

The Ideal Train Timetabling Problem

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20th CONFERENCE OF THE INTERNATIONAL FEDERATION OF OPERATIONAL RESEARCH SOCIETIES



I F O R S





International Federation of Operational Research Societies

The Art of Modeling **BARCELONA 2014**

13th-18th July

July 17, 2014

Purely commercial rail passenger services in Europe

	Market closed for commercial national rail passenger services.
	Open access, but no external RUs providing commercial national rail passenger services .
	Open access with external RUs providing commercial national rail passenger services.
	AT and CZ: commencing end of 2011, external RUs providing purely commercial national rail passenger services.



Liberalisation – Overview

Liberalisation time line

1 January 1993

Access for international groupings providing international services and for international combined transport goods service providers

15 March 2003

Access to the Trans-European Rail Freight Network for international freight services

1 January 2006

Access to the entire EU rail network for international freight services

1 January 2007

Access to the entire EU rail network for all types of rail freight (including domestic)

1 January 2010

Access to the infrastructure in all EU Member States for the purpose of operating international passenger services (cabotage permitted)

? December 2019

Access to the infrastructure in all EU Member States for all rail services, including domestic passenger services

Public Sector – Accessibility/Mobility

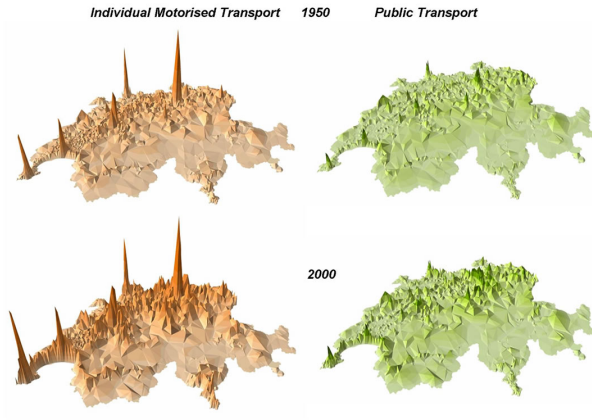


Figure : Mobility evolution in Switzerland¹

¹ – source: *Entwicklung der MIV und OV Erreichbarkeit in der Schweiz: 1950-2000*; Ph. Frohlich, M. Tschopp and K.W. Axhausen

Private Sector



Increase profits

Market Settings

Travel Time is the same



Serve Different Destinations



Better Quality

Departure / Class / Service	Duration	Stops	Train Number	Class
06:27 - 09:53	3:25	4	9907	Smart
Base	83.50 €	111 €	123.50 €	
Economy	55 €	71 €	111 €	
Low Cost	36 €	57 €		
Prezzo Rate	47.50 €			
A/R Ingiornata	80 €	97.50 €		

Better Price



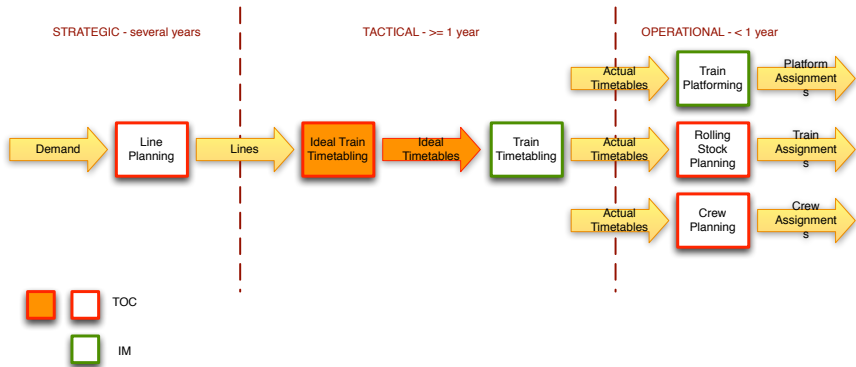
Better Departure Times

Goal: Better Timetables!

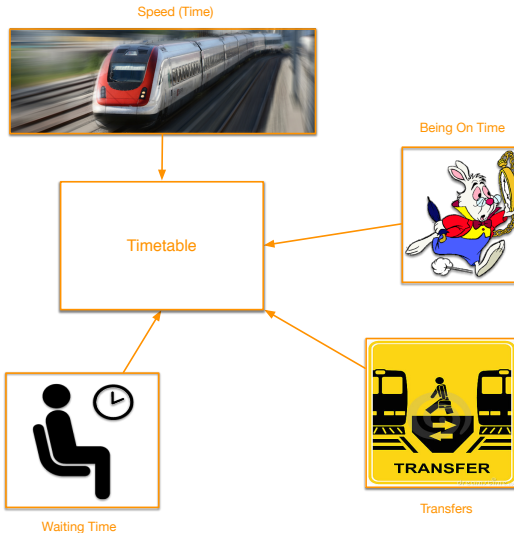
Halt	An	Ab	Gleis	Beleg.
St. Gallen		09:11	1	1.# 2.#
Gossau SG	09:18	09:19	4	1.# 2.#
Flawil	09:23	09:24	1	1.# 2.#
Uzwil	09:29	09:30	3	1.# 2.#
Wil	09:38	09:39	2	1.# 2.#
Winterthur	09:56	09:58	3	1.# 2.#
Zürich Flughafen	10:11	10:13	3	1.# 2.#
Zürich HB	10:23	10:32	17	1.# 2.#
Neubaustrecke				
Bern	11:28	11:34	4	1.# 2.#

- How to measure goodness of a timetable?
- Timetable design in the literature
 - **non-cyclic**: using so called "ideal timetables"
 - **cyclic**: does not take into account anything
- In the industry – historical

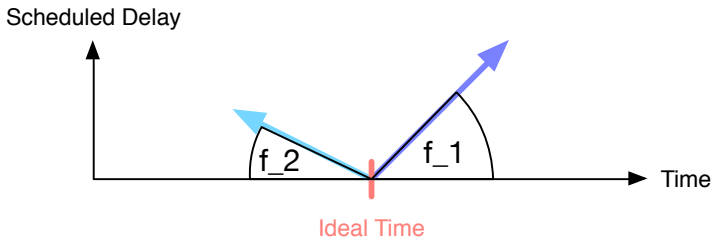
Update of Planning



How to Measure Quality of a Timetable?



Being on Time



- Scheduled delay times value of time (Arnott et al. (1990))

The Rest

Speed (Time)

- Running time multiplied by the value of time (Axhausen et al. (2008))

Waiting Time

- Waiting time multiplied by the value of waiting time (Wardman (2004))

Transfers

- Minimum transfer time multiplied by the number of transfers and the value of waiting time (Wardman (2004))

References



- Arnott, R., de Palma, A. and Lindsey, R. (1990). Economics of a bottleneck, *Journal of Urban Economics* **27**(1): 111 – 130.
- Axhausen, K. W., Hess, S., König, A., Abay, G., Bates, J. J. and Bierlaire, M. (2008). Income and distance elasticities of values of travel time savings: New swiss results, *Transport Policy* **15**(3): 173 – 185.
- Wardman, M. (2004). Public transport values of time, *Transport Policy* **11**(4): 363 – 377.

Ideal Timetable



The ideal timetable consists of such train departures that the passengers' global costs are minimized, i.e. the fastest most convenient path to get from the origin to the destination traded-off by a timely arrival to the destination for every passenger.

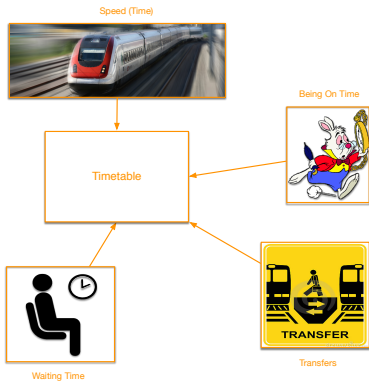
Inputs

- $i \in I$ – set of origin-destination pairs
- $t \in T$ – set of time steps t in the planning horizon
- $t' \in T^i$ – set of ideal times for OD pair i
- $l \in L$ – set of operated lines
- $v \in V^l$ – set of available vehicles on line l
- $p \in P^i$ – set of possible paths between OD pair i
- $l \in L^p$ – set of lines in the path p
- r_i^{pl} – running time between OD pair i on path p using line l
- h_i^{pl} – time to arrive from the starting station of the line l to the origin of the pair i
- $D_i^{t'}$ – demand between OD i with ideal time t'
- m – minimum transfer time
- c – cycle
- q_1 – value of the waiting time
- q_2 – value of the in vehicle time
- f_1 – coefficient of being early
- f_2 – coefficient of being late

Decisions

- $C_i^{t'}$ – the total cost of the passengers with ideal time t' between OD pair i
- $W_i^{t'}$ – the total waiting time of the passengers with ideal time t' between OD pair i
- $W_i^{t',p}$ – the total waiting time of the passengers with ideal time t' between OD pair i using path p
- $W_i^{t',pl}$ – the waiting time of the passengers with ideal time t' between OD pair i on the line l that is part of the path p
- $X_i^{t',p}$ – 1 – if the passengers with ideal time t' between OD pair i choose path p ; 0 – otherwise
- $S_i^{t'}$ – the final scheduled of the passengers with ideal time t' between OD pair i
- $S_i^{t',p}$ – scheduled delay of the passengers with ideal time t' between OD pair i traveling on the path p
- d_v^l – the departure time of a train v on the line l
- $y_i^{t',plv}$ – 1 – if the passengers with ideal time t' between OD pair i on the path p take the train v on the line l ; 0 – otherwise
- z_v^l – frequency within cyclicity

Objective



$$\min \sum_{i \in I} \sum_{t' \in T^i} D_i^{t'} \cdot C_i^{t'}$$

Pricing Constraints

$$\begin{aligned}
 C_i^{t'} &= q_1 \cdot w_i^{t'} + q_1 \cdot m \cdot \sum_{p \in P} x_i^{t'p} \cdot (|L^p| - 1) \\
 &+ q_2 \cdot \sum_{p \in P} \sum_{l \in L^p} r_i^{pl} \cdot x_i^{t'p} + q_2 \cdot \sum_{p \in P} s_i^{t'}, \quad \forall i \in I, \forall t' \in T^i, \\
 w_i^{t'} &\geq w_i^{t'p} - M \cdot (1 - x_i^{t'p}), \quad \forall i \in I, \forall t' \in T^i, \forall p \in P^i, \\
 w_i^{t'p} &\geq \sum_{l \in L^p \setminus 1} w_i^{t'pl}, \quad \forall i \in I, \forall t' \in T^i, \forall p \in P^i,
 \end{aligned}$$

$$\begin{aligned}
 w_i^{t'pl} &\geq \left((d_v^l + h_i^{pl}) - (d_{v'}^{l'} + h_i^{pl'} + r_i^{pl'} + m) \right) \\
 &- M \cdot (1 - y_i^{t'plv'}) - M \cdot (1 - y_i^{t'plv}), \quad \forall i \in I, \forall t' \in T^i, \forall p \in P^i, \forall l \in L^p : \\
 &l > 1, l' = l - 1, \forall v \in V^l, \forall v' \in V^{l'},
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$$w_i^{t'pl} \geq M \cdot \sum_{l \in L^p} y_i^{t'pl} (V^{l+1}), \quad \forall i \in I, \forall t' \in T^i, \forall p \in P^i,$$

$$s_i^{t'} \geq s_i^{t'p} - M \cdot (1 - x_i^{t'p}), \quad \forall i \in I, \forall t' \in T^i, \forall p \in P^i,$$

$$s_i^{t'p} \geq f_2 \cdot \left((d_v^{|L|} + h_i^{|L|} + r_i^{p|L|}) - t' \right) - M \cdot (1 - y_i^{t'p|L|v}), \quad \forall i \in I, \forall t' \in T^i, \forall p \in P^i, \forall v \in V^{|L|},$$

$$s_i^{t'p} \geq f_1 \cdot \left(t' - (d_v^{|L|} + h_i^{|L|} + r_i^{p|L|}) \right) - M \cdot (1 - y_i^{t'p|L|v}), \quad \forall i \in I, \forall t' \in T^i, \forall p \in P^i, \forall v \in V^{|L|},$$

Pricing Constraints

$$c_i^{t'} = q_1 \cdot w_i^{t'} + q_1 \cdot m \cdot \sum_{p \in P} x_i^{t'p} \cdot (|L^p| - 1)$$

$$+ q_2 \cdot \sum_{p \in P} \sum_{l \in L^p} r_i^{pl} \cdot x_i^{t'p} + q_2 \cdot \sum_{p \in P} s_i^{t'}, \quad \forall i \in I, \forall t' \in T^i,$$

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$$s_i^{t'p} \geq f_1 \cdot \left(t' - (d_v^{|L|} + h_i^{|L|} + r_i^{p|L|}) \right) - M \cdot (1 - y_i^{t'p|L|v}), \quad \forall i \in I, \forall t' \in T^i, \forall p \in P^i, \forall v \in V^{|L|},$$

Pricing Constraints

$$\begin{aligned}
 c_i^{t'} &= q_1 \cdot w_i^{t'} + q_1 \cdot m \cdot \sum_{p \in P} x_i^{t'p} \cdot (|L^p| - 1) \\
 &+ q_2 \cdot \sum_{p \in P} \sum_{l \in L^p} r_i^{pl} \cdot x_i^{t'p} + q_2 \cdot \sum_{p \in P} s_i^{t'p}, \quad \forall i \in I, \forall t' \in T^i, \\
 w_i^{t'} &\geq w_i^{t'p} - M \cdot (1 - x_i^{t'p}), \quad \forall i \in I, \forall t' \in T^i, \forall p \in P^i, \\
 w_i^{t'p} &\geq \sum_{l \in L^p \setminus 1} w_i^{t'pl}, \quad \forall i \in I, \forall t' \in T^i, \forall p \in P^i,
 \end{aligned}$$

$$\begin{aligned}
 w_i^{t'pl} &\geq \left((d_v^l + h_i^{pl}) - (d_{v'}^{l'} + h_i^{pl'} + r_i^{pl'} + m) \right) \\
 &- M \cdot (1 - y_i^{t'plv'}) - M \cdot (1 - y_i^{t'plv}), \quad \forall i \in I, \forall t' \in T^i, \forall p \in P^i, \forall l \in L^p : \\
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 w_i^{t'} &\geq w_i^{t'p} - M \cdot (1 - x_i^{t'p}), \quad \forall i \in I, \forall t' \in T^i, \forall p \in P^i, \\
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Feasibility Constraints

$$\sum_{p \in P^i} x_i^{t'p} = 1, \quad \forall i \in I, \forall t' \in T^i,$$

$$\sum_{v \in V^{l+1}} y_i^{t'plv} = 1, \quad \forall i \in I, \forall t' \in T^i, \forall p \in P^i, \forall l \in L^p,$$

$$d_v^l - d_{v-1}^l = c \cdot z_v^l, \quad \forall l \in L, \forall v \in V : v > 1,$$

domain

constraints

Feasibility Constraints

$$\sum_{p \in P^i} x_i^{t'p} = 1, \quad \forall i \in I, \forall t' \in T^i,$$

$$\sum_{v \in V^{l+1}} y_i^{t'plv} = 1, \quad \forall i \in I, \forall t' \in T^i, \forall p \in P^i, \forall l \in L^p,$$

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domain *constraints*

Feasibility Constraints

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$$d_v^l - d_{v-1}^l = c \cdot z_v^l, \quad \forall l \in L, \forall v \in V : v > 1,$$

domain *constraints*

Feasibility Constraints

$$\sum_{p \in P^i} x_i^{t'p} = 1, \quad \forall i \in I, \forall t' \in T^i,$$

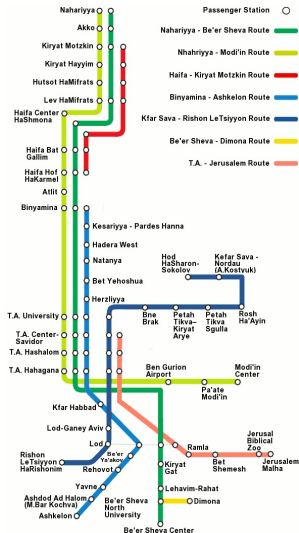
$$\sum_{v \in V^{l+1}} y_i^{t'plv} = 1, \quad \forall i \in I, \forall t' \in T^i, \forall p \in P^i, \forall l \in L^p,$$

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domain

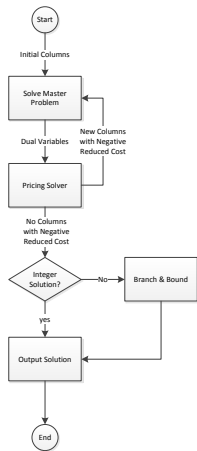
constraints

Case Study – Israel



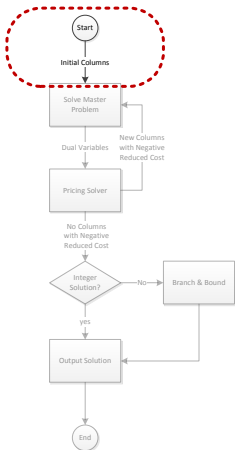
- OD Matrix for an average working day (Sunday to Thursday) in Israel during 2008
- 47 Stations
- 2162 ODs
- 36 (unidirectional) lines
- 389 trains
- Min. transfer – 4 mins
- VOT – 21.12 NIS per hour

Too Heavy – Branch-and-Price Framework



- Initial Solution
- Column Generation – Lower Bound
- Branch and Bound – Optimal Integer Solution

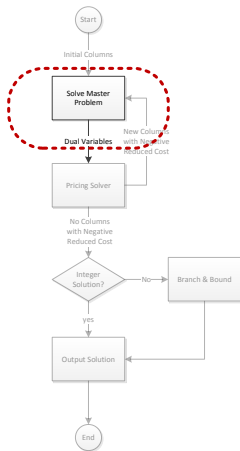
Initial Solution



The screenshot shows a mobile application titled 'Train Timetable' at 11:31. The interface displays a list of train routes and their schedules:

Route	Destination	Time	Station
Petah Tikva	Kiryat Arye towards South	11:25	Nahariyya
		11:31	Hod HaSharon
		11:45	Akko
Tel Aviv - University	towards North	12:11	Hod HaSharon
		12:11	Binyamina
		12:25	Nahariyya
		12:31	Hod HaSharon
		12:45	Haifa Center HaShmona

Master Problem (MP)

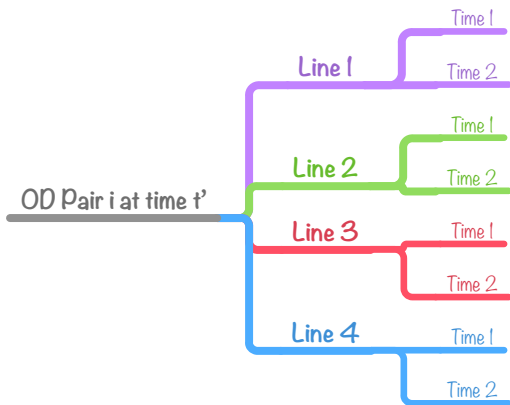
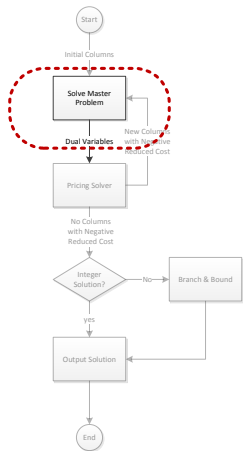


Idea

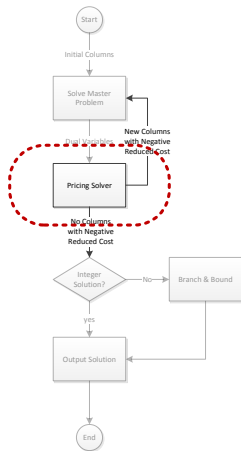
- Formulated as a Set Partitioning Problem
- Decision variables are relaxed, solution space restricted
- Consists of the feasibility constraints

▶ Go to model

MP – Columnn



Sub-Problem (SP)



Idea

- run for each OD pair i , each ideal time $t' \in T^i$ and each path $p \in P^i$ separately
- Consists of the pricing constraints

► Go to model

Dual Variables

- $\alpha_i^{t'}, \beta_j^t$

Results – 6 a.m. to 8 a.m.

Current [NIS]	Column Generation					Cyclic	Non-Cyclic
	# iter.	time	Ω_1	lb [NIS]	decrease [%]	ub [NIS]	ub [NIS]
220 279.32	2	48m 19s	228 859	132 371.26	40	209 791.62	204 770.06
218 564.49	2	48m 15s	230 941	131 582.85	40	207 717.86	202 209.67
219 271.56	2	46m 12s	233 627	131 420.41	40	207 886.98	201 949.28
217 644.50	2	48m 01s	228 849	130 494.59	40	208 275.88	203 133.08
213 957.08	2	45m 00s	226 772	127 586.58	40	205 328.12	199 417.99

Conclusions

- New planning phase based on the demand
- In line with the new market structure
- Can handle both non- and cyclic timetables
- Takes care of the connections, in the current practice:
 - non-cyclic – does not exist
 - cyclic – always imposed
- Returns ideal timetables, its cost and the routings of the passengers

Future Work

- Unsolvable using exact method
- Brute force algorithm to evaluate current timetable
- Heuristic to solve the whole day



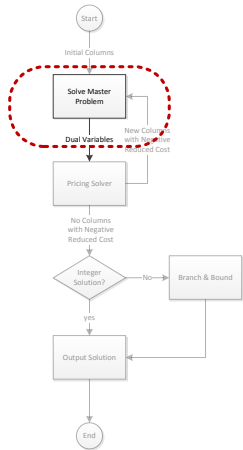
Thank you for your attention.

MP – Inputs

[◀ Go back](#)

- $a \in \Omega$ – set of all possible assignments
- $i \in I$ – set of origin-destination pairs
- $t \in T$ – set of all time steps
- $t' \in T^i$ – set of times that there is a demand between OD i
- $l \in L$ – set of operated lines
- c – cycle
- C_a – cost of the assignment a
- D_a – demand using assignment a
- n_l – number of available train units on line l
- $B_a^{it'}$ = $\begin{cases} 1 & \text{if OD pair } i \text{ at time } t' \text{ is assigned in assignment } a, \\ 0 & \text{otherwise.} \end{cases}$
- E_a^{lt} = $\begin{cases} 1 & \text{if the assignment } a \text{ is using line } l \text{ at time } t, \\ 0 & \text{otherwise.} \end{cases}$

MP – Decisions



λ_a - $\begin{cases} 1 & \text{if assignment } a \text{ is a part of the solution,} \\ 0 & \text{otherwise.} \end{cases}$

x_l^t - $\begin{cases} 1 & \text{if there is a train scheduled on line } l \text{ at time } t, \\ 0 & \text{otherwise.} \end{cases}$

MP

$$\begin{aligned} \min \quad & \sum_{a \in \Omega} D_a \cdot C_a \cdot \lambda_a \\ & \sum_{a \in \Omega} B_a^{it'} \cdot \lambda_a = 1, & \forall i \in I, \forall t' \in T^i, \\ & \sum_{a \in \Omega} E_a^{lt} \cdot \lambda_a \leq x_l^t, & \forall l \in L, \forall t \in T, \\ & \sum_{t \in T} x_l^t \leq n_l, & \forall l \in L, \\ & \sum_{t''=t}^{\min(t+c, T)} x_l^{t''} \leq 1, & \forall l \in L, \forall t \in T, \\ & \lambda_a \in \{0, 1\}, & \forall a \in \Omega, \\ & x_l^t \in \{0, 1\}, & \forall l \in L, t \in T. \end{aligned}$$

MP

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MP

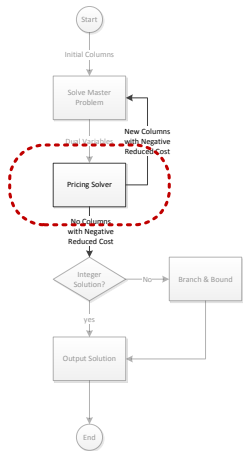
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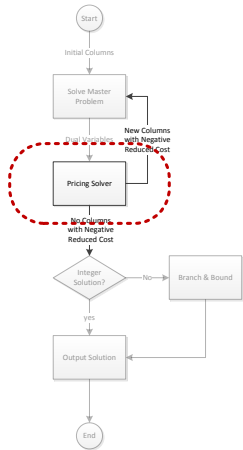
SP – Inputs

◀ Go back



- i – the origin destination pair
- t' – ideal travel time for OD pair i
- p – path of the sub-problem
- $t \in T$ – set of all time steps
- $l \in L^p$ – the sequence of lines used to get from the origin to the destination
- r_l – running time of line l
- h_l – running time to get from the starting station of the line l to the first station on the same line included in the current path
- m – the minimum transfer time
- q_1 – the value of time spent waiting
- q_2 – the value of time spent in vehicle
- f_1 – coefficient of being early
- f_2 – coefficient of being late

SP – Decision Variables



β_l^t – $\begin{cases} 1 & \text{if line } l \text{ is used at time } t, \\ 0 & \text{otherwise.} \end{cases}$

w – the total waiting time of the passengers

w_l – the waiting time of the passengers when transferring to line l

s – scheduled delay of the passengers

C – the cost of the passengers

SP – Model

$$\min C - \left(\alpha + \sum_{l \in L} \sum_{t \in T} \beta_l^t \cdot \text{beta}_l^t \right)$$

$$C = \left(q_1 \cdot w + q_1 \cdot m \cdot (|L| - 1) + q_2 \cdot \sum_{l \in L} r^l + q_2 \cdot s \right)$$

$$\sum_{t \in T} \text{beta}_l^t = 1, \quad l \in L^P,$$

$$w = \sum_{l \in L \setminus 1} w_l,$$

$$w_l \geq (t \cdot \text{beta}_l^t + h_l) - (t'' \cdot \text{beta}_{l-1}^{t''} + h_{l-1} + r_{l-1} + m), \quad \forall l \in L^P : l > 1, \forall t, t'' \in T :$$

$$t \geq t'' + h_{l-1} + r_{l-1}$$

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$$s \geq f_2 \cdot \left((t \cdot \text{beta}_{|L|}^t + h_{|L|}) - t' \right), \quad \forall t \in T,$$

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domain constraints

SP – Model

$$\min C = \left(\alpha + \sum_{l \in L} \sum_{t \in T} \beta_l^t \cdot \text{beta}_l^t \right)$$

$$C = \left(q_1 \cdot w + q_1 \cdot m \cdot (|L| - 1) + q_2 \cdot \sum_{l \in L} r^l + q_2 \cdot s \right)$$

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domain *constraints*

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