Cognitive mechanism in synchronized motion:
An internal predictive model for manual tracking control
(Special session)

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Abstract—Many daily tasks involve spatio-temporal coordination between two agents. Study of such coordinated actions in human-human and human-robot interaction has received increased attention of late. In this work, we use the *mirror paradigm* to study coupling of hand motion in a leader-follower game. The main aim of this study is to model the motion of the follower, given a particular motion of the leader. We propose a mathematical model consistent with the internal model hypothesis and the delays in the sensorimotor system. A qualitative comparison of data collected in four human dyads shows that it is possible to successfully model the motion of the follower.

Fig. 1. Two subjects playing mirror game in leader-follower setup.

I. Introduction

Adaptability of the human motion control system is a well studied fact. Adaptations is needed against external perturbations such as change in load, internal dynamics, e.g., change in muscle strength over time, or in sensory feedback guiding the motion. Many studies have given a qualitative account of the mechanisms behind such adaptations under perturbations of force fields [1], distorted visual feedback [2] and changing coordination patterns with another subject [3]. A common result among these studies is that the subjects tend to re-learn the task, over time, even in the presence of perturbations, and to un-learn it slowly when the perturbation is removed.

In this paper we focus on developing an adaptive model for quantitative and qualitative assessment of coordinated motion generation in a "mirror game" [4], [5]. The mirror game is an experimental paradigm for studying the interaction between two subjects while they try to mirror each other's motion (see Fig. 1). Possible modalities in such a setup are Leader-Follower (LF) or Joint Improvisation (JI). In this study we will focus on the LF modality where one of the subjects is designated to lead the motion and the other is instructed to synchronize with the leader.

The rest of the paper is organized as follows. In the next section, we review other models explaining adaptive motor behavior in humans. In Section III we describe our setup and the data collection procedure. Section IV describes our mathematical model and we present the results in Section V. Discussion and conclusion are presented in Section VI and Section VII respectively.

II. Related Work

There are a large number of activities that are characterized by joint actions performed by a dyad of subjects. Although there is a rich literature on characterizing motion and control patterns in single subject experiments, joint actions have been studied only lately. First, we review how adaptation has been studied for subjects performing solo tasks. One line of research has been dedicated to internal models and estimating the sensory delay and its effect on the motor output. Smith predictor [6] is one such mechanism which maintains an internal model of the dynamics combined with an estimate of sensory delay. Results from [7], [2] also suggest the presence of a delay component in the internal process for motion generation. Kelso [8] derived a coupled oscillator based model to explain the fact that in their finger-tapping experiments, anti-phase motion falls into synchronized motion after a certain frequency. Phase and frequency locking behaviors even in the absence of visual feedback observed in [9] suggests some form of *social-memory*. Several other studies [10], [11] have suggested that the presence of a forward internal model might be to overcome the effect of time delays.

Joint activities have been rarely studied, mainly due to the lack of an experimental paradigm. Noy et. al. [4] adapted the mirror-game - a fundamental practice in improvisational theater - as an experimental system for studying joint interactions between two subjects. In this paper, we use the same setup to develop a mathematical model to explain the follower's behavior in the LF modality. We take the established delayed-internal-model hypothesis and use tools from regression analysis and dynamical systems to realize our model. Our model is consistent with the hypothesis that the adaptation in motor behavior is a direct result of update in the internal model [11]. In other words, in the LF setting, the follower incrementally builds a model of the leader's motion and executes its own motion by using forward prediction based on the internal model. We also incorporate sensory delay in our model which is hypothesized to be nullified by the forward prediction.
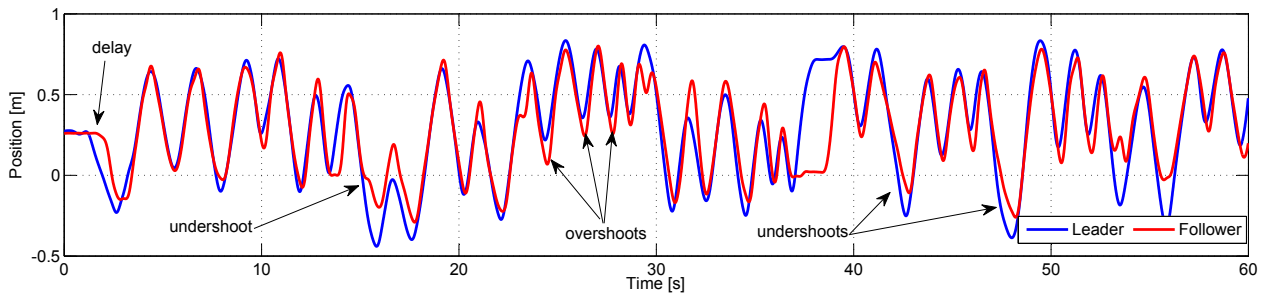
Fig. 2. A sample of experimental recording from Leader-follower setup in mirror game. Arrows indicate noticeable patterns in the follower's behavior.
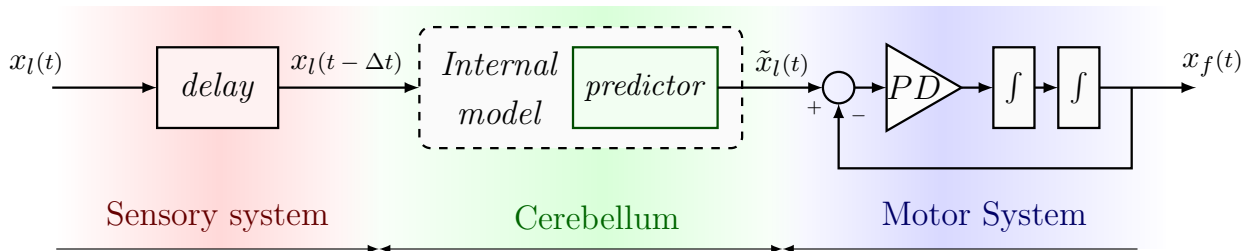

Fig. 3. Structural representation of the proposed cognitive mechanism and its relation to the neuroscientific view.

III. MATERIALS AND METHODS

The mirror game setup for this study is as follows. The two subjects are seated facing each other with a set of parallel strings passing between them (Fig. 1). On each string, we place a ball that the subject can hold and move along the string. The players are instructed to play the game such that the designated follower should follow the designated leader's hand motion on the string. The leader is instructed to make "rich" motions (namely successive oscillations with variation in amplitude and frequency) while taking into account the follower's capacity to follow (i.e. slow down when needed to allow the follower to catch up). A total of eight subjects paired in four different dyads participated in this study. Each dyad made three trials at the game, each one lasting 60 seconds. The data consist of the 1D end-effector positions (the two balls position on the strings) recorded at a rate of 100 samples per second.

IV. PROPOSED MODEL

To model the behavior of the follower in this leader-follower setup, we first take a qualitative approach. In Fig. 2, one sample of leader-follower motion is illustrated. At the beginning, the follower shows an expected delay in his motion. Up to $15sec$, the tracking is satisfactory until the leader "*suddenly*" changes the location of the oscillation, and the follower shows a tendency to oscillate according to the last observed max and min points. This creates an interesting pattern in the follower's motion; i.e undershooting and overshooting. These observations imply that the follower uses an internal model which helps him/her to compensate for the delay. This forward modeling, however, worsens the tracking when the leader suddenly changes the dynamics of the motion and hence no longer matches the follower's expected model. In this case, the follower must build again an internal model of the leader's motion. In the following, we

consider these observations to drive our modeling approach.

A. Internal model

In this section we propose a cognitive mechanism (Fig. 3) to explain the follower's hand motion; i.e. a mathematical model which accurately describes the main qualitative features of the data. Receiving the leader's trajectory $(x_l(t))$ as the input, this cognitive mechanism generates the follower's trajectory $(x_f(t))$ as the output. This mechanism can be seen as three sub-systems placed in series: *sensory system*, *internal modeling*, and the *motor system*. Physiological sensory-motor systems have significant feedback delays. In our proposed mechanism, this is represented by a delay in the *sensory* sub-mechanism. The *internal model* plays the role of the cerebellum in controlling for the timing of the agent's response. We adapt this internal model to the delayed perceived motion $(x_l(t - \Delta t))$ and by forward integration, we estimate the follower's current position $(\tilde{x}_l(t))$. This estimation of current position is used as the set-point for the *motor system* where the motor system is represented by a 2nd-order closed-loop control system.

Relying on the delayed sensory data $(x_l(t - \Delta t))$ and using it as the set-point for the motor system limits human tracking performance. In contrast, modeling the leader's motion and using this to predict the leader's motion would not only improve tracking performance, but also lower control effort (no need to focus continuously our attention to visual feedback). There are myriad of studies attesting that humans benefit from these two aspects of forward modeling ([10], [11]). In our cognitive mechanism, we use a dynamical system to model and predict the leader's motion; see Fig. 4. The parameters of the dynamical system are updated by using previous data-points falling into a *memory window* (of
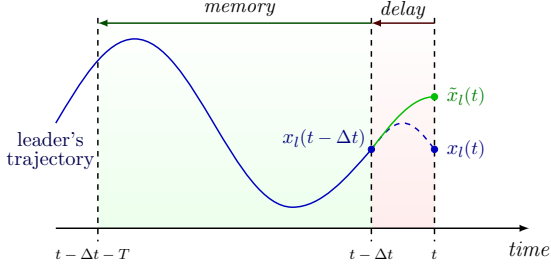
Fig. 4. Memory and delay as two fundamental model parameters. Memory controls the amount of previous data being used for updating, while delay controls prediction horizon.

length T). Once the model is updated, it is used to predict the current leader's position based on the leader's position and velocity at $t - \Delta t$. Memory length T and delay Δt are the model's parameters. These parameters can be tuned to achieve a desired behavior.

It is desirable to select a model of control which can capture the qualitative properties of the data, while producing dynamics close to humans. The oscillatory nature of the task and the smoothness of human motion can be realized by a 2nd-order dynamical system of the form:

$$\ddot{\tilde{x}}_l = a + b\tilde{x}_l + c\dot{\tilde{x}}_l \tag{1}$$

where the dynamical system parameters (a, b, and c) can be updated to model the leader's motion. Being linear with respect to parameters is a great advantage for this model as it enables us to use a simple learning method; i.e. Minimum-Least-Square (LS) method. In this method, the regressor matrix is

$$X = \begin{bmatrix} 1 & x_l(t - \Delta t) & \dot{x}_l(t - \Delta t) \\ 1 & x_l(t - \Delta t - dt) & \dot{x}_l(t - \Delta t - dt) \\ \vdots & \vdots & \vdots \\ 1 & x_l(t - \Delta t - T) & \dot{x}_l(t - \Delta t - T) \end{bmatrix} \tag{2}$$

and the target vector is

$$y = \begin{bmatrix} \ddot{x}_l(t - \Delta t) \\ \ddot{x}_l(t - \Delta t - dt) \\ \vdots \\ \ddot{x}_l(t - \Delta t - T) \end{bmatrix} \tag{3}$$

Therefore, using LS method, the parameter vector ($p = [a\ b\ c]^T$) is calculated as follows:

$$p = (X^T X)^{-1} X^T y \tag{4}$$

Having the model parameters updated, we can integrate forward to compensate the delay and predict the leader's current position ($\tilde{x}_l(t)$). During forward integration, we can saturate position, velocity, and acceleration based on the assumption that these quantities are limited on the leader's side. These saturations prohibit the model from generating fast, and unreasonable motions. Finally, the prediction ($\tilde{x}_l(t)$) is used as the desired trajectory for a 2nd-order dynamical system where a hand-tuned PD-controller performs
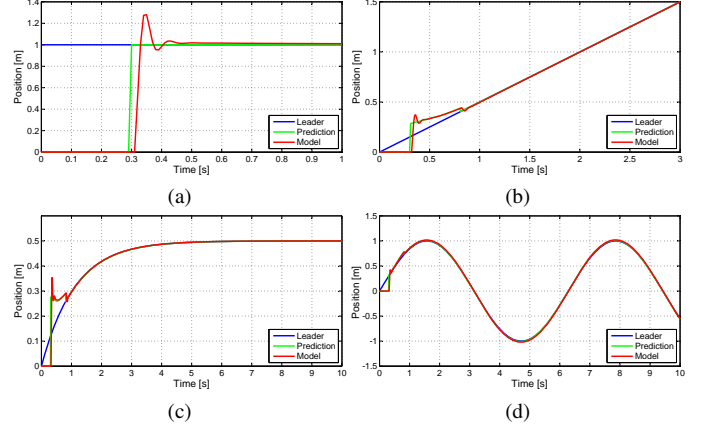

Fig. 5. Performance of our model to follow typical trajectories. The first three trajectories cause singularity for the LS method. In these simulations, we have $dt = 0.01s$, $\Delta t = .3s$, $T = .5$, $\mu_0 = 10$, and $\alpha = 1$. For discrete PD controller, we have $K_p = K_d = .5$

satisfactorily; see Fig. 3.

Using LS method, we should consider multiple local minimums case; i.e. singularities. To study this, we consider different possible scenarios where the columns of the regressor matrix (X) can have linear dependency:

- $\alpha 1 + \beta x_l = 0$: constant position
- $\alpha 1 + \beta \dot{x}_l = 0$: constant velocity
- $\alpha x_l + \beta \dot{x}_l = 0$: 1st-order dynamics

To overcome this, we use damped (regularized) LS method where we adaptively change the damping according to the regressor matrix condition number:

$$\begin{cases} \kappa(X) = \frac{\lambda_{max}(X)}{\lambda_{min}(X)} \\ \mu = \mu_0(1 - e^{-\alpha\kappa(X)}) \\ p = (X^T X + \mu I)^{-1} X^T y \end{cases} \tag{5}$$

One special case for our model is when $a = b = c = 0$ which means $\ddot{x} = 0$. This leads to predictive First-Order-Hold. Using damped LS pushes the solutions to this case which works fine for the aforementioned singular cases when the sampling time (dt) is small enough.

In Fig. 5, the performance of our method is tested for typical trajectories; i.e. different dynamics of motion. In Fig. 5a, we consider the constant position case. Based on the delay, during the first 0.3 seconds of the simulation, we do not have sensory input and it is not reasonable to move. After $0.3s$, the model is updated and its prediction is tracked by the 2nd-order dynamics motor system. Dealing with discrete PD controller, we have a lag of 2 samples between prediction and command generation by the model. For the constant velocity case (Fig. 5b) and first-order dynamics (Fig. 5c), the model performs satisfactorily. Having few data-points in the beginning of each simulation makes the prediction unreliable. Finally, in Fig. 5d, the proposed model is tested against sinusoidal trajectory. It is interesting to note that
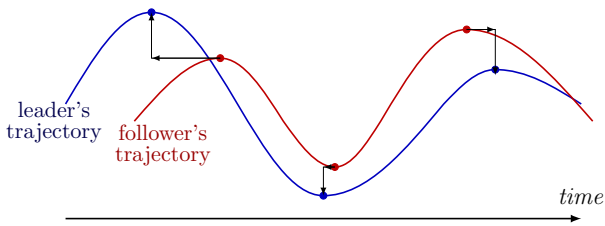
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Fig. 6. Extraction of temporal and spatial error from zero-velocity points. Horizontal arrows show temporal errors and vertical arrows show spatial error.

this trajectory is dynamically consistent with our model, and setting the damping in LS to zero will lead to $p = [0 \ \omega^2 \ 0]$ where ω is the oscillation frequency. Moreover, Fig. 5 shows that the proposed method can also follow the changes in the motion dynamics; e.g. from oscillation to reaching.

V. RESULTS

In order to tune our model to human performance, especially to account for the observed overshoot and undershoot, we focus on zero-velocity points. Two types of error can be extracted from zero-velocity points: *temporal error* (as used in [4]) and *spatial error*. For each zero-velocity point in the follower's trajectory, we consider the time-nearest zero-velocity instant of the leader's trajectory but with the same direction (minimum or maximum); see Fig. 6. Then we calculate temporal/spatial error based on the time/position difference.

With the features presented above, we study the human performance in Fig. 7. Interestingly, for the temporal error, most of the probability mass is present in the region with positive error. This shows that the follower's trajectory is lagging most of the time. The distribution in the negative part of temporal error shows that the follower sometimes changes the direction of motion sooner than the leader. This, again, speaks in favor of an internal model that guides the change in motion direction. Another interesting property of this graph is that almost all the delays are below $300ms$. We can use this observation and fix the delay in our model to $\Delta t = 300ms$.

The distribution of spatial error is very close to a normal distribution with mean $0.005m$ and standard deviation of $0.13m$. Similar distributions can be achieved by tuning our proposed model on the data-set. It is desirable to determine a set of parameters that match best the model-follower and human-follower distributions. To do this, we must extract feature vectors from the temporal and spatial distributions. We do this by counting the frequency of data-points in the following bins.

Here, w_h^t and w_h^s are the coarse representation of the temporal and spatial error distribution for human performance. Same
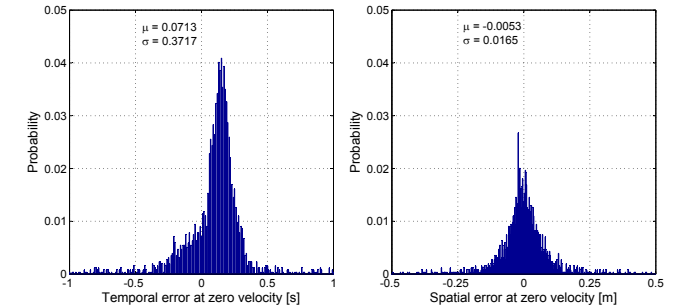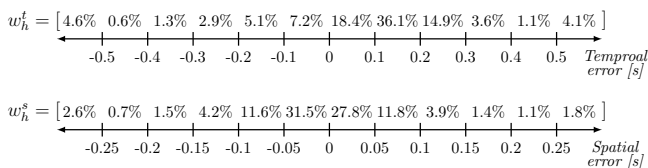

Fig. 7. Probability distribution of temporal and spatial error in zero-velocity points for human follower in mirror game setup.
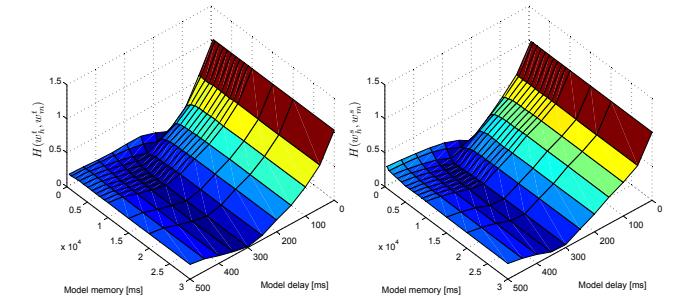

Fig. 8. Grid search for delay and memory parameters. Statistical difference between human and model follower in (left) temporal and (right) spatial error.
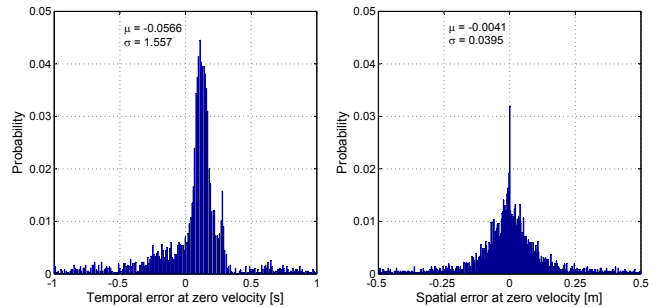

Fig. 9. Probability distribution of temporal and spatial error at zero-velocity points for model-follower in the mirror game setup with tuned parameters ($300ms$ for delay and $5s$ for memory).

feature vectors can be extracted for any model; i.e. w_m^t and w_m^s. Having these feature vectors, to tune model parameters, we can use the following cost function; i.e. Hellinger statistical distance:

$$H(w_h, w_m) = \frac{1}{\sqrt{2}} \parallel \sqrt{w_h} - \sqrt{w_m} \parallel_2 \qquad (6)$$

where w_h and w_m represent the human and model temporal or spatial error distribution.

Now that we formalized our tuning problem as a multi-objective optimization, we search for the best combination of our model's parameters; i.e. *delay* and *memory*. This search is illustrated in Fig. 8. As it can be seen, these cost functions are consistent with each other. Moreover they are more sensitive to changes in the *delay* parameter than in the *memory* parameter. The best choice for *delay* is $300ms$ which is consistent with our previous hypothesis from Fig. 7.
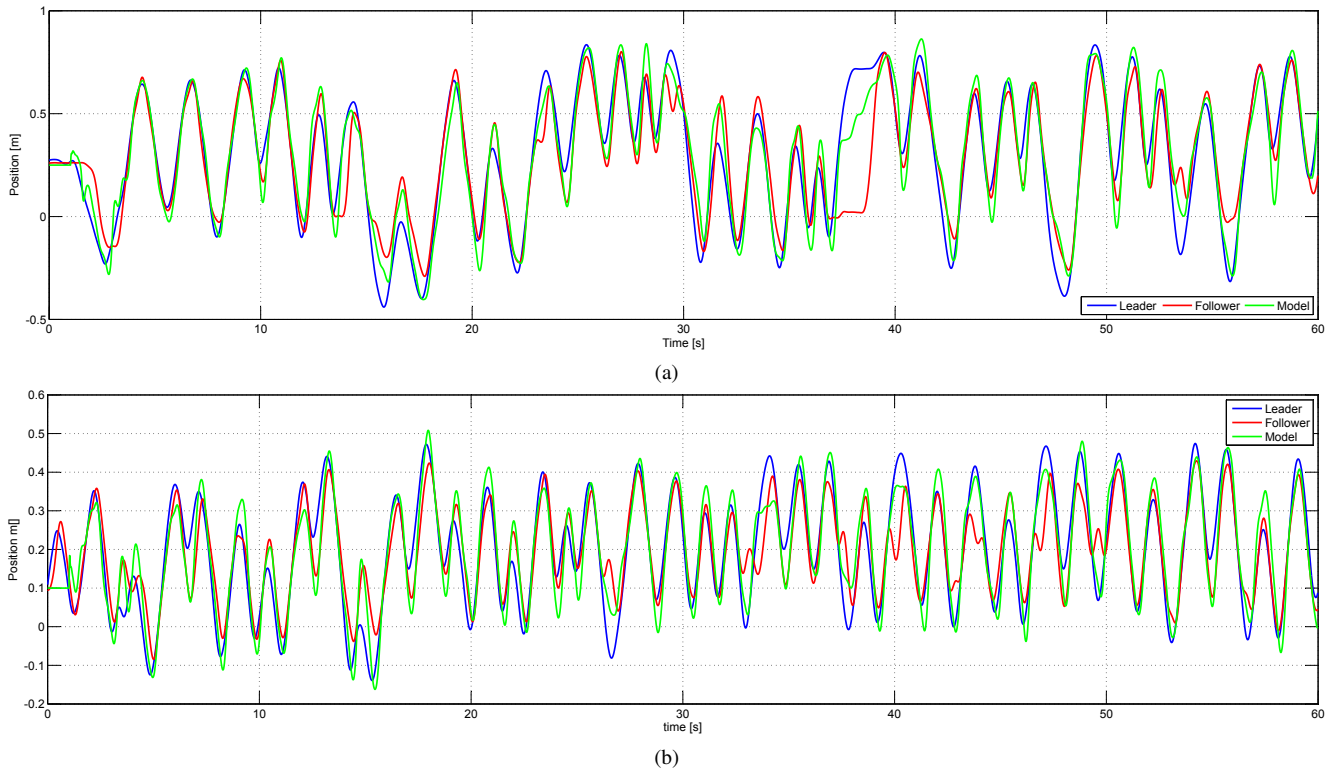
(a)

(b)

Fig. 10.   Model performance for Leader-follower setup.

This delay is also in agreement with the previous studies in human reaction time to changes in direction [12]. Having $300ms$ for *delay*, these cost functions are almost insensitive to *memory*. Here, we pick $5s$ for *memory* where the cost functions exhibits less sensitivity to the other parameter; i.e. robustness.

The temporal and spatial error distribution of our model with tuned parameters are illustrated in Fig. 9. As it can be seen, the temporal and spatial error distribution graphs are highly similar to those for human-follower in Fig. 7. In both graphs, the maximum and cut-off delay (a point which contains 90% of the temporal error distribution) are alike. In both human and the model, the temporal distribution in the negative part (where the follower switches direction sooner than the leader) is also similar.

Fig. 10 shows the performance of the model on the experimental data (leader's trajectory). Our model's output matches the human-follower trajectory most of the time. It however accounts relatively poorly for the overshoots and undershoots. This is likely due to the fact that these are caused by previous max and min points of trajectory and our dynamical model can only model the offset (center of oscillations). Improving our model to take these into account, we might be able to create these pattern more accurately. In the next section, we discuss some interesting extensions to our model that can create more human-like over and undershoots.
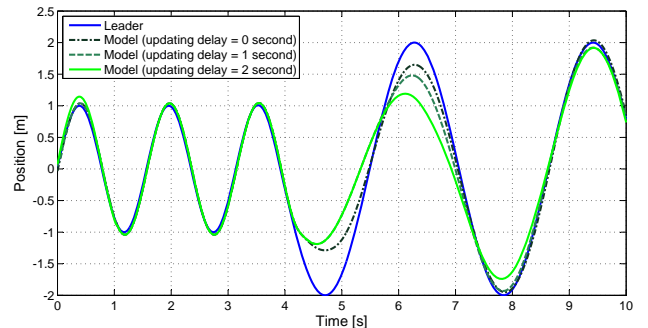

Fig. 11.   Performance of the proposed method with delayed update for the internal model.

VI.   DISCUSSION

Here we discuss different approaches to create more drastic over and undershoot behaviors. To test these approaches in a more controlled situation, we consider a synthesized trajectory for the leader; i.e. a sinusoidal trajectory (with frequency of $4rad/s$ and amplitude of $1m$) which smoothly changes to a different one ($2rad/s$ and $2m$).

Delay in updating: A delay can be added to the memory window (as shown in Fig. 4) to have a outdated internal model. This means instead of using the data in $[t - \Delta t - T \quad t - \Delta t]$ interval, we use $[t - \Delta t - T - T_u \quad t - \Delta t - T_u]$ interval where T_u controls the delay in updating. As shown in Fig. 11, by increasing this parameter, the follower behavior will be more similar to the observed motion for a longer time which creates a more drastic undershoot.
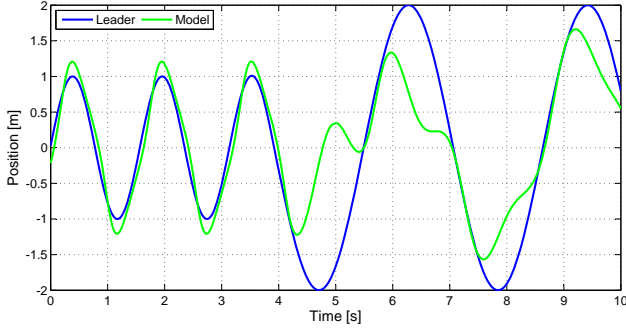
Fig. 12. Performance of the proposed method using nonlinear terms in internal model dynamics.
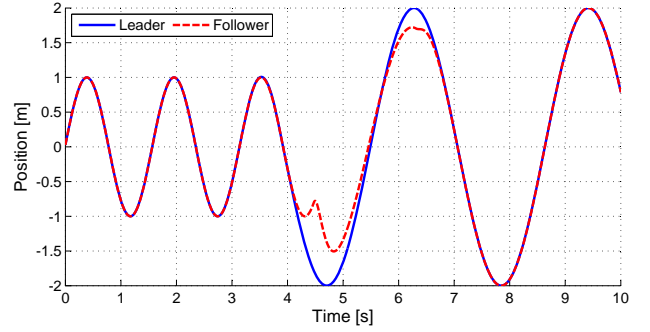

Fig. 13. Switching between an autonomous internal dynamics and tracking.

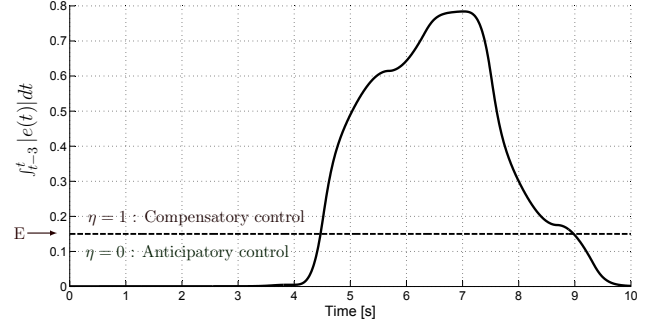

Fig. 14. Switching between anticipatory and compensatory behavior based on the integration of the tracking error.

Nonlinear terms: Using nonlinear terms in the internal dynamics (Eq. 1) can improve under and overshooting. For example using a nonlinear spring ($\ddot{\tilde{x}}_f = a + b\tilde{x}_f^3 + c$), pulls far-displaced trajectories faster to the center of oscillation compared to linear one. Performance of our method with nonlinear spring is illustrated in Fig. 12. As it can be seen, undershoots are more drastic compared to the linear spring and the behavior around $5s$ is similar to human recovering; e.g. undershoot around $27s$ in Fig. 10b. However, adaptation to new trajectory is very slow compared to human performance.

Forced dynamical systems: In the proposed method, the leader's position and velocity is always being used for forward prediction which imposes high control effort. Here, we propose a more autonomous model. This method is developed upon the assumption that human-follower switches between an autonomous dynamics (internal model) and tracking. Therefore, we propose the following dynamics.

$$\ddot{x}_f(t) = f(x_f(t)) + \eta(t)g(e(t)) \tag{7}$$

In this model, $f(x(t))$ represents the internal dynamics which can be adapted to the leader's trajectory. Tracking of the leader's trajectory can be achieved by $g(e(t))$ where $e(t) = x_l(t) - x_f(t)$. Switching between relying on the internal model and tracking is simulated by $\eta(t)$. A simple choice for these functions can be:

$$\begin{cases} f(x(t)) = -kx(t) \\ g(e(t)) = K_p e(t) + K_d \dot{e}(t) \\ \eta(t) = \begin{cases} 1 & \text{if } \int_{t-T}^{t} |e(t)| dt > E \\ 0 & \text{otherwise} \end{cases} \end{cases} \tag{8}$$

Our choice of $f(x)$ is the most simple oscillatory system as well as PD-controller for tracking dynamics ($g(e)$). Switching between internal model and tracking happens when the integral of the error over last T seconds goes over a threshold E. Performance of such dynamics for a sinusoidal trajectory is shown in Fig. 13. The frequency and amplitude of this reference trajectory is smoothly changing from $4rad/s$ to $1rad/s$ and from $1m$ to $2m$. For this simulation, $K_p = 60$, $K_d = 15$, $T = 3s$, and $E = 1.5ms$. Setting $k = 16N/m$ consistent with the trajectory is sufficient for good tracking behavior until the leader changes its frequency and amplitude. This sudden change in leader's trajectory creates an error between leader and follower which triggers the tracking

dynamics ($\eta = 1$); see Fig. 14. Reducing the error and tuning $k = 4N/m$ manually consistent with new frequency makes the internal model sufficient for tracking. Transitory behavior of this model is quite similar to human performance; e.g. undershoot around $27s$ in Fig. 10b. Moreover, Fig. 14 shows how the integration of error triggers the compensatory control and how the absence of tracking error triggers back the anticipatory control; i.e internal model. Switching between anticipatory ($f(x)$) and compensatory ($g(e)$) control has been proposed and studied in action coordination in groups and individuals [13]. Evaluation of this model –forced dynamical system– for the experimental data requires parameter tuning which lays out of this paper scope.

VII. CONCLUSION

In this paper, we introduced a cognitive mechanism capable of producing human-like motions for the mirror game setup. We built this model based on qualitative assumptions and observations from human data recordings. Moreover, using quantitative methods, we tuned our model's parameters to fit the human data. We showed that simple dynamical models can be used explain and reproduce the follower's motion in the LF setting of the mirror game. We also proposed a number of avenues to improve the performance of the model, so as to account for the adverse side-effects (overshoot and undershoot) of forward models.

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