

Number of draws

Number of generated paths

Lai & Bierlaire (EPFL)
CNL and sampling of alternatives
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### Model I: true $G_i$ — MH $\theta = 0.5$

<table>
<thead>
<tr>
<th></th>
<th>10 draws</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_L$ (-0.5)</td>
<td>-0.443</td>
<td>0.0163</td>
<td>27.3</td>
<td>3.48</td>
<td></td>
</tr>
<tr>
<td>$\beta_{SB}$ (-0.1)</td>
<td>-0.0647</td>
<td>0.0427</td>
<td>1.51</td>
<td>0.826</td>
<td></td>
</tr>
<tr>
<td>$\mu_m$ (1.5)</td>
<td>1.56</td>
<td>0.0340</td>
<td>45.8</td>
<td>1.72</td>
<td></td>
</tr>
</tbody>
</table>

Estimation time: 1362 seconds

<table>
<thead>
<tr>
<th></th>
<th>40 draws</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_L$ (-0.5)</td>
<td>-0.479</td>
<td>0.0156</td>
<td>30.8</td>
<td>1.34</td>
<td></td>
</tr>
<tr>
<td>$\beta_{SB}$ (-0.1)</td>
<td>-0.0720</td>
<td>0.0393</td>
<td>1.83</td>
<td>0.713</td>
<td></td>
</tr>
<tr>
<td>$\mu_m$ (1.5)</td>
<td>1.51</td>
<td>0.0322</td>
<td>47.0</td>
<td>0.367</td>
<td></td>
</tr>
</tbody>
</table>

Estimation time: 4648 seconds
**Model I: true \( G_i \) — MH \( \theta = 0.01 \)**

<table>
<thead>
<tr>
<th>Draws</th>
<th>Est.</th>
<th>Std err.</th>
<th>t-test(0)</th>
<th>t-test(true)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10 draws</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \beta_L ) (-0.5)</td>
<td>-0.535</td>
<td>0.0174</td>
<td>30.8</td>
<td>2.01</td>
</tr>
<tr>
<td>( \beta_{SB} ) (-0.1)</td>
<td>-0.132</td>
<td>0.0545</td>
<td>2.42</td>
<td>0.580</td>
</tr>
<tr>
<td>( \mu_m ) (1.5)</td>
<td>1.41</td>
<td>0.0355</td>
<td>39.8</td>
<td>2.47</td>
</tr>
<tr>
<td>Estimation time: 1612 seconds</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>40 draws</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \beta_L ) (-0.5)</td>
<td>-0.544</td>
<td>0.0160</td>
<td>33.9</td>
<td>2.76</td>
</tr>
<tr>
<td>( \beta_{SB} ) (-0.1)</td>
<td>-0.130</td>
<td>0.0410</td>
<td>3.16</td>
<td>0.726</td>
</tr>
<tr>
<td>( \mu_m ) (1.5)</td>
<td>1.41</td>
<td>0.0322</td>
<td>43.8</td>
<td>2.85</td>
</tr>
<tr>
<td>Estimation time: 4914 seconds</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Model I: comments

- Trade-off between dispersion (low $\theta$) and number of draws
- Lower value of $\theta$ requires more draws
- $\theta = 0.5$, 40 draws: parameters are correctly estimated
- First sampling scheme is validated
- No specific guideline for $\theta$ and $R$
Approximating $\bar{b}$ and $|C|$

Protocol

- For $\bar{b}$: generate $\mathcal{D}$ using MH with 100 draws and $\theta = 0.01$
- For $|C|$: generate 10000 paths using random walk
- Repeat 100 times
- Compute the empirical mean and standard error

Results

<table>
<thead>
<tr>
<th></th>
<th>True</th>
<th>Mean</th>
<th>Std err</th>
<th>t-test(true)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{b}$</td>
<td>0.688</td>
<td>0.684</td>
<td>0.0023</td>
<td>1.62</td>
</tr>
<tr>
<td>$</td>
<td>C</td>
<td>$</td>
<td>170</td>
<td>169.8</td>
</tr>
</tbody>
</table>
Model II

Protocol

- Denominator: $\mathcal{D}$ generated with MH (40 draws, $\theta = 0.5$)
- Expansion factor: $\mathcal{D}'$ MH with various values
Model II: 100 draws ($t$-test vs true value)

Validation on synthetic data

Sampling protocol for $\mathcal{D}'$: $\theta = 0.5$

<table>
<thead>
<tr>
<th></th>
<th>$w^G$</th>
<th>$w^F$</th>
<th>$w^L$</th>
<th>$w = 1$</th>
<th>$\beta_L$</th>
<th>$\beta_{SB}$</th>
<th>$\mu_m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mod. II</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>2.48</td>
<td>0.910</td>
<td>2.02</td>
</tr>
<tr>
<td>Mod. III</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>19.4</td>
<td>0.221</td>
<td>1.06</td>
</tr>
</tbody>
</table>

Sampling protocol for $\mathcal{D}'$: $\theta = 0.01$

<table>
<thead>
<tr>
<th></th>
<th>$w^G$</th>
<th>$w^F$</th>
<th>$w^L$</th>
<th>$w = 1$</th>
<th>$\beta_L$</th>
<th>$\beta_{SB}$</th>
<th>$\mu_m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mod. II</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>4.61</td>
<td>0.303</td>
<td>4.70</td>
</tr>
<tr>
<td>Mod. III</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>18.9</td>
<td>0.634</td>
<td>3.63</td>
</tr>
</tbody>
</table>
Validation on synthetic data

Model II: 200 draws ($t$-test vs true value)

<table>
<thead>
<tr>
<th></th>
<th>Mod. II</th>
<th>Mod. III</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$w^G$</td>
<td>$w^F$</td>
</tr>
<tr>
<td>$\beta_L$</td>
<td>0.578</td>
<td>10.5</td>
</tr>
<tr>
<td>$\beta_{SB}$</td>
<td>0.513</td>
<td>0.194</td>
</tr>
<tr>
<td>$\mu_m$</td>
<td>1.36</td>
<td>5.02</td>
</tr>
</tbody>
</table>

Sampling protocol for $\mathcal{D}'$: $\theta = 0.5$

<table>
<thead>
<tr>
<th></th>
<th>Mod. II</th>
<th>Mod. III</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$w^G$</td>
<td>$w^F$</td>
</tr>
<tr>
<td>$\beta_L$</td>
<td>3.51</td>
<td>3.84</td>
</tr>
<tr>
<td>$\beta_{SB}$</td>
<td>0.173</td>
<td>0.119</td>
</tr>
<tr>
<td>$\mu_m$</td>
<td>9.11</td>
<td>8.65</td>
</tr>
</tbody>
</table>
### Validation on synthetic data

**Model II: 300 draws (t-test vs true value)**

#### Sampling protocol for $D'$: $\theta = 0.5$

<table>
<thead>
<tr>
<th></th>
<th>$w^G$</th>
<th>$w^F$</th>
<th>$w^L$</th>
<th>$w = 1$</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_L$</td>
<td>0.981</td>
<td>3.62</td>
<td>0.703</td>
<td>0.981</td>
<td>19.3</td>
</tr>
<tr>
<td>$\beta_{SB}$</td>
<td>0.428</td>
<td>1.34</td>
<td>0.537</td>
<td>0.428</td>
<td>0.0052</td>
</tr>
<tr>
<td>$\mu_m$</td>
<td>2.28</td>
<td>3.12</td>
<td>1.70</td>
<td>2.28</td>
<td>1.66</td>
</tr>
</tbody>
</table>

#### Sampling protocol for $D'$: $\theta = 0.01$

<table>
<thead>
<tr>
<th></th>
<th>$w^G$</th>
<th>$w^F$</th>
<th>$w^L$</th>
<th>$w = 1$</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_L$</td>
<td>0.809</td>
<td>0.0271</td>
<td>1.02</td>
<td>5.05</td>
<td>18.5</td>
</tr>
<tr>
<td>$\beta_{SB}$</td>
<td>0.565</td>
<td>0.780</td>
<td>0.480</td>
<td>0.564</td>
<td>0.654</td>
</tr>
<tr>
<td>$\mu_m$</td>
<td>1.66</td>
<td>0.650</td>
<td>1.84</td>
<td>5.19</td>
<td>3.01</td>
</tr>
</tbody>
</table>
Comments

- $\theta = 0.5$ seems again the most appropriate
- Model II outperforms Model III (no correction, no expansion factor)
- New expansion factor is the most appropriate (already good results with 100 draws)
- $\mu_m$ seems to be the most sensitive parameters
Validation on synthetic data

$t$-tests with $w^L$ and $\theta = 0.5$
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4. Validation on synthetic data
5. Case study with real data
Tianhe region (CBD) of Guangzhou (China)
Data

Network
- 208 nodes
- 662 links
- 24 major roads
- 34 arterial streets
- 32 minor streets
- 57 signalized intersections

GPS traces from taxis
- 7 ODs
- 740 trips
Case study with real data

Model

Utility

\[ V_i = \beta_L \text{Length}_i + \beta_{\text{ARR}} \text{ArteryRoadRatio}_i + \beta_S \text{Signal}_i. \]

Cross-nested logit

- Two nests: \( \mu \): non-artery roads, \( \mu_{mA} \): artery roads
- \( \alpha_{im} = \ell_m / L_i \)

MH sampling

| \( \theta \) | \( |D| \) | \( \theta \) | \( |D| \) |
|----------|------|----------|------|
| 0.005    | 29   | 0.0025   | 3813 |
| 0.004    | 54   | 0.0023   | 5624 |
| 0.003    | 201  | 0.002    | 7766 |
| 0.0028   | 2036 | 0.001    | 9836 |
Estimation results (with Matlab, Intel i5 with 4GB RAM, one processor)

\[ \theta = 0.003 \]

<table>
<thead>
<tr>
<th>Model II</th>
<th>Est.</th>
<th>Std. err.</th>
<th>t-test (0)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta_L )</td>
<td>-1.58</td>
<td>0.0566</td>
<td>27.9</td>
</tr>
<tr>
<td>( \beta_{ARR} )</td>
<td>8.09</td>
<td>0.636</td>
<td>12.7</td>
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<tr>
<td>( \beta_S )</td>
<td>-0.513</td>
<td>0.267</td>
<td>1.91</td>
</tr>
<tr>
<td>( \mu_m )</td>
<td>3.90</td>
<td>0.117</td>
<td>33.3</td>
</tr>
<tr>
<td>( \mu_{mA} )</td>
<td>2.22</td>
<td>0.257</td>
<td>8.62</td>
</tr>
</tbody>
</table>

Number of observations: 740 trips from 7 OD
Null log likelihood: -3.4078e+03
Final log likelihood: -1.9206e+03
Estimation time: 22.32 hours
Conclusion

Contributions

- Application of sampling of alternative for MEV and route choice
- New expansion factor
- Validity check: synthetic data
- Feasibility check: real data
- Heavy, but tractable

Future work

- Investigate other nesting structures
- Different ways to approximate $G_i$
- Estimation of $\alpha_{im}$ (?)


Bibliography II


