Numerical Modelling Approaches for Existing Masonry and RC Structures

Alexandre A. Costa, Bruno Quelhas and João P. Almeida

Abstract Assessment of existing buildings making use of numerical simulation methods, even under the hypothesis of full knowledge of current conditions and materials, it is not an easy and straightforward task due to the limitations and complexities of such analysis tools. In this chapter, a discussion of different approaches for the simulation of structural response is introduced and applied to two of the most common building typologies: masonry structures and reinforced concrete frames. Following a brief introduction of the problematic, an overview of different modelling possibilities for masonry structures is presented. Afterwards, choices made during numerical modelling are discussed, based mainly on the finite element method. Moreover, the problematic of different modelling techniques is addressed, where some paths and best practices are suggested. The last section is devoted to the response simulation of reinforced concrete structures. Efficient frame elements and sectional models, which allow capturing an extended range of elastic and inelastic response, are analysed first. Strut-and-tie modelling is then recalled as a powerful analysis tool, and its application to the assessment of old buildings is studied.

Keywords Masonry • Modelling strategies • Finite elements • Strut-and-tie • Frame elements • Sectional fibre analysis

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1 On the Structural Response of Existing Constructions

The study of existing constructions and their actual behaviour, even for gravity loads, is a difficult task, being however a key point on current engineering practices and scientific research.

When dealing with existing constructions and their numerical simulation, the next points should be respected and taken into consideration, in order to minimize uncertainties and increase the confidence on the developed numerical models, namely:

(i) Overall characterization: all the structural elements must be surveyed (vertical and horizontal ones, including foundations and roof) for a correct definition of the elements' geometry and constituent materials, as well as existing loads applied on the structure;
(ii) Detailed survey of the structural elements: quantification of reinforcement on reinforced concrete (RC) elements and corresponding distribution (amount of longitudinal reinforcement, stirrups diameter and spacing, etc.), detection of design and/or construction errors, eventual cracking pattern and apparent causes (overburden, design errors, foundation settlement, etc.);
(iii) Construction detailing (especially for seismic assessment): connection between horizontal and vertical elements, structural detailing at beam-column joints, connection between perpendicular load-bearing walls and existence of out-of-plane devices (for masonry structures only) such as tie-rods, floor connections to the walls, RC ring beams, etc.

After the first stage of inspection and diagnosis, which is essential for the development of the numerical model on the actual conditions of the structure, a first problematic may arise related to material characterization. Current codes are in general not very helpful since they mainly focus on the design of new structures, while poor guidance is offered to the analysis of existing buildings.

Since the numerical simulation of structural behaviour is directly related to the material properties, the second stage should cope with in situ material testing to estimate boundaries of their mechanical characteristics. Despite some current attempts made towards this purpose (e.g. [1–4]), the amount of information and experimental tests required to obtain a good confidence level may lead to an unrealistic number of experiments (e.g. core drilling) to perform on single elements or the whole structure, disrespecting its integrity or even its future usage.

On the other hand, the material characterization of existing masonry structures is a major problem because current non-destructive techniques (NDT, as sonic tests or tomography) and even minor-destructive (as the flat-jack technique) may not give relevant results (usually only the Young's modulus) when applied to multi-leaf stone masonry walls or irregular masonry. Only destructive testing methods, as in situ vertical or diagonal compression tests, may give pertinent results (as maximum compressive strength) for the numerical model. Great care

should be taken when dealing with existing constructions and unknown materials, belonging to the engineer the decision on material properties and material safety factors to be used in the model. Improper preparation of technicians for a correct assessment of existing structures leads usually to erroneous and/or too intrusive interventions, also correlated with the analytical and/or numerical approach followed.

Modal analysis is an easy and straightforward technique which may reduce the uncertainties of numerical modelling within the elastic range of the structure. Indeed, a good numerical model should be always calibrated based on the eigenfrequencies and mode shapes resulting from experimental modal identification, which can be carried out using simple ambient vibration tests.

On RC structures, this permits to infer the current load conditions of the structure, the influence of the surrounding constructions on its dynamic properties (if dealing with adjoining buildings), but more important than that, its current elastic modulus and current dynamic mass, as well as the influence of masonry infills in the global behaviour.

Modal analysis is even more important for masonry structures because it allows, in addition to the calibration of Young's modulus and current masses, the perception of existing connections between all horizontal and vertical elements. In this manner, it is possible to understand the current loading conditions of the structure, and also to simulate a strengthening intervention to cope with eventual deficiencies. Concerning the seismic behaviour, the evaluation of the existence of connections between horizontal and vertical elements (yielding the so-called "box behaviour", which modal analysis can help to identify), and despite the connections' unknown efficiency for higher excitation levels, is mandatory for a thorough structural assessment.

In the work presented by Ilharco et al. [5], the modal identification permitted to observe the most vulnerable part of an existing three-storey building, Fig. 1a, as well as to identify the absence of an efficient connection between horizontal (timber floors, not represented in the figures) and vertical elements (load-bearing masonry walls), Fig. 1b. Local mode shapes were observed in both parallel façades without frequency or phase correlation, meaning that the floor was not

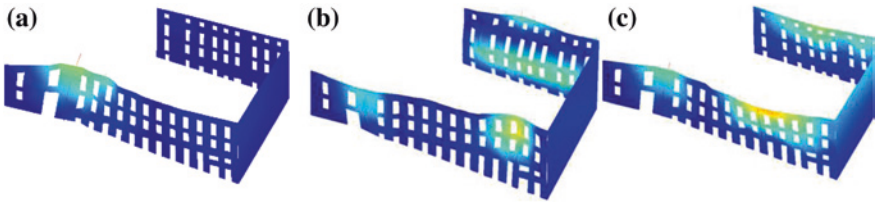

Fig. 1 Local mode shapes of an existing building [5]: **a** major vulnerable area; **b** lack of connection between timber floors and exterior masonry wall; **c** good connection and in-plane stiffness of timber roof

connecting both façades at that level, and mobilized only portions of the façades. On the other hand, good connection at the roof was observed as well as a mode shape governed by its in-plane behaviour, Fig. 1c, connecting parallel façades of the building. It is possible to conclude the above because both façades are mobilized in the same direction and with equal frequency of vibration, meaning that the roof was acting as an effective connection. This information was important for the design of the strengthening intervention. In general, the design of rehabilitation and/or strengthening techniques should thus be based on a comprehensive and exhaustive characterization of the structures' current conditions.

Despite experimental advances on material characterization and assessment of actual structural conditions, as expressed in the previous lines, the engineering experience and judgment plays the main role when dealing with rehabilitation interventions, criticizing and eventually rejecting the results obtained with the numerical models.

As an attempt to minimize the influence of engineering judgment and experience in the final decisions, the following paragraphs present some recommendations and modelling strategies to perform better and more efficient numerical simulations of existing constructions.

2 Numerical Assessment of Existing Masonry Structures

Masonry is one of the oldest structural material still in use; it has been applied on a huge diversity of constructions throughout the world. It is by nature a heterogeneous material whose components present a quite unknown geometry and a high mechanical variability. The structural behaviour of masonry depends on several factors such as member geometry; the characteristics of its texture; the physical, chemical and mechanical properties of its components and finally, the characteristics of masonry as a composite material [6].

All of the above mentioned factors make the analysis of the stone masonry mechanical behaviour a very complex matter. This is why it is of great interest the development and calibration of effective modelling and analysis strategies capable of predicting the behaviour of stone masonry structures, in particular under cyclic loads for seismic assessments. For this, it is necessary to accurately characterize this type of material through experimental testing. However, achieving a good characterization of masonry structures, detailed enough to be used confidently on the simulation, is, most of the times, a very demanding task, both in terms of cost and time [7].

The mechanical in situ characterization through non- and minor-destructive tests (sonic tests, flat jacks, etc.) gives precious information, almost without damaging the buildings, but with arguable global representation, while performing destructive tests on existing structures, either in situ or by removing samples large enough to be representative, is most often not possible especially when the structures have a high cultural value. As so, laboratory tests on masonry specimens representative of real constructions appear as a feasible alternative.

All the previous testing techniques are very useful to characterize masonry within its linear or non-linear range, to be used with different analysis methods.

2.1 Analysis Methods

To analyse the behaviour of existing masonry structures, there are nowadays several methods and computational tools that are based on different theories and strategies, resulting in different levels of complexity, different calculation times and, of course, different costs.

Simple analytical models may be used to assess the existing conditions of masonry structures, such as the simple rule of thumb, limit equilibrium analysis, arch theory, kinematic analysis, among others.

On the other hand, the analysis of existing masonry structures concerning seismic behaviour can be performed mainly through four different methods: (i) linear static (or simplified modal), (ii) linear dynamic (typically multimodal with response spectrum), (iii) non-linear static ("pushover") and (iv) non-linear dynamic.

When opting for one method of analysis one must have clearly defined the desired type of analysis, its objectives and also the knowledge of the advantages and limitations of the available tools, bearing in mind that more complex analysis are not necessarily synonymous of better results [8]. Above all, an analysis must be informed and planned in order to maximize its simplicity. A practical analysis of existing masonry structures implies great simplifications in the creation of the model geometry; the technician responsible for the analysis has to assess what is or is not important for a given analysis.

Nowadays the technological and scientific development permits the execution of increasingly complex analysis in increasingly shorter times, often enhancing the detail and size of a model. However, this may cause loss of objectivity and

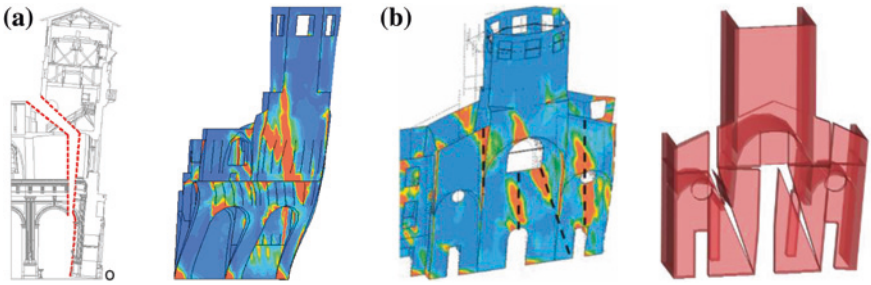

Fig. 2 Comparative analyses, using different methods, of the Cathedral of Santa Maria Assunta, Reggio Emilia (Italy) [9]: **a** Out-of-plane mechanism of the frontal façade; **b** In-plane mechanism of the frontal façade

result on a huge amount of information, rendering more difficult the analysis of the structural behaviour.

Given the large number of parameters of which depends the analysis of existing masonry structures and the degree of uncertainty and lack of knowledge that surrounds them, it is not appropriate and/or correct to propose a single method of analysis. In addition, on the process of choosing a method of analysis one must take into consideration the limitations and advantages of each method and the objectives of the analysis. As such, it is correct to propose analyses of a same problem using different methods, and the comparison between the results of each method can increase the degree of confidence in the obtained results. Several authors combined different modelling strategies, on the analysis of this type of structures, such as limit analysis and numerical modelling (Fig. 2). The numerical models allow individualizing macro-elements and their respective lines of collapse, as well as the formation of mechanisms on a structure, whose vulnerability and safety can be analysed through simplified limit analysis.

2.2 Modelling Strategies

Historically, structures began to be designed using simple rules of thumb based on the workers' experience. This method, although quite basic, was used in the construction of big and important structures such as bridges. After this, static graphics started being used; it is a quite simple method, which allows solving graphically the structural problems.

Later, methods based on the concept of limit analysis began to be employed. These methods assume that a structure is collapsing and compares the state of collapse with the actual condition of the structure, thus defining its structural safety. The main advantage of this type of analysis is the combination of the knowledge on the mechanisms and failure loads with the simplified implementation on practical computational tools. The number of required material parameters is thus reduced to a minimum, which is convenient as this type of parameters have a high degree of uncertainty, being very difficult to find reliable information [10].

Only recently, due to the high (and ever increasing) capacity of modern computers to solve numerical problems, it became possible to simulate the response of materials, such as masonry, considering their non-linear structural behaviour. Several methods, such as the Finite Element Method (FEM), were then implemented. The basic unit that characterizes this method, the finite element, usually does not represent a structural member but rather one of its sub-parts. Thus, the FEM can be applied to simulate separately the behaviour of various materials of the elements that compose the assembly (micro-modelling), or to simulate in a homogenized and continuous way the global behaviour of a composite material (macro-modelling).

Apart from the micro- and macro-modelling techniques, there is also the so-called homogenized modelling. In this type of approach the structure is composed

by a finite repetition of an elementary cell, and the masonry is seen as a continuum whose constitutive relations are derived from the characteristics of its individual components (blocks and mortar), and from the geometry of the elementary cell. In recent years, some of these homogenization techniques have been developed and applied by several researchers (e.g. [11, 12]).

Other modelling methods widely used are the Structural Element Models (SEM), the Discrete Element Method (DEM) and its new formulation, and the Discontinuous Deformation Analysis (DDA). These last two methods are extensively used on rock mechanics, which in many cases is quite similar to masonry problems. They are also quite useful on the analysis of the failure mechanisms of masonry structures.

2.3 Finite Element Method

The Finite Element Method (FEM) is one of the most used approaches for the modelling of structures. It offers a widespread variety of possibilities concerning the description of the masonry structures within the frame of detailed nonlinear analysis. Most of modern possibilities based on the FEM fall within two main approaches: modelling at the micro level, considering the material as discontinuous, or at the macro level. Hybrid models can also be created, which have considerable interest when, for example, one intends to analyse in detail a specific structural element within a more complex structure.

2.3.1 Micro-modelling

Some authors, such as Costa et al. in [13] on the analysis of stone masonry structures through the FEM, used a detailed micro-modelling approach reducing the masonry to its basic components (joints, blocks and infill), Fig. 3. The units and the mortar at joints are described using continuum finite elements, whereas the

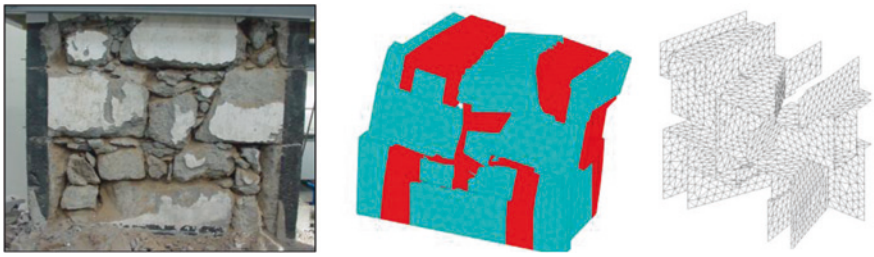

Fig. 3 Micro-modelling using finite elements—replica of an existing wall from Azores [13]

unit-mortar interface is represented by discontinuous elements accounting for potential crack or slip planes.

This type of detailed modelling has a greater degree of accuracy; however, it is inevitably accompanied by an increase in the calculation time and modelling effort, which makes this simulation strategy unpractical for common engineering practice. Nonetheless, it is suitable for the study of localized areas and effects, provided that there is detailed knowledge of the geometry and composing elements and materials. It is particularly adequate to describe the local response of the material. Elastic and inelastic properties of both unit and mortar should be realistically taken into account.

The detailed micro-modelling strategy leads to very accurate results, but requires an intensive computational effort. This drawback is partially overcome by the simplified micro-models. Some authors (e.g. [14, 15]) opt for this simplified micro-modelling strategy, which is characterized by the combination, or omission, of certain constituents, allowing to drastically reducing the computation time without a great loss of accuracy.

The primary aim of the micro-modelling approaches is to closely represent masonry based on the knowledge of the properties of each constituent and the interface. The necessary experimental data for calibration must be obtained from laboratory tests on the constituents and small masonry samples.

2.3.2 Macro-modelling

Still in the field of finite element modelling, some authors opt for the macro-modelling using macro mechanical models, also known as homogeneous or continuous, in which all elements of an assembly of materials are incorporated into a continuum, wherein it is established a relation between the average extensions and stresses of the masonry. These relations are obtained by adopting a phenomenological point of view or by using homogenization techniques.

Macro-modelling is probably the most popular and common approach due to its lesser calculation demands. In practice-oriented analyses on large structural members or full structures, a detailed description of the interaction between units and mortar may not be necessary. In these cases, macro-modelling, which does not make any distinction between units and joints, may offer an adequate approach to the characterization of the structural response.

The macro-models have been extensively used with the aim of analysing the seismic response of complex masonry structures, such as arch bridges (e.g. [16]), historical buildings (e.g. [17]), and mosques and cathedrals (e.g. [18, 19]).

The smeared crack scalar damage models or other similar models, such as those presented in [20, 21], are often used in macro-modelling of masonry. This type of models, where the damage is defined in a given point by a scalar value which defines the level of material degradation (ranging from the elastic state until collapse), and the cracking is considered as distributed along the structure, are commonly used in the modelling/analysis of reinforced concrete structures (e.g. [22]) or large volumes of

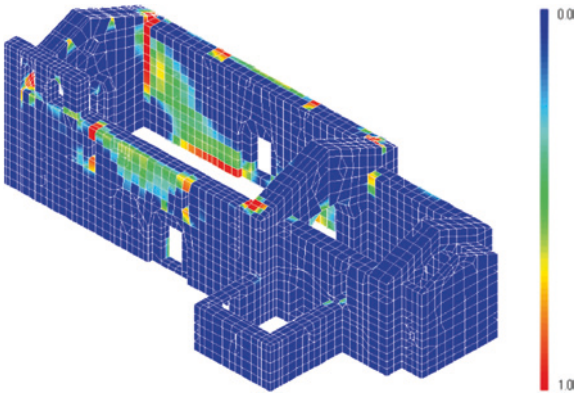

Fig. 4 Macro-modelling of the Gondar church using Finite Elements based on a damage model [25], Tensile damage map (d^+)

concrete (e.g. [20]). Such models were later adapted for the use in masonry buildings through the work of different authors, such as [23, 24] or [25] (Fig. 4).

In terms of applicability, it is a type of modelling clearly suitable when factors such as time, simplicity of modelling and computational capacity are crucial. Further, it is oriented for the everyday use in the analysis of real structures and when there is the need of maintaining a balance between accuracy and speed/efficiency [8].

2.4 Modelling Problematic

For what concerns the available analysis methods, the linear elastic analysis is the most commonly used in current practice, due to its advantages in terms of computational time. However, its application to masonry structures is, in principle, inadequate because it does not take into account the non-tension response or non-linear sliding-shear, among other essential features of masonry behaviour. It must be noted that, due to its very limited capacity in tension, masonry shows a complex non-linear response even at low or moderate stress levels. Moreover, simple linear elastic analysis cannot be used to simulate masonry strength responses, typically observed in arches and vaults, characterized by the development of sub-systems working in compression. Attempts to use linear elastic analysis to design arches may result in very conservative or inaccurate approaches. Linear elastic analysis is not useful, in particular, to estimate the ultimate response of masonry structures and should not be used to conclude on their strength and structural safety.

Notwithstanding, linear elastic analysis has been used, with partial success, as an auxiliary tool assisting in the diagnosis of large masonry structures. Easy availability and reduced computer costs have promoted its use, in spite of the mentioned limitations, before the development and popularization of more powerful computer applications. Some examples are the studies of San Marco's Basilica in

Venice by [26], the Metropolitan Cathedral of Mexico [27], the Tower of Pisa [28], the Colosseum of Rome [29] and the Church of the Güell Colony in Barcelona [30], among many others. In all these cases, the limitations of the method were counterbalanced by the very large expertise and deep insight of the analysts.

Whenever possible, it is preferable to individually analyse the elements of a structure, simplifying its geometry and reducing the computation effort. In this case it may be advisable to use 2D models instead of the 3D ones. In cases where it is considered important to use the complete model of the structure, it is advised the use of mixed complexity, detailing (from the geometrical and/or material behaviour point of view) the areas of major interest, with higher influence on the overall behaviour of the structure or critical areas that present particular problems to be analysed.

For what regards modelling with finite elements, in the case of continuous elements such as masonry walls, it is frequent to use shell elements, since these present advantages in terms of creation of the geometrical model and of computational effort. However, certain precautions need to be taken into account and several difficulties arise when using this type of element, such as:

- the use of this type of element does not allow considering, in a direct manner, the load or support eccentricity effect, since the shells are aligned with the axis. In cases where there are large variations in thickness on a structure, it is important to take into account this phenomenon, in particular, when the floors are supported only in one of the leaves of a multiple-leaves wall panel;
- the use of such elements makes it impossible to consider directly the phenomenon of leaves' separation, i.e. it can only be used when this type of phenomenon is not important to analyse the problem, or when the structural element in question is monolithic;
- in finite elements, the stress distribution along the thickness of a wall is linear, which may deviate from reality, at a local level;
- in such elements, the stiffness in the areas of intersection of two shells (e.g. facade angles) is not always well represented.

The use of volumetric elements allows reproducing, in a more realistic way, the intersection zones of structural elements. By using this approach it is possible to evaluate the stresses in the thickness of a wall; however, more than one element to discrete the mesh along the thickness has to be used, otherwise the errors will be large, and the greater the thickness of the actual element the greater the error. By contrast, the use of volumetric elements makes the creation of the geometric model more complex and time consuming.

The modelling of the links between elements of a masonry structure is almost always one of the most important numerical modelling problems of historical structures. This difficulty arises not only from the difficulty in defining them appropriately, but also from the choice of numerical models capable of realistically characterizing these links. For a question of simplicity, in most cases the connections are considered as continuous and fixed, but this type of assumption is only valid in certain cases and it is the responsibility of the technician in charge of the analysis to evaluate this assumption in each case.

In the particular case of the support conditions of a structure on the soil, most of the times the structure is considered as fixed at the base. However, this assumption is only valid in the cases where the soil presents good quality and the wall foundation can be considered as properly fixed to the soil. Otherwise, the soil should be modelled as well, for example with springs at the base featuring equivalent stiffness characteristics to the ones presented by the considered type of soil.

In the modelling of wooden roofs reinforced with metal rods, only the tensile resistance must be considered on the ties, using appropriate models in order to render the analysis more realistic.

3 Reinforced Concrete Structures

Today, most (if not all) structural engineering offices use software—typically based on the Finite Element Method as mentioned in the previous section, but not only—to assess the structural condition of old RC buildings and rehabilitation interventions that may be required. The present section deals with some of the numerical models that are often used for such intent.

The number of non-linear mechanisms that describe the behaviour of RC in its cracked state is countless. It ranges from compression and tension softening to creep, shrinkage, core confinement effects, aggregate interlock, rebar bond slip, dowel action, tension stiffening, scale effects, and so on. The role of each of those mechanisms in the strength and deformation capacity of the structure is highly dependent on the type and magnitude of the applied loads (in the static, cyclic or dynamic regimes), as well as on the geometrical and mechanical member properties. Despite the current fast development of software for RC analysis, it is still virtually impossible to build a numerical model that simultaneously accounts for all the aforementioned mechanisms and load case scenarios. Consequently, the engineer is faced with the choice of selecting one (or a few) models that can provide a satisfactory insight into the problem at hand. That decision becomes yet more delicate in the context of assessment of old RC buildings, wherein additional physical phenomena—associated to material deterioration and other damage sources—may have to be simulated too.

Likewise, the selection of the modelling methods to present in these few pages is far from straightforward. Nevertheless, the very fact that RC buildings, from the force flow viewpoint, are essentially frame structures justifies addressing beam-column elements in the next section. Not only they are indispensable in almost every RC building model, but quite often also the only component used in assembling large models. The latter element can be suitably supplemented by a multipurpose analysis tool that, although applicable to the whole RC structure, is typically used near statistical or geometrical discontinuities: the strut-and-tie method, discussed in the subsequent section. Its versatility implies successful adaptation to the specificities of old buildings, in addition to modelling of strengthening systems. Finally, some closing remarks on other relevant physical phenomena and applicable simulation techniques are briefly addressed.

3.1 The Fundamental Role of Frame Elements

Among the major sources of deformation on reinforced concrete structures, member bending is typically the most important and should thus be always considered in the corresponding models. Other mechanisms of deformation, including member shear, second-order effects, anchorage slip, etc., are briefly referred in Sect. 3.3.

RC simulation should provide tools for reliable conventional assessment of existing buildings (at the serviceability and ultimate states). However, in earthquake-prone regions, their seismic vulnerability is often the conditioning factor of the overall assessment. Therefore, an ideal member model should be resourceful enough to capture the flexural behaviour of reinforced and prestressed concrete members in its various stages of linear and non-linear behaviour, both under static and cyclic loading. The present section recalls efficient models for sectional and element analysis that achieve the previous goal.

Due to the inherent 1D character of any beam theory, its displacement field can be decomposed into a component along the beam longitudinal axis, and another in the cross-section. The latter include the assumptions that make each beam theory unique, leading to a specific set of beam local compatibility and equilibrium equations, as well as their boundary counterparts.

In particular, the well-known Euler–Bernoulli hypothesis of plane sections remaining plane and perpendicular to the longitudinal axis gives rise to the most widely used beam theory for simulation of member flexural behaviour. Therein, the distribution of uniaxial normal strains is uniquely defined throughout the cross-section by the strain at the reference axis and the curvature (or curvatures about the two axes, if bi-axial bending is considered). The sectional behaviour can be best simulated by discretising the section into layers (or fibres—s in Fig. 5, for bi-axial bending), wherein it is assumed that the strain in each layer (fibre)

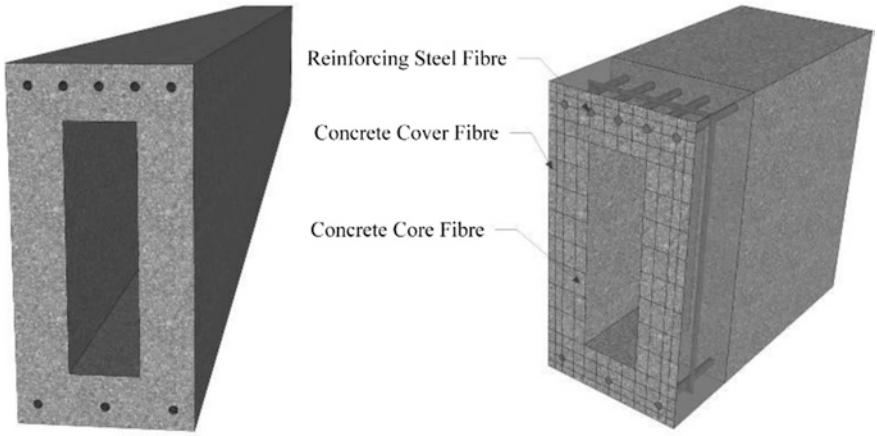

Fig. 5 Rectangular hollow RC section (*left*) and discretisation into concrete/steel fibres (*right*)

is uniform and equal to the strain at the centre of that layer (fibre). Making use of appropriate concrete and steel strain–stress relations, the corresponding normal stress is obtained. Their sectional integration is obtained by summation throughout the layers (fibres), yielding the axial force and moment(s) [31]. An advantage of the latter method is that the axial force-bending moment(s) interaction in both directions is directly simulated, as well as the post-peak softening response.

Alongside the sectional response, the other crucial point in the simulation of member behaviour is the choice of the element model. Shortly, the solution of the previously mentioned compatibility or equilibrium equations leads to two distinct types of frame finite element formulations: the classical displacement-based (DB), for long the most widely employed in structural analysis software, and the force-based (FB) approach, which now has stable and efficient state determination algorithms [32]. The former enforce a linear curvature and constant axial strain along the element. On the other hand, the latter assume a constant approximating polynomial for the axial force and a linear shape function for the bending moment. Under nodal element loading, it is thus apparent that the features of the FB formulation allow for a strict verification of equilibrium along the length, which holds irrespective of material constitutive behaviour. The DB element, on the other hand, only provides exact response for linear elastic behaviour. Furthermore, it can also be shown that span loading can be directly and exactly accounted for in FB formulations, unlike DB, and that no shear-locking phenomena exists. The first sensible consequence of the abovementioned features is that practitioners should give preference to FB over DB elements.

The strict verification of equilibrium is an advantage of fundamental relevance whenever the member response simulation involves highly non-linear material behaviour, such as under seismic loads (where cyclic uniaxial material laws must be utilized) or blast loads. It also implies that only one FB element is required to model each structural member, whilst a refined mesh of DB elements is called for to attain results of a comparable level of accuracy. Nevertheless, FB elements can also be very effectively employed to model structural behaviour in earlier stages of post-cracking response. The model's exact consideration of distributed loading, for instance, proves of the utmost significance for the computation of deflections for service load checks, wherein gravity and other span live loads control.

Consider a simply supported, uniformly loaded, eccentrically prestressed concrete beam. While the load is low, the beam will be curved upward. A single FB element with a layered section approach can be used to estimate the short-term camber. The long-term upward deflection—accounting for creep, shrinkage and relaxation—can also be straightforwardly predicted with the same model through adequate modification of the strain–stress relationships of the prestressing strands and concrete. For an increasing value of the uniformly distributed load, failure will eventually occur at mid-span, and once again the previous simple finite element scheme can be used to compute the corresponding deflection. Many strengthening techniques can also be modelled with the current method.

However, the element response also depends on the integration scheme and number of integration points where the sectional response is computed. The use

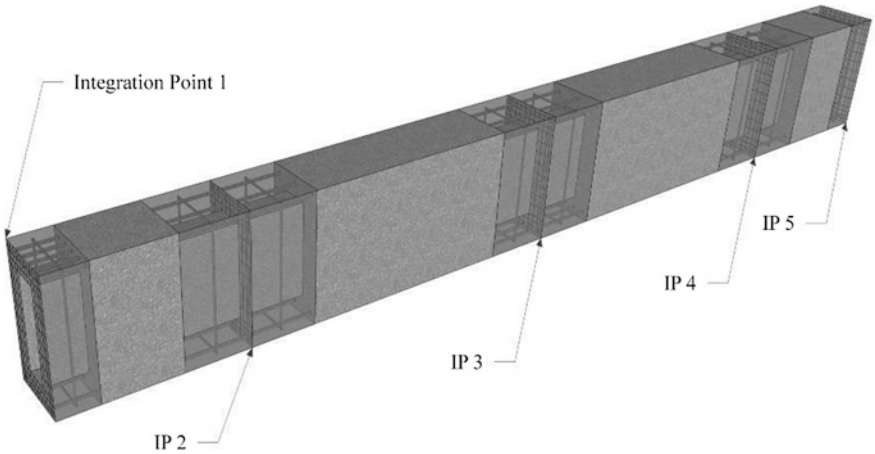

Fig. 6 Recommended lower bound for FB elements: five-point Gauss–Lobatto integration

of a Gauss–Lobatto integration scheme is advised as it controls the end sections of the element, which are privileged locations for the occurrence of inelastic behaviour. The number of integration points to consider is a question of required numerical accuracy: five can be roughly considered as the lower bound—see Fig. 6, but seven or even more may be advisable on occasion.

Finally, it is noted that if softening branches of the moment–curvature relation are attained (in one or more integration points), a pathological numerical behaviour known as strain-localisation takes place. As a consequence, the model results begin losing their physical meaningfulness and regularization techniques should be applied to restore it [33].

3.2 Strut-and-Tie in Assessment and Rehabilitation

It is most likely that the standards on which the design of an 'old' RC building was based no longer comply with current state-of-the-art principles, in terms of modelling and analysis methods, as well as detailing rules. Furthermore, it is also very probable that all computations were carried out by hand. Therefore, the influence of computer-aided methods of structural simulation—fundamentally connected to the development of the Finite Element Method—cannot be found.

Although many regular RC buildings can be entirely modelled by frame elements (henceforth denoted as B-regions, from Bernoulli or beam), there are also, in general, statical or geometrical discontinuity zones depicting two or three-dimensional stress states that require further special-purpose analysis and verification. In those so-called D-regions (from disturbed or discontinuity), the assumptions of the previously studied layered analysis do no longer apply since

plane sections do not remain plane, transverse strains may not be negligible, etc. Additionally, old structures often evidence distinct effects of damage, such as cracking patterns, deterioration of concrete, rebar corrosion, etc., and it would thus be most helpful to bring such information inside the modelling and analysis framework process. Finally, within the context of rehabilitation, simulation of possible strengthening solutions is also frequently required.

The so-called strut-and-tie modelling (STM) is a versatile tool that rationally tackles the aforementioned points (D-regions, damage and strengthening) and whose relevance has been recognized by incorporation into major international codes for structural concrete. Since the famous works by Marti [34] and Schlaich [35], almost 30 years ago, the STM has progressively established as a powerful analysis and design method—now increasingly computerized—that has replaced the empirical approaches historically applied in the design of D-regions.

Figure 7a depicts an example of division between B-regions—analysed in the previous section—and D-regions in a frame, assumed under in-plane loading. Typically, D-regions are areas under point or reaction loads, openings, re-entrant corners, frame joints, etc. Based on Saint–Venant's principle, the dimensions of a D-region can be taken within a distance corresponding to the largest of the depth or width of the member to either side of the disturbance, see Fig. 7b. Once D-regions are identified they can be isolated for analysis purposes.

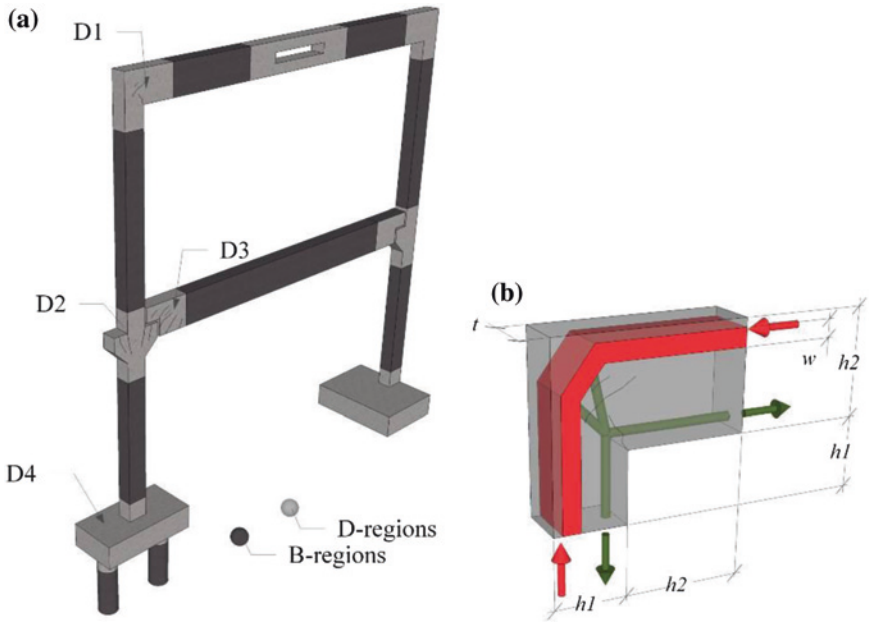

Fig. 7 **a** Illustrative identification of *B*-and *D*- regions in cracked frame structure; **b** STM for region *D1*, under gravity loads, with indication of geometrical properties

STM is based on the theory of plasticity and requires sketching, for each D-region, an internal truss (which consists of struts, ties and nodes) representing a statically admissible field. The latter has to be in equilibrium: (i) externally, with the applied loading and reactions (boundary forces from supports or adjacent B-regions); (ii) internally, at each node. The method produces a lower bound solution and therefore conservative predictions of the ultimate capacity of D-regions are obtained.

In general, ties represent one or multiple layers of reinforcing or prestressing steel. A strut represents a concrete compressive stress field whose centreline is in the same direction of the predominant principal stresses. It can have distinct shapes (prismatic, bottle-shaped, fan-shaped) [35], but in STM a uniform distribution is typically assumed. The effective cross-sectional area of the strut is given by $A_c = w \times t$, where t is the member thickness and w is the critical effective width, both indicated in Fig. 7b. Finally, nodes are regions where forces are transferred between struts and ties.

The stress limit of a strut (also known as effective strength), f_{cu}, is obtained from the uniaxial concrete compressive strength, f_c', and the so-called effectiveness factor, ν ($\leq$1.0), as follows: $f_{cu} = \nu\, f_c'$. The purpose of the previous effectiveness factor ν is to account for the limited deformation capacity of concrete, its post-peak softening behaviour, the strength reduction caused by the shape of the strut's stress field and disturbances due to cracks and tensile strains, and still the strength degradation that may arise during cyclic loading. The effectiveness factor can also consider strength enhancements from confinement, as that provided by transversal reinforcement. Assuming a uniformly distributed stress field, the capacity of the strut is given simply by $F_{cu} = A_c\, f_{cu}$.

On the other hand, the stress in the ties is limited by the yield strength of ordinary reinforcing steel, f_y, or prestressing steel, f_{py}. Hence, the capacity of the tie is computed by $F_{tu} = f_y\, A_s + \Delta f_p\, A_{ps}$, where A_s is the area of rebars, A_{ps} is the area of prestressing steel, and Δf_p is the difference between f_{py} and the installed prestressing steel stress.

It should be noted that more than one STM may be envisaged for each load case, as long as equilibrium is satisfied. However, attention is required in order to respect the limited deformation capacity of concrete. This is the most time-consuming phase of the process, often requiring iterations to optimize the model and satisfy stress limit criteria. The devised truss should stand as an idealization of the actual flow of tensile and compressive forces in the region. Whilst experience is advantageous, the following tips can also prove useful: (i) loads tend to reach the support through the shortest path, (ii) the nodal angle between struts and ties should be reasonably large to minimize strain incompatibilities caused by strut shortening and tie lengthening in almost the same direction, (iii) based on the principle of minimum strain energy after cracking and the smaller deformability of concrete struts in comparison to steel ties, Schaich et al. [35] proposed to select STMs that minimize the total length of the ties. However, other less subjective aids can be used in sketching the STM, as follows.

To start with, the engineer can and should take advantage of previously existing STMs associated with similar regions (under analogous loading conditions) that can be found in the literature, guidelines and standards. A common alternative is

to arrange the STM according to the elastic principal stresses produced by a finite element model, which will arguably satisfy both serviceability and ultimate limit states of D-regions; the deviation between the STM and the elastic solution should be kept within 15 degrees [35]. More recently, techniques for automatic generation of STMs, based on system performance criteria and topology optimization of continuum structures, have been developed [36, 37]. They are a few examples of the great increase in STM software, which is generalizing.

One should also look at the particulars of analysing existing members and at how damage inspection of old buildings can be used to calibrate appropriate STMs. The review of the building construction plans is invaluable, since they contain clear indications on the physical placement of the steel reinforcement and hence the centroid of the modelled ties can be accurately determined. In their absence, monitoring of crack patterns assumes extra pertinence. It is known that principal compressive stresses are parallel to cracks, which are caused by the orthogonal principal tensile stresses; therefore, struts should be set along the cracks' direction, and possibly centred in the cracked region width. Figure 7a and b, as well as Fig. 8a and c, help understanding the aforementioned relation. On the other hand, clear signs of concrete degradation, evidence of rebar corrosion or steel–concrete bonding problems, should also be duly accounted for in the STM. That can be achieved by readjustment of the truss or by an appropriate reduction (or even elimination) of the capacity of the concerned struts and ties.

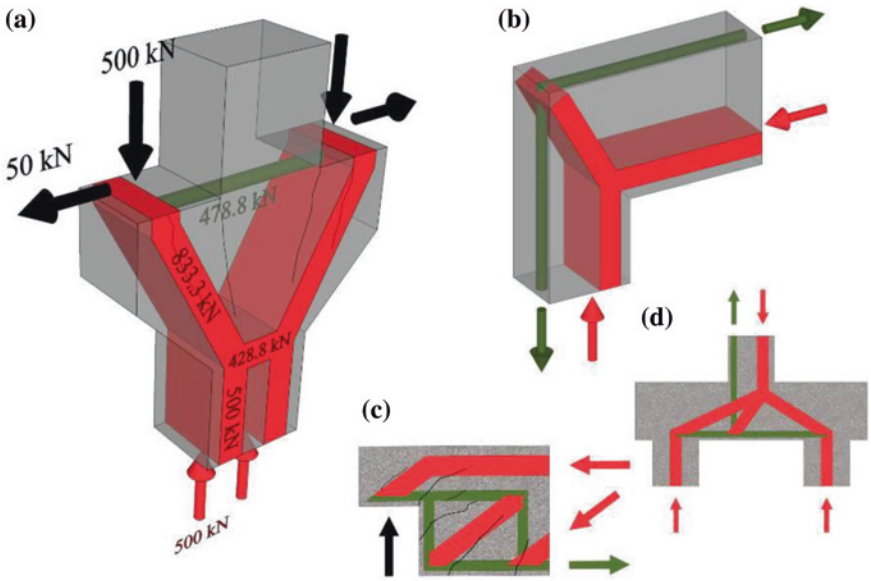

Fig. 8 **a** STM for region *D2* of Fig. 7, with sample calculations, **b** Alternative STM for region *D1*, under seismic loads, **c** STM for region *D3*, **d** STM for region *D4*

The selection of the effective width w of a strut is usually computed so that the strut's capacity is larger than the force it carries. On the other hand, there is also a tie effective width that can be computed during the analysis of the node in order to satisfy the nodal bearing capacity. Several methods for construction and verification of nodes, which won't be addressed, have been proposed up to now, including the hydrostatic approach [34] and the modified hydrostatic approach [38].

Figure 8a shows a straightforward computation of sample truss member forces; it is noted that other more complex STMs may be challenging in that they may imply solving statically indeterminate systems. Figure 8b illustrates the necessity that can arise, on occasion, of sketching a different truss when a distinct load case is considered.

Finally, it is highlighted that the STM can also be used in the rehabilitation context to assess the efficiency of a specific strengthening technique. Using the same underlying principles of static admissibility and flow of forces, STM—eventually combined with frame fibre analysis—can readily simulate common retrofitting solutions such as RC or steel member jacketing, steel bracing, infill strengthening, use of fibre reinforced polymers, etc.

3.3 Other Relevant Modelling Tools

From the major sources of deformation on reinforced concrete buildings, member bending is typically the most important and is therefore always considered by linear and non-linear beam-column models, such as those of Sect 3.1. However, besides normal stresses, shear stresses are also present in frame members that may result in diagonal cracking. The latter can lead to premature member failure unless adequate reinforcement has been provided.

In frame elements, a Timoshenko or a higher-order beam theory must be considered to simulate shear deformation, and the corresponding governing equations solved either with a displacement-based or force-based formulation. At the sectional level, the most simplified approaches use approximate shear force-shear deformation relationships. More detailed frame models employs 2D or 3D material models for reinforced concrete to characterize the layer (fibre) behaviour; however, the assumed imposition of a sectional shear-strain profile is unable to respect local equilibrium throughout the whole range of non-linear analysis. Therefore, the correct coupling between normal and shear stresses cannot be reproduced in a direct way, which impairs the theoretical soundness of such models. Several iterative procedures with simplifying assumptions have been proposed by different authors [39], but further developments are still required in this complex field of research.

On the opposite end of the modelling spectrum one can find detailed 2D or 3D finite element models. Advanced concrete models are required, for instance based on non-linear elasticity, theory of plasticity, fracture, microplane models, etc. They have been used by the research community to reproduce the behaviour of RC walls and piers with relevant shear deformations, but are not a practical tool for

practitioners. Additionally, they call for a demanding combination of expertise in numerical modelling, as well as powerful processing capabilities.

Second-order effects can also be relevant in assessing serviceability and failure limit states. For each type of structural element (beam, plate, shell, solid…) there is a wealth of theoretical models to account for such effect. In view of the discussion in Sect. 0, it is worth mentioning the appropriateness of the co rotational approach for non-linear geometrical analysis of force-based elements [40].

Fixed end rotations, caused by slip of longitudinal steel reinforcing bars along their embedment length in beam-column joints, or footings, can be another important mechanism of member deformation. In terms of contribution to the total lateral member displacement, it may represent, on occasion, almost as much as the flexural component during inelastic response [41]. It is possible to find in the literature different simulation methods and pseudo-empirical expressions addressing this issue, but again there is room for more insightful research on the mechanics of its non-linear performance.

Finally, a thorough assessment of the condition of old buildings may require assessing the influence of additional physical phenomena, for which adequate models can in general be found. One may wish to consider, for instance, creep and shrinkage in concrete, corrosion of reinforcement (which affects not just the rebars, but bond, cracking and spalling of concrete), the deformability of beam-column joints and foundations, crack widths and propagation, among others.

4 Final Remarks

Some guidance and paths were shown in the previous sections, as well as state-of-the-practice tools and solutions for current engineering problems that researchers and engineers may face while developing numerical models for masonry and RC structures. It is thus expected that some uncertainties related to modelling issues were mitigated, contributing to an improvement of the overall quality of the numerical model and subsequent results.

Nevertheless, one of the foremost remarks is that there is no unique solution for the modelling of an existing structure. The usage of different modelling approaches, such as sub-assemblages and detailed finite element analysis of components of a structure, together with macro analysis of the complete construction, increases the global knowledge of structural performance and confidence on the developed numerical model.

The selection of the adequate tools, methods and elements is a crucial step, to the success of the model, as thoroughly discussed for masonry and RC structures. Moreover, the inspection of the current conditions of the structure is of meaningful importance for tuning the numerical model, and therefore to obtain reliable final assessment results. For masonry structures, materials and connections should be checked with particular care, while for RC buildings, concrete cracking, rebar corrosion, and other damage sources should be identified for the definition of appropriate material properties of beam-column models, as well as configuration and parameters of strut-and-tie approaches.

For the overall interpretation of the results, the engineering judgment and experience will play a decisive role, yet the guidelines presented herein are expected to reduce modelling uncertainties and assist in the final decision making process.

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