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Interaction between in-plane shear forces and transverse bending moments in concrete bridge webs

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Abstract

This paper investigates a reliable design approach for reinforced concrete panels subjected to the combined action of in-plane shear and out-of-plane bending moments, which develop in webs of box-girder bridges and similar structures. Neglecting this interaction is not consistent with the actual structural behaviour at failure and can potentially lead to an insufficient design. Existing design approaches (based on the rigid-plastic stress field (RPSF) method) account for this interaction by reducing the shear strength as a function of the transverse moment and predict a substantial reduction of the shear strength. The proposed multi-layered elastic-plastic stress field (ML-EPSF) approach represents an equilibrium solution (satisfying the static and kinematic theorem of the theory of plasticity) with larger shear strength, especially in presence of small transverse moments. The ML-EPSF model confirms the basic principle of the RPSF interaction models, but shows that its underlying hypotheses are however very conservative and might need some adjustments.

1 Introduction

The design of structural concrete elements subjected to shear and longitudinal bending has been extensively studied over the past century. A topic where less research efforts have been devoted to is the interaction of web shear and transverse bending which is present in numerous structures (such as box-girder bridges as shown in Fig. 1 left). Neglecting their interaction and performing independent analyses of the in-plane shear and the out-of-plane moment, and then summing the required shear reinforcement is too simplistic and not consistent with the actual behaviour of the web at failure. It gives potentially rise to excessive amounts of reinforcement and to inconsistent superposition of concrete compressive stresses. The latter might even lead to unsafe results in case of high shear forces.

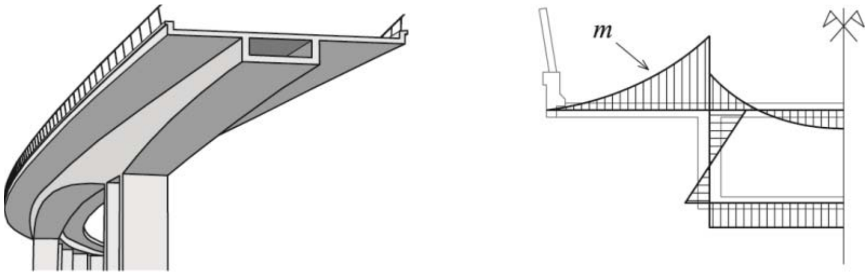


Fig. 1 Left: Box-girder bridge. Right: transverse bending moment diagram for uniform load

A number of consistent design approaches have been proposed [1-5] based on equilibrium conditions and accounting for the interaction between in-plane shear and transverse bending, while respecting the plastic strength of the materials (approaches based on the lower bound theorem of the theory of plasticity). These methods base the analysis and design of such members on rigid-plastic stress-fields (RPSF) [6]. They allow accounting for the influence of transverse moments on the shear strength by means of shear-transverse bending (V-m) interaction diagrams.

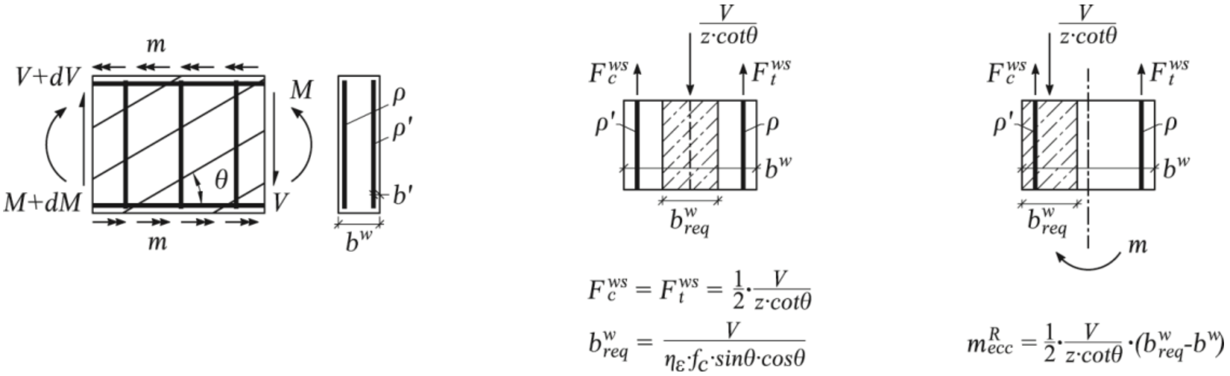
RPSF provide conservative estimates of the strength of a member [6]. A more general procedure for automatic development of stress fields has also recently been proposed [7] based on the elastic-plastic stress field (EPSF) method. This method accounts for equilibrium and compatibility conditions and also respects the plastic strength of the materials, leading thus to complete solutions according to the theory of plasticity.

In this paper, an application of the EPSF method to the problem of panels and beams subjected to shear and transverse bending is investigated. This is done by considering a multi-layered finite element, in which every layer satisfies the conditions of the EPSF. It allows a detailed modelling of the longitudinal shear behaviour and a better understanding of the interaction between in-plane shear and transverse bending moment. In addition, this paper presents an overview on the design criteria available in the literature and compares them to the proposed multi-layered elastic-plastic procedure.

2 Interaction models for in-plane shear and transverse bending

2.1 Rigid-plastic interaction models

Over the past forty years several design approaches have been proposed, in particular by Thürlimann [1], Menn [2], Stucchi [3] and Gaspar [4]. These methods are based on the static theorem (lower bound) of the theory of plasticity and can be applied for design of new structures and the assessment of existing ones. The lower bound theorem requires that the internal stress field is in equilibrium with the acting forces and that the limit states for both, steel and concrete, are not violated. Based on these considerations, simplified models for the ultimate resistance (V_R, m_R) of web sections subjected to shear and transverse bending have been established.



$$F_c^{ws} = F_t^{ws} = \frac{1}{2} \cdot \frac{V}{z \cdot cot\theta}$$

$$b_{req}^w = \frac{V}{\eta_\varepsilon \cdot f_c \cdot sin\theta \cdot cos\theta}$$

$$m_{ecc}^R = \frac{1}{2} \cdot \frac{V}{z \cdot cot\theta} \cdot (b_{req}^w - b^w)$$

Fig. 2 Left: Rigid-plastic stress field with a transverse bending moment *m*. Centre: Equilibrium of a web segment in pure shear. Right: Subjected to a small transverse bending moment

Fig. 2 (left) illustrates the fundamental stress field used by the above mentioned authors to develop their rigid-plastic interaction models. The tensile strength of concrete is neglected and the concrete compressive stress distribution is modelled as a uniform stress block. The angle of inclination of the compression field (θ) and the reduction factor to account for transverse cracking (η_ϵ) are assumed constant throughout the thickness of the web (b_w). The strength reduction factor accounting for transverse cracking (η_ϵ) is usually assigned a constant value (typically 0.6 for webs in shear) as more refined formulations of it (e.g. [8]) require the analysis of the strain state of the element.

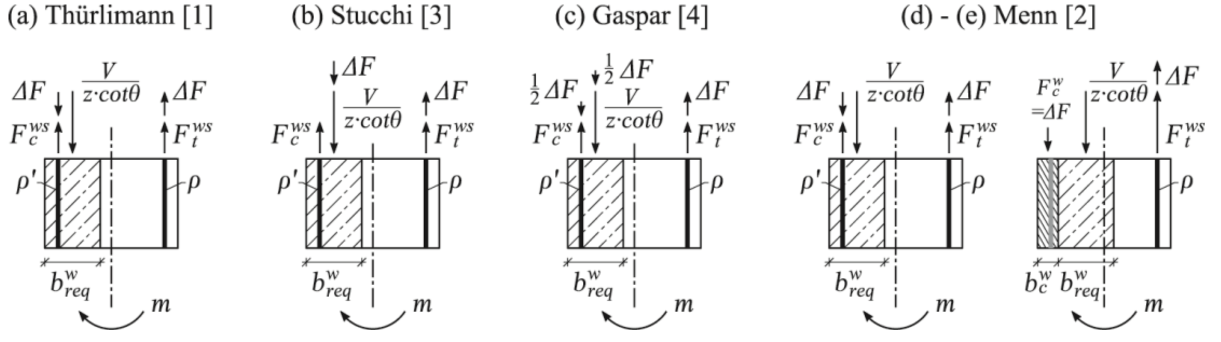

(a) Thürlimann [1] (b) Stucchi [3] (c) Gaspar [4] (d) - (e) Menn [2]

Fig. 3 Equilibrium of a web segment subjected to a high transverse bending moment

The shear-transverse bending interaction models derived from this stress field are established on the basis of the following principle. Up to a certain level of transverse bending ($m < m_{R,ecc}$), the resultant of the concrete compressive force ($V/zcot\theta$) is shifted to the flexural compression side. Thus the equilibrium of moments can be respected without changing the stirrup forces F_c^{ws} and F_t^{ws}, Fig. 2 (right). This phenomenon had already been observed by Kaufmann and Menn [9]. The eccentricity of the resultant is limited by b^{w}_{req}, i.e. the minimum web width required to resist the shear force V, Fig. 2 (centre).

For higher levels of transverse bending ($m > m_{R,ecc}$) the authors propose different stress fields that allow equilibrating the supplementary transverse moment. Thürlimann [1] assumes that an additional moment (Δm) can be resisted by changes in the stirrup tensions, Fig 3(a). Stucchi [3] recommends increasing the tensile force in the stirrups on the flexural tensile side by ΔF. This tensile force is compensated by an increase of the compression force acting on the shear strut, Fig. 3(b). For a more realistic behaviour of the web, Gaspar [4] considers both of the two previous ideas. His model equilibrates the additional tensile force ΔF, due to bending, at the same time by an increase of the concrete compression and a decrease of the stirrup force F_c^{ws}, Fig. 3(c). Menn [2] proposes to analyse the web in two situations: (i) when shear predominates, (ii) when the transverse bending moment predominates. In the first situation, like for Thürlimann, the additional moment is balanced by variations in stirrup forces, Fig. 3(d). In the second situation, the tensile force F_c^{ws} vanishes and is replaced by a compressive force, F_c^{w}, acting on the outer layer of the flexural compression face. As a consequence, the resultant of the concrete compressive force due to shear ($V/z\cot\theta$) is shifted back to the centreline, Fig. 3(e).

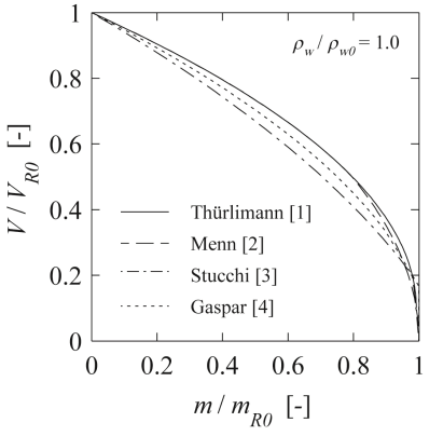


Fig. 4 Left: shear-transverse bending interaction diagrams for $\theta = 45°$, $\eta_\varepsilon = 0.6$, $\rho/\rho' = 1.0$, $b'/b_w = 0.1$

The comparison of the different rigid-plastic interaction models from the literature is shown in Fig 4, where the following values are used for the normalization of the plot.

$$\rho_{w0} = \frac{\eta_\varepsilon \cdot f_c}{f_y} \cdot \sin^2(\theta) \tag{1}$$

$$V_{R0} = \rho_{w0} \cdot f_y \cdot b_w \cdot z \cdot \cot(\theta) \tag{2}$$

$$\omega_0 = \tfrac{1}{2}\,\rho_{w0} \cdot \frac{b_w}{b_w - b'} \cdot \frac{f_y}{f_c} \tag{3}$$

$$m_{R0} = \omega_0 \cdot (b_w - b')^2 \cdot f_c \cdot (1 - \tfrac{1}{2}\omega_0) \tag{4}$$

The overall behaviour of the models is very similar. Stucchi's model is the most conservative, whereas Thürlimann's and Menn's models are more favourable, especially for larger bending moments. Overall, the effect of the transverse bending moments on the shear strength is quite significant. For example, a ratio $m/m_{R0} = 0.3$ leads to a loss of up to 20% of the initial shear strength. One can ask whether the interaction between shear and transverse bending is indeed so significant, or whether it might be a consequence of the hypotheses of these models.

One of the most debatable assumptions of all previous models is the constant inclination θ of the compression field throughout the thickness of the member. According to the stress field method, a vertical compressive force, due to transverse bending, acting on an inclined compression field tends to increase the inclination of the strut. This would mean that the inclination θ should not be the same for small and large transverse bending moments. Furthermore, the intensity of the vertical compressive force varies throughout the thickness of the web (larger on the outer layers compressed by the transverse bending) which suggests that θ changes likewise. In the same context, the hypothesis of a constant value for η_ε, as well as the value itself (e.g., $\eta_\varepsilon = 0.6$) is arguable. According to Vecchio and Collins [8], the concrete compressive strength reduction factor η_ε depends on the transverse strain. The latter is influenced by the vertical compression induced by the transverse bending moment. Thus, like for θ, the factor η_ε varies throughout the thickness of the web and depends on the intensity of the transverse moment.

A multi-layered elastic-plastic model has been developed to investigate the validity of rigid-plastic models for shear design in the presence of a transverse bending moment.

2.2 Multi-layered elastic-plastic interaction model

An alternative approach for development of suitable stress fields in reinforced concrete structures has recently been developed by Fernández Ruiz and Muttoni [7]. This approach allows considering compatibility conditions for determining the stress field in a structure. As a consequence, at failure, the lower bound theorem of the theory of plasticity is respected (the resulting stress field is in equilibrium with the applied loads and respects the yield criteria of the materials) as well as the upper bound theorem of the theory of plasticity (failure with a mechanism compatible with the boundary conditions). These stress fields, called as elastic-plastic stress fields (EPSF), thus provide the exact solutions according to the theory of plasticity (as both theorems are simultaneously respected).

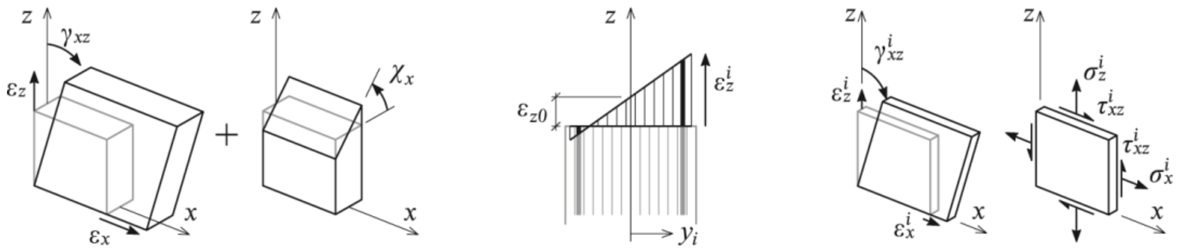


Fig. 5 Multi-layered shear panel with transverse bending moment

For the case investigated in this paper, this technique can be applied as follows: The width of the web is divided into a finite number (n) of concrete layers, which each behave according to the EPSF method. Two additional layers are introduced for steel at the location of the shear reinforcement. The deformation of each layer is defined by a separate strain field (ε_x^i, ε_z^i, γ_{xz}^i). Thus the effect of the transverse bending moment is accounted for by varying the vertical deformation of every layer with respect to the Bernoulli-Navier hypothesis of plane sections ($\varepsilon_z^i = \varepsilon_{z0} + \chi\cdot y_i$, where χ is the curvature associated to the transverse bending moment). The principal strains, (ε_1^i, ε_2^i), are determined using Mohr's circle, which allows computing the principal stresses (assuming that the principal strain directions are parallel to the principal stress directions). In accordance with the EPSF method the stress-strain relationship for both concrete and steel are considered elastic-perfectly plastic and the concrete tensile strength is neglected. The latter is corrected by a parameter η_ε to take into account the influence of transverse strain on the concrete compression strength. This factor is computed individually for each layer using the formula proposed by Vecchio and Collins [8]:

$$\eta_\varepsilon^i = \frac{1}{0.8+170\cdot\varepsilon_1^i} \le 1.0 \tag{5}$$

The resulting stresses in concrete (σ_1^i, σ_2^i), respectively (σ_x^i, σ_z^i, τ_{xz}^i), and in steel (σ_z^{s1}, σ_z^{s2}), are then integrated over the width of the web to obtain the shear force V and the transverse bending moment m corresponding to the applied strain field. This equilibrium of forces (vertical compression acting on the concrete layers and tensile forces in the stirrups) is attained by performing an iteration on the value of the vertical deformation ε_{z0} to reach $N_z = 0$.

$$V = \frac{b_w\cdot z}{n}\cdot \sum_{i=1}^{n} \tau_{xz}^i \tag{6}$$

$$m = \frac{b_w}{n}\cdot \sum_{i=1}^{n} y_i\cdot\sigma_z^i + \frac{1}{2}\cdot\rho_w\cdot b_w\cdot(y_{s1}\cdot\sigma_z^{s1} + y_{s2}\cdot\sigma_z^{s2}) \tag{7}$$

$$N_z = \frac{b_w}{n}\cdot \sum_{i=1}^{n} \sigma_z^i + \frac{1}{2}\cdot\rho_w\cdot b_w\cdot(\sigma_z^{s1} + \sigma_z^{s2}) \tag{8}$$

The application of the EPSF method on to a multi-layered element allows determining a stress field in equilibrium with the combined action of in-plane shear and out-of-plan bending. Some results are presented in Fig 6. The shear-transverse bending interaction diagrams Fig 6 (top left), computed for various shear reinforcement ratios, confirm that in the range of small transverse bending moments the shear strength decreases only slightly. Furthermore, the diagrams show that the longitudinal strain ε_x has a non-negligible effect on the overall resistance of the element (this is not considered in rigid-plastic models). Fig. 6 (bottom) gives indications on various parameters of the multi-layered elastic-plastic stress field (ML-EPSF) on the cross section of the element. For small transverse bending moments, the behaviour of the element is close to pure shear behaviour: shear stresses $\tau_{xz}^{\ i}$ are uniformly distributed over the entire width of the web and the angles of inclination of the compression fields θ_i are constant. With increasing transverse bending moments, the shear stresses increase towards the flexural compression side and the inclination in the outer layers increases significantly (reaching 90° in some cases). For very high levels of transverse bending, the outer layers behave as in pure flexion and only a small amount of shear is transferred by the internal layers. The diagrams of vertical stress σ_z and shear stress τ_{xz} confirm this observation. These observations validate the basic principle of the rigid-plastic interaction models, i.e. a shift of the resultant shear force towards the surface of the web, and Menn's model for predominant transverse bending (assuming the presence of an additional compression zone at the outmost layer of the web). However, the hypothesis on the constant inclination of the compression field (for any value of m and throughout the whole thickness of the web) is very different from what is observed with the ML-EPSF method. The same remark also applies to the concrete compressive strength reduction factor η_ε.

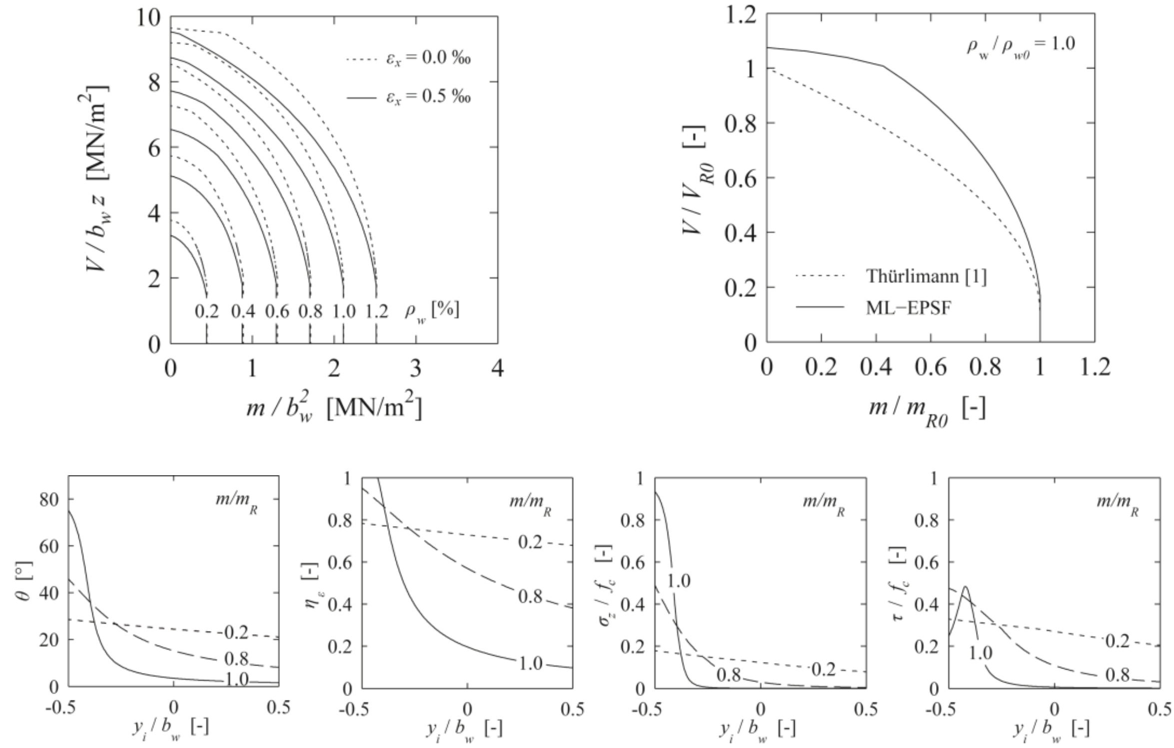


Fig. 6 Top left: Shear-transverse bending interaction diagram computed by the ML-EPSF method, with $f_c = 30$ MPa, $f_y = 500$ MPa, $b'/b^w = 0.1$ and $\rho/\rho' = 1.0$. Bottom: state of stress in the width of the web for $\rho_w = 0.8\%$. Top right: Comparison between interaction models: Thürlimann [1] with $\eta_\varepsilon = 0.6$, $\alpha = 45°$ and the ML-EPSF method with $\varepsilon_x = 0.5$‰

The comparison between the rigid-plastic model proposed by Thürliman [1] and the ML-EPSF method is plotted in Fig 6 (top right). The overall behaviour shows that the ML-EPSF method predicts a weaker interaction between shear and transverse bending which results in a substantial increase of the shear strength. For low levels of transverse bending ($m/m_{R0} < 0.4$), the reduction of the shear strength is almost negligible. The difference between those two models is mainly due to the fact that in RP models the values for η_ε and θ are constant and chosen to be an the safe side (for example for beams $\eta_\varepsilon = 0.6$ and $25° < \theta < 45°$), whereas in the ML-EPSF method these values are computed directly from the static and kinematic compatibility conditions. Thus, in the RP models, the positive effect of the bending compression on the strut inclination and on η_ε is neglected, which leads to conservative values for the shear strength.

3 Conclusion and outlook

This paper presents an investigation on the interaction between in-plane shear and out-of-plane bending moment in reinforced concrete webs and panels. Some interaction models from the literature, based on the rigid plastic stress field (RPSF) method, are analysed. A more consistent interaction model, based on a multi-layered elastic-plastic stress field (ML-EPSF) approach, is proposed and compared to the models from the literature. The main conclusions are the following:

- The RPSF interaction models predict a strong interaction between shear and transverse bending moments, which leads to a large reduction of the shear strength in the presence of transverse bending moments.
- The ML-EPSF considers a more realistic behaviour and leads to equilibrium solutions with larger shear strengths than those predicted by RPSF. The reduction of the shear strength due to small transverse bending is almost negligible, which is particularly interesting for the assessment of existing structures.
- The ML-EPSF approach confirms the basic principal of the RPSF interaction models, but it also shows that some of the hypotheses underlying these models are strong simplifications.

The next steps of the dissertation are:

- Collection and investigation of additional experimental test results from the literature.
- Comparison of the test results to RPSF and ML-EPSF approaches
- Exploration of the sensitivity of the ML-EPSF method to different parameters (f_c, f_y and ρ/ρ').
- Development of a simplified interaction model based on the results of the ML-EPSF method.
- Implementation of the ML-EPSF model into the existing finite element program for EPSF analysis.

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