Plasma turbulence in the tokamak scrape-off layer

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Theory of Fusion Plasmas
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Scrape-off layer physics crucial for magnetic fusion

Heat load to PFCs, rotation, impurities, L-H transition...

How do we develop 1st principles understanding of SOL dynamics?
Simple problem: inner wall limited (pol. ×-section)
Ballooning turbulence with $k_\theta \rho_s \approx 0.1 \sim 1 \text{cm}^{-1}$
Gaussian in near SOL, intermittent in far SOL
Fluctuation level $\mathcal{O}(1)$, skewed PDF
Power balance $\rightarrow$ exponentially decaying profiles

\[ \nabla \cdot \Gamma_\perp + \nabla \cdot \Gamma_\parallel = 0 \]

Turbulence

Sonic flows towards PFCs
Some of the questions that must be addressed...

✓ What mechanism sets the turbulence levels?
✓ What instability drives the perpendicular transport?
✓ What is the qualitative effect of finite $T_i$?
✓ How does the SOL width change with parameters?
✓ Can we reconcile theory, simulations, and experiments?
✓ What are the effects of neutrals? [C. Wersal, P-22 Thursday]
✓ How is toroidal rotation generated in the SOL? [Loizu, PoP 2014]
× Is SOL transport related to the density limit? [LaBombard, NF 2005/08]
× How is the SOL coupled with the closed flux surface region?
A tool to simulate SOL turbulence

Global Braginskii Solver (GBS) [Ricci, PPCF (2012)]

- Drift-reduced Braginskii equations
  \[ \frac{d}{dt} \ll \omega_{ci}, k_{\perp}^2 \gg k_{\parallel}^2 \]
- Evolves \( n, \phi, V_{||e}, V_{||i}, T_e, T_i \) in 3D
- Global, flux-driven, no separation between equilibrium and fluctuations
- Power balance between plasma outflow from the core, turbulent transport, and parallel losses
- Scalable \( \rho_\star \) up to medium size tokamak (e.g. TCV, C-Mod)
Drift-reduced Braginskii equations to describe the SOL

\[
\frac{\partial n}{\partial t} = -\frac{\rho^*}{B} [\phi, n] + \frac{2}{B} [nC(T_e) + T_e C(n) - nC(\phi)] - n\nabla_\parallel v_{\parallel e} - v_{\parallel e} \nabla_\parallel n
\]

\[
\frac{\partial \tilde{\omega}}{\partial t} = -\frac{\rho^*}{B} [\phi, \tilde{\omega}] - \nu_{\parallel i} \nabla_\parallel \tilde{\omega} + \frac{B^2}{n} \nabla_\parallel j_{\parallel} + \frac{2B}{n} C(p) + \frac{B}{3n} C(G_i), \quad \tilde{\omega} = \nabla^2_\perp (\phi + \tau T_i)
\]

\[
\frac{\partial}{\partial t} \left( v_{\parallel e} + \frac{m_i}{m_e} \frac{\beta_e}{2} \psi \right) = -\frac{\rho^*}{B} [\phi, v_{\parallel e}] - v_{\parallel e} \nabla_\parallel v_{\parallel e} + \frac{m_i}{m_e} \left[ \nu j_{\parallel} / n + \nabla_\parallel \phi - \frac{\nabla_\parallel p_e}{n} - 0.71 \nabla_\parallel T_e - \frac{2}{3n} \nabla_\parallel G_e \right]
\]

\[
\frac{\partial v_{\parallel i}}{\partial t} = -\frac{\rho^*}{B} [\phi, v_{\parallel i}] - v_{\parallel i} \nabla_\parallel v_{\parallel i} - \frac{2}{3} \nabla_\parallel G_i - \frac{1}{n} \nabla_\parallel p
\]

\[
\frac{\partial T_e}{\partial t} = -\frac{\rho^*}{B} [\phi, T_e] - v_{\parallel e} \nabla_\parallel T_e + \frac{4}{3} \frac{T_e}{B} \left[ \frac{7}{2} C(T_e) + \frac{T_e}{n} C(n) - C(\phi) \right] +
\]

\[
+ 2 \left\{ T_e \left[ 0.71 \nabla_\parallel v_{\parallel i} - 1.71 \nabla_\parallel v_{\parallel e} \right] + 0.71 T_e (v_{\parallel i} - v_{\parallel e}) \frac{\nabla_\parallel n}{n} \right\} + D_{\parallel T_e} (T_e)
\]

\[
\frac{\partial T_i}{\partial t} = -\frac{\rho^*}{B} [\phi, T_i] - v_{\parallel i} \nabla_\parallel T_i + \frac{4}{3} \frac{T_i}{B} \left[ C(T_e) + \frac{T_e}{n} C(n) - C(\phi) \right] +
\]

\[
+ 2 \left\{ T_i \left( v_{\parallel i} - v_{\parallel e} \right) \frac{\nabla_\parallel n}{n} - \frac{2}{3} T_i \nabla_\parallel v_{\parallel e} - \frac{10}{3} \frac{T_i}{B} C(T_i) + D_{\parallel T_i} (T_i) \right\}
\]

+ Sheath BCs consistent with PIC simulations [Loizu, PoP (2012)]
Parameters, normalizations, coordinates

- Coordinate system: $(\theta, r, \varphi) \rightarrow (\text{poloidal}, \text{radial}, \text{toroidal})$

- Equations expressed in normalized units:
  - $L_\perp \rightarrow \rho_s$
  - $L \parallel \rightarrow R$
  - $\nu \rightarrow c_s$
  - $t \sim \gamma^{-1} \rightarrow R/c_s$

- The dimensionless code parameters are as follows:
  - $\rho_* = \rho_s/R$
  - $\nu = e^2 n R / (m_i \sigma \parallel c_s)$
  - $\beta_e = 2 \mu_0 p_e / B^2$
  - $q \approx (r/R) B_\varphi / B_\theta$

- Simplified notation in analytical expressions:
  - $p_0 = \langle p \rangle_t, \ t \gg \gamma^{-1}$
  - $L_p = -\langle p/\partial_r p \rangle_t$
Poloidal cross sections showing SOL turbulence
Modes saturate due to pressure non-linearity

We observe in simulations [Ricci, PoP (2013)]:

- Mode saturation caused by local pressure non-linearity

\[ \partial_r p_1 \sim \partial_r p_0 \rightarrow \frac{p_1}{p_0} \sim \frac{\sigma_r}{L_p} \]

- Radial eddy length is mesoscopic [Ricci, PRL (2008)]

\[ \sigma_r \approx \sqrt{L_p/k_\theta} \]

- Turbulent flux dominated by radial $E \times B$ convection

\[ \Gamma_1 = \rho_*^{-1} \left\langle p_1 \frac{\partial \phi_1}{\partial \theta} \right\rangle \]
Saturation model yields $E \times B$ turbulent flux

Gradient removal hypothesis

$$\frac{p_1}{p_0} \approx \frac{\sigma_r}{L_p}$$

$$\Gamma_1 \approx \rho_\star^{-1} \langle p_1 \partial_\theta \phi_1 \rangle$$

$$\partial_t p = -\rho_\star^{-1} [\phi, p]$$

$$\partial_\theta \phi_1 = \gamma \left( \frac{p_1}{p_0} \right) \left( \rho_\star L_p \right)$$

$$\Gamma_1 \sim p_0 \left( \frac{\gamma}{k_\theta} \right)_{\text{max}}$$
Self-consistent prediction of pressure gradient length

In steady state, $\nabla \cdot \Gamma_1$ balances parallel losses $\sim \nabla_{\|} \cdot (p v_{\|} e)$, hence

$$L_p \approx \frac{q}{c_s} \left( \frac{\gamma}{k_\theta} \right)_{\text{max}}$$

- Results in iterative scheme to predict $L_p$ self-consistently:
  - Compute $\gamma = f(L_p, k_\theta, \rho_*, q, \nu, \hat{s}, m_i/m_e)$
    - Vary $L_p$ until LHS = RHS using secant method
Excellent agreement between theory and simulations

$L_p$ predicted using self-consistent procedure [Halpern, NF (2014)]

![Graph showing the comparison between $L_p$ predicted using the self-consistent procedure and the simulation results. The graph shows a strong linear correlation with an $R^2$ value of 0.94. The data points fall closely along the line of best fit.](image)

GBS sims.: $\rho^{-1}_* = 500–2000$, $q = 3–6$, $\nu = 0.01–1$, $\beta = 0–3 \times 10^{-3}$
Dominant instability depends principally on $q$, $\nu$, $\hat{s}$, $T_i/T_e$

- Build instability parameter space using reduced models
  → gradient removal theory, linear dispersion relations
- Verify results using GBS non-linear simulations [Mosetto, PoP (2013)]

Which instability drives $\perp$ transport?
- Inertial/Resistive Ballooning modes/Drift Waves?
Dominant instability depends principally on \( q, \nu, \hat{s}, T_i/T_e \)

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- Which instability drives \( \perp \) transport?
  - \textit{Inertial/Resistive Ballooning modes}/Drift Waves?
Presence of RBMs verified in TCV SOL sims

- $(\tilde{n}, \tilde{\phi})$ phase difference, joint $(\tilde{n}, \tilde{\phi})$ pdf [Halpern, NF (2014)]

Curvature-driven, non-adiabatic mode $\rightarrow$ RBMs
Addition of finite $T_i$ weakens adiabatic coupling

- Analysis extended to include $T_i$ effects [Mosetto, PoP (submitted)]
- Joint ($\tilde{n}, \tilde{\phi}$) pdf in GBS sims with $\tau = 1, \tau = 4$

RBM component is enhanced by finite $T_i$
SOL width in RBM regime scales with $\rho_\star$, $q$

- SOL width obtained analytically with RBMs [Halpern, NF 2013/14]:

$$\gamma_b = \sqrt{2/(\rho_\star L_p)}$$

$$L_p = q \left( \frac{\gamma}{k_\theta} \right)_{\text{max}}$$

$$k_b = \sqrt{(1 - \alpha)/(\nu \gamma_b)/q}$$

- Our simple model leads to a dimensionless scaling:

$$L_p = \left[ 2\pi \rho_\star (1 - \alpha)^{1/2} \frac{\alpha_d}{q} \right]^{-1/2}$$

Machine size

$\alpha = q^2 \beta/(\rho_\star L_p)$

Electromagnetic effects

$\alpha_d = \nu^{-1/2}(\rho_\star L_p)^{1/4}/q$

Collisionality vs connection length
Parallel dynamics physics in agreement with simulations

- Verify saturated RBM theory with GBS EM simulations
  - $\rho_*^{-1} = 500$, $\beta_e = 0-3 \times 10^{-3}$, $\nu = 0.01-1$, $q = 3, 4, 6$

(Contours of $L_p$ given by theory, symbols are GBS simulations)
GBS simulations confirm size-scaling up to TCV size
Dimensionless scaling follows GBS simulation data

Comparison carried out over wide range of parameters \((\rho_*, q, \beta, \nu)\)
Good agreement with SOL width measurements

\[ L_p \approx 7.2 \times 10^{-8} q^{8/7} R^{5/7} B_\phi^{-4/7} T_{e0}^{-2/7} n_{e0}^{2/7} (1 + T_i/T_e)^{1/7} \text{ [ m ]} \]

Exp. data:
- G. Arnoux
- I. Furno
- J.P. Gunn
- J. Horacek
- M. Kočan
- B. Labit
- B. LaBombard
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Intermission

- We discussed a theory describing SOL turbulent dynamics
  - Turbulent saturation mechanism
  - Non-linear instability driving \( \perp \) transport
  - SOL width scaling with plasma parameters
  - Verified with non-linear simulations
  - Compared against data from several machines
- Some experimental data disagrees with theory

Carry out detailed comparison with these experiments
An ideal testbed for simulation-experiment comparison

- Inner-wall limited Ohmic C-Mod discharges [Zweben, PoP (2009)]
- $R = 0.67 \text{m}$, $a = 0.20 \text{m}$, $B = 2.7, 3.8 \text{T}$, $\kappa = 1.2$
- Density scan at each value of $B$
- Characterize C-Mod SOL turbulence using GPI diagnostic, and compare with GBS results
  - Low $\beta$, no $T_i$ or $\tilde{B}$ diagnostics $\rightarrow$ simple electrostatic, cold ion model
  - $\delta D_\alpha D_\alpha$, pdf moments, $\tau_{auto}$, $L_r$, $L_\theta$, $v_r$, $v_\theta$, $P(k_\theta)$, $P(\omega)$

Very stringent test!
Gas-puff imaging of C-Mod SOL

Phantom 710 high-speed camera at 400’000fps [S.Zweben, J.Terry]
δD_α/D_α diagnostic for GBS

Using DEGAS modeling of GPI emissivity, model D_α fluctuations

- Emissivity locally parametrized as \( E \propto T_e^\alpha n_e^\beta \), use H656 line
- Fluctuations modelled as \( \delta D_\alpha/D_\alpha \approx \alpha(T_e, n_e) \tilde{T}_e + \beta(T_e, n_e) \tilde{n} \)

- Simulate finite GPI resolution (3 × 3mm + 2.5μs smoothing), B-field tilt respect to sensors (8mm poloidal smoothing)
$\delta D_\alpha/D_\alpha$ synthetic diagnostic results

- Left to right: $\tilde{n}$, $\delta D_\alpha/D_\alpha$, $\delta D_\alpha/D_\alpha$ (diode), $\delta D_\alpha/D_\alpha$ (full)

High $k_\theta$ modes strongly damped by smoothing
Large $\delta D_\alpha / D_\alpha$ fluctuations, skewed PDF

- $\delta D_\alpha / D_\alpha$ level increases with SOL, $\sim 30\%$ in far SOL
- Skewness $\sim 1 \rightarrow$ blobs (?)
- Moment profiles robust with plasma parameters
Large $\delta D_\alpha / D_\alpha$ fluctuations, skewed PDF

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Quantitative comparison using shaded area (GPI sensors)
GBS agrees with [Zweben PoP 2009] within error bars

- Compare GBS radial/poloidal average against GPI data
- Shot-to-shot variation indicated with error bars
- GBS gives good match for $\frac{\delta D_\alpha}{D_\alpha}$ and higher moments
- Previous gyrofluid simulations gave $\frac{\delta D_\alpha}{D_\alpha} \approx 5-10\%$
Typical spatial, temporal turbulent scales give reasonable agreement

- Compute $\tau_{auto}$, $L_{rad}$, $L_{pol}$ using 2 point correlations functions $C_{ij}$

$$C_{ii}(\tau_{auto}) = \frac{1}{2}$$

$$L = 1.66 \frac{\delta x}{\sqrt{-\ln C_{ij}(t = 0)}}$$

- Good match for $L \sim 1.5\text{cm}$, $\tau_{auto}$ underpredicted by $\sim 2$
Propagation velocities

- Obtain $v_{rad}$, $v_{pol}$ from time lag that maximizes correlation between two neighboring points separated by $\delta_x \rightarrow v = \delta_x / \tau$
- Good agreement in $v_{rad} \rightarrow$ poloidal mode structure
- Large mismatch in $v_{pol} \rightarrow$ resolution smoothing in GBS data?
Spectral power vs wavenumber of $\delta D_\alpha / D_\alpha$

- From FFT of $\delta D_\alpha / D_\alpha$ in $\theta$, then average over $r$, $t$
- Significant drop at $k_{pol} = 125 \text{m}^{-1}$ high $k$ due to smoothing
- Unsmoothed $\delta D_\alpha / D_\alpha$ has same power law scaling as GPI
Spectral power vs frequency of $\delta D_\alpha / D_\alpha$

- From FFT of $\delta D_\alpha / D_\alpha$ in $t$, then average over $t$, $r = 2 \pm 0.2\text{cm}$
- GPI measurements and GBS show same asymptotic behavior
Summary and outlook

- Towards first principles understanding of SOL width:
  - ✓ Non-linearly saturated RBMs, enhanced with $T_i$ effects
  - ✓ SOL width scales with $\rho_\star$, $q$, collisionality
  - ✓ Simple analytical scaling agrees with experimental data

- Detailed comparison between GBS and C-Mod discharges
  - ✓ $L_p$, $\delta D_\alpha/D_\alpha$ pdf moments, $L_{rad}$, $L_{pol}$, $v_{rad}$, $P(\omega)$, $P(k_{pol})$
  - × $\tau_{auto}$, $v_{pol}$ → under/overpredicted by factor $\sim 2$

- Next: 2 $L_p$’s profile structure using 2014 C-Mod discharges
  - ▶ More advanced simulation model → $T_i$, shaping
  - ▶ Mirror langmuir probe → high res. profiles, $(n, \phi)$ phase
Thank you for your attention!
Properties of the SOL

- $L_{fluc} \sim \langle L \rangle_t$
- $n_{fluc} \sim \langle n \rangle_t$
- Collisional magnetized plasma
- Low frequency modes $\omega \ll \omega_{ci}$
- Open field lines
Sheath BCs from kinetic approach [Loizu, PoP (2012)]

- **COLLISIONAL PRESHEATH (CP)**
  - Quasi-neutral, IDA holds
  - Potential drop $\sim 0.5 T_e$ over $\sim L$
  - Ions accelerated to $v_s = c_s \sin \alpha$

- **MAGNETIC PRESHEATH (MP)**
  - Quasi-neutral, IDA breaks
  - Potential drop $\sim 0.5 T_e$ over $\sim \rho_s$
  - Ions accelerated to $v_s = c_s$

- **DEBYE SHEATH (DS)**
  - Non-neutral, IDA breaks
  - Potential drop $\sim 3 T_e$ over $\sim 10 \lambda_D$
  - Ions accelerated to $v_s > c_s$
Extra slides: Summary of the BC

\[
\nu_{\|i} = c_s \left(1 + \theta_n - \frac{1}{2} \theta_{T_e} - \frac{2\phi}{T_e} \theta_{\phi}\right)
\]

\[
\nu_{\|e} = c_s \left(\exp(\Lambda - \eta_m) - \frac{2\phi}{T_e} \theta_{\phi} + 2(\theta_n + \theta_{T_e})\right)
\]

\[
\frac{\partial \phi}{\partial s} = -c_s \left(1 + \theta_n + \frac{1}{2} \theta_{T_e}\right) \frac{\partial \nu_{\|i}}{\partial s}
\]

\[
\frac{\partial n}{\partial s} = -\frac{n}{c_s} \left(1 + \theta_n + \frac{1}{2} \theta_{T_e}\right) \frac{\partial \nu_{\|i}}{\partial s}
\]

\[
\frac{\partial T_e}{\partial s} \approx 0
\]

\[
\omega = -\cos^2 \alpha \left[ (1 + \theta_{T_e}) \left(\frac{\partial \nu_{\|i}}{\partial s}\right)^2 + c_s (1 + \theta_n + \theta_{T_e}/2) \frac{\partial^2 \nu_{\|i}}{\partial s^2} \right]
\]

where \(\theta_A = \frac{\rho_s}{2 \tan \alpha} \frac{\partial x A}{A}\), and \(\eta_m = e(\phi_{mpe} - \phi_{wall}) / T_e\). [Loizu et al PoP 2012]
Resistive ballooning modes destabilized by EM effects

- Starting from reduced MHD, obtain simple dispersion relation

\[
\gamma^2 \left( \nu + \frac{\beta e_0 \gamma}{2 k^2} \right) = 2 \frac{R}{L_p} \left( \nu + \frac{\beta e_0 \gamma}{2 k^2} \right) - \frac{k^2_{\|}}{k^2_{\perp}} \gamma
\]

- Neglecting ideal ballooning mode, the resistive branch gives

\[
(\gamma^2 - \gamma^2_b) k^2_{\perp} = -\gamma \left( \frac{1 - \alpha}{q^2 \nu} \right)
\]

and we identify \( \gamma \sim \gamma_b = \sqrt{2R/L_p} \) and \( k_b \sim \sqrt{(1 - \alpha)/(\nu \gamma_b)/q} \)