Applications of Model Reduction Techniques in Aerospace Combustors

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In this paper we consider some practical applications of model reduction methods in unstable gas turbine and rocket combustion. We explore a set of promising methods for reducing computational burden in large scale LES calculations of unsteady turbulent combustion, as well as define scenarios in which model reduction can be advantageous. Three combustors will be considered here, that have been well documented in recent times. Reduction of the time taken to perform parametric unsteady simulations of a given geometry will be discussed first. Next, we will present our experiences in the use of reduced models as surrogate representations of fuel injectors, and demonstrate multi-injector simulations in which reduced basis models inter-operate with one another and with full numerical computations. This paper will be an empirical exploration of these ideas, while the formal mathematical implications will be discussed in a separate paper.

I. Introduction

In a recent paper (Munipalli et al.[15]) we discussed some prospects for formal computational model reduction in combustion dynamics simulations, and presented promising methods with preliminary results. By “model reduction” we imply the reduction of numerical effort needed in performing a certain simulation without changing the physical or mathematical model. Rather, reduced basis methods (RBMs[16]) exploit the low-dimensional nature of solution spaces using which seemingly complex numerically computed field data can be reconstructed. This is different from methods in which simplified variants of the governing equations are solved (e.g., ref. [12]). Recent RBM results have indicated many orders of magnitude reduction in computational effort in a number of scenarios of physical interest (refs [4, 5, 10]).

Turbulent combustion calculations based on LES are the most promising method to model the dynamics of gas turbine and rocket combustion chambers. These simulations can save a great deal of risk and expense in full scale testing of future combustor designs. However, the computational problem size encountered here is usually very large, on account of the fine mesh resolution needed to capture mixing and other phenomena, physical complexity, and tedious nonlinear calculations associated with chemistry and the equation of state. In ref. [15] we suggested ways in which RBM techniques can be applied to such computations, and noted the limitations posed by the extremely nonlinear nature of the governing equations. Nonlinearity primarily limits the efficiency and stability of

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most popular reduced basis strategies in literature. There are many successful RBM variants (refs [5,7,9,10,17]) in literature which seek to overcome these limitations. However, each new physical application poses unique challenges, and both the method and the manner in which model reduction is to be affected must be customized to procure the maximum benefit.

In this paper we will present three combustor studies to which we have been applying RBM procedures. We will highlight the types of computational gains that are sought, describe the methods that are used, and analyze the performance of the reduced model. We shall focus on an empirical exploration of methods and defer formal mathematical proofs to a forthcoming publication. Emphasis on instabilities related to injector design (ref. [3]) dominate our interest in this subject, due to the often extreme parametric sensitivity of the injector, and the computational effort needed to model the dynamics of a large multi-injector combustor.

II. Mathematical Apparatus

We present the outline of the procedure described earlier in ref. [15] below, and defer a discussion on method updates, and improvements to the final version of the paper. We begin with a system of conservation laws (the Favre filtered Navier-Stokes equations solved in LES [11]):

$$\frac{\partial Q(x,t)}{\partial t} = \overbrace{-AQ(x,t)}^{\text{Linear}} - \overbrace{F(Q(x,t))}^{\text{Non-linear}}$$

where AQ is a sum of linear fluxes and source terms, and F is the nonlinear flux of these quantities (including source terms which accounts for species production and depletion rates and so forth). Q is then approximated as Q_{RBM} using a modal expansion (Galerkin technique) in terms of modes ψ_n as: $Q_{RBM}(x,t) = \sum_{n=1}^{N} Q_{Rn}(t)\psi_n(x)$, where the modes are obtained such that this expansion reduces an appropriately defined numerical error: $\left\| Q(x,t) - Q_{RBM}(x,t) \right\|$ as the number of modes in this expansion is increased. The coefficients Q_R are obtained by solving a system of 1st order ordinary differential equations (stated here for the case when the modes are orthogonal to one another):

$$\frac{d\,Q_{Rn}(t)}{d\,t} = -AQ_{Rn}(t) - \psi_n(x)F\left(\sum_m \psi_m(x)Q_{Rm}(t)\right)$$

The calculation proceeds in two parts: The first is performed "*offline*", when the full set of equations is solved at a set of equal intervals in time, and solution snapshots are assembled. These snapshots can be used to construct a set of orthogonal basis functions ψ_n using techniques such as proper orthogonal decomposition (POD). (See, e.g., [13] for details about this procedure). These basis functions provide a representation of the solution snapshots in the following sense. If we consider a matrix V_n whose columns are the basis functions, we can "project" the set of snapshots $Q(x,t)$ into a space spanned by these basis functions, and write:

$$Q(x,t) \approx V_n(x)Q_R(t)$$

This expression is approximate if we do not include the entire set of basis functions. When we select the basis functions with the largest eigen values, we effectively compress the snapshot data into a lower dimensional representation. The discrete version of the projected ODE set will be (we assume there are N modes and K cells in the computational domain):

$$\underbrace{\frac{d\,Q_R(t)}{d\,t}}_{\mathrm{N \times 1}} = \underbrace{V_n^T A V_n(x)}_{\mathrm{N \times N}} Q_R(t) + \underbrace{V_n^T}_{\mathrm{N \times K}} F\underbrace{(V_n(x)Q_R(t))}_{\mathrm{K \times K}}$$

The second part in RBM is an "*online*" calculation which involves the solution to the ODEs mentioned above. These equations are solved to obtain temporal coefficients of the basis functions, which by requirement are far fewer than the full number of unknowns, as mentioned earlier. An efficiency problem arises immediately due to the nonlinear fluxes. The last term in the above equation shows that the operation count to evaluate the nonlinear flux is unchanged from the full model and scales with the number of cells in the domain K. Chaturantabut et al. [7] present a method to overcome this inefficiency, developing a Discrete Empirical Interpolation Method (DEIM), which has been used successfully in more general RBM scenarios by Drohmann et al [9]. Their approach is as follows.

Approximate the nonlinear flux in terms of modes $U(x)$ as follows: $F(Q(x,t)) \approx U(x)c(t)$.

In the above, the modes $U(x) = \{u_1(x), \ldots u_m(x)\}$ are obtained from a space spanned by the POD modes describing the nonlinear flux *F*. A greedy algorithm (Empirical Interpolation Method, EIM), selects a subset of this modal decomposition to reduce the size of the F vector to m (<< N). We simply state the final expression for the reduced model (where *P* is a selection matrix which picks the elements of U above that are needed):

$$\underbrace{\frac{dQ_R(t)}{dt}}_{N \times 1} = \underbrace{V_n^T A V_n(x)}_{N \times N} Q_R(t) + \underbrace{V_n^T U(P^T U)^{-1}}_{N \times m \text{ (precomputed)}} F(\underbrace{P^T V_n(x) Q_R(t)}_{m \times 1})$$

This last equation is the final form of the reduced basis model of the nonlinear conservation law. There are several possible variants of this approach. The method can be extended to problems with multiple parameters, wherein a POD-type approach is used to model (compress) the time variation, and a greedy algorithm is used to determine the modes that are parameter dependent. Time varying parametrized functions can be represented at discrete instances of time t_k, where $k = 1 \ldots K_{max}$ as time samples are selected in the range $t_k=\{0 \ldots T\}$ using a slightly rewritten version of the earlier expansion: $Q^k(x,\mu) \approx V_n(x) Q_R^k(\mu)$, where *k* is a time-index, and the amplitudes Q_R are now time trajectories for a given parameter vector μ.

We shall also consider alternative approaches (e.g., GNAT, ref. [5]), and some recent robust variants of the DEIM approach (e.g., ref. [1]).

III. Demonstrations and key results

We will consider three combustor models in this study:

1. NASA Lean Direct Injection (LDI) injector element (fig. 1): Developed as a low NOx injector for gas turbine engines, the LDI design is susceptible to thermoacoustic instabilities. Both experimental and computational LES studies (e.g., ref. [14]) have been conducted. We are building reduced models with and without Lagrangian models for a spray injection of fuel for this injector. In the final paper we will present the reduced basis surrogate model (RBSM) of the injector, and explore a multi-injector implementation based on reusing the RBSM in an array (e.g., see fig. 1). Performance when compared with full simulations will be documented, subject to appropriate representations of the interaction processes. A Kerosene-air combustion model is used. The effect of the flow upon the spray as well as a two-way coupled approach will be presented separately (a preliminary one-way interaction model of a flow RBM with spray is shown in fig. 2).
2. A shear-coaxial injector element used in studies at the technical university of Munich (ref. [6], fig. 3) will be used to develop RBMs of the combustor. Long period integration of the reduced basis model will be used to demonstrate stability characteristics of different model reduction approaches.
3. The Continuously Varying Resonance Combustor (CVRC, ref [18], fig. 4): We will present a reduced basis model of the CVRC subject to parametric variations in upstream flow conditions. A multi-injector setup of the CVRC will be modeled. Chemistry model will comprise of gaseous methane and hydrogen.

Computational efficieny: We will present our approach to efficient storage of solution snapshots and linear algebra operations in handling large datasets. Reduction in simulation time vs. loss in stability and/or accuracy will be discussed.

Background of the methods to be used, with relevant references has been presented in ref. [15]. In this paper we shall mainly deal with applications and our experiences in using these methods. Given the nature of the nonlinearities encountered here, and the frequency rich content of the data, we recognize that a formal mathematical treatment may have to occur in several important stages. We shall outline some of these difficulties and defer the details to a future publication.

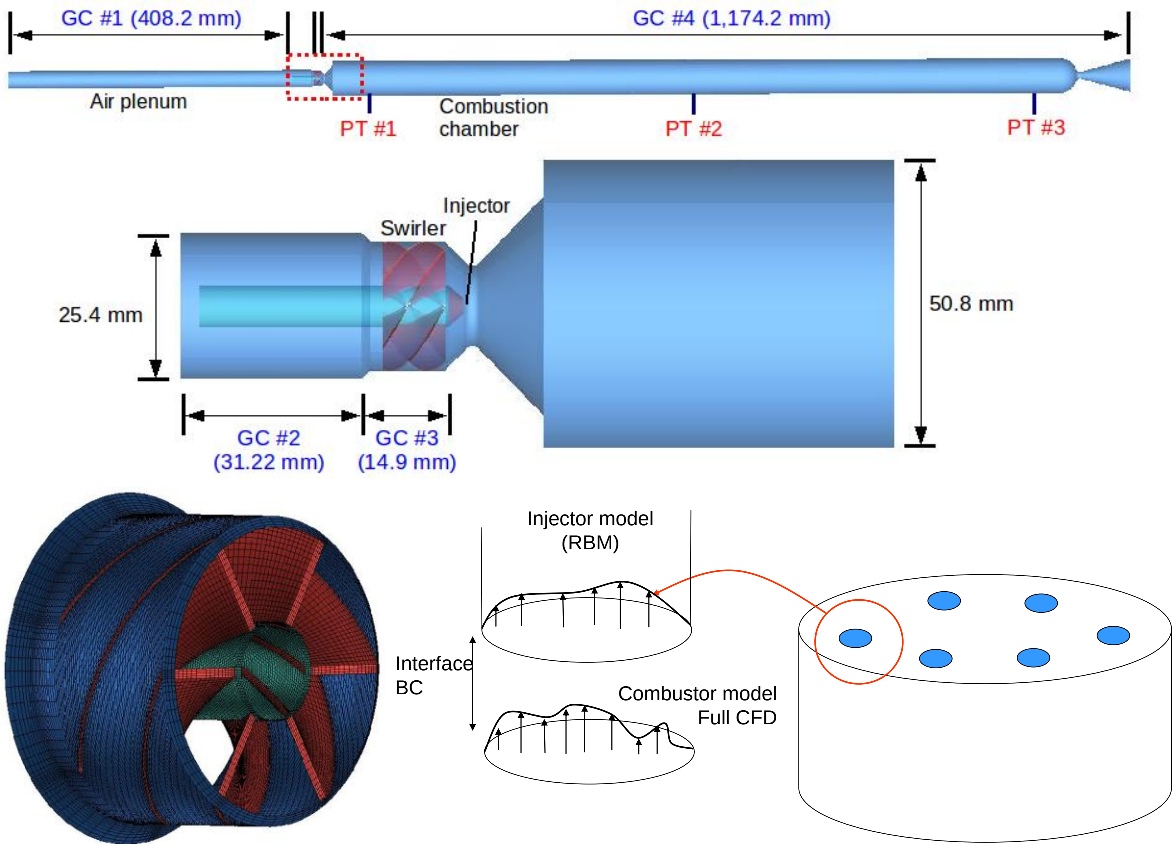


Figure 1: Lean Direct-Injection gas turbine injector element geometry (see Kim et al. [14]), and a schematic of a multi-injector setup

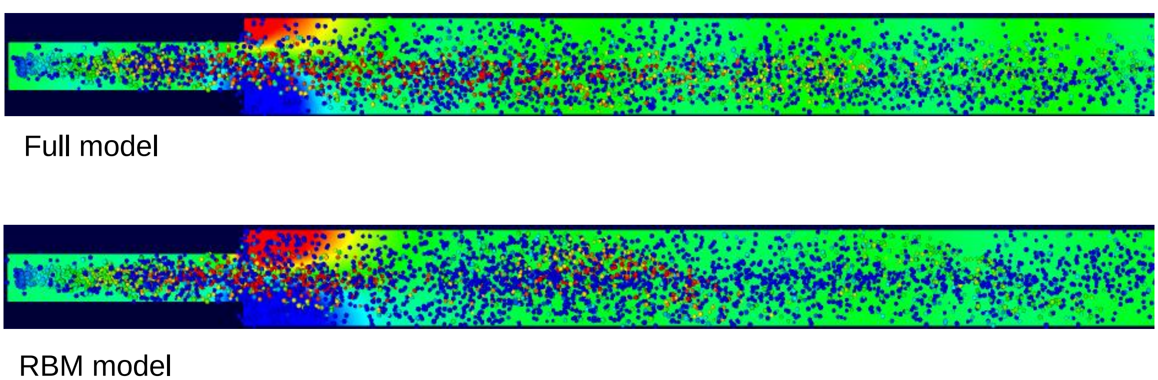
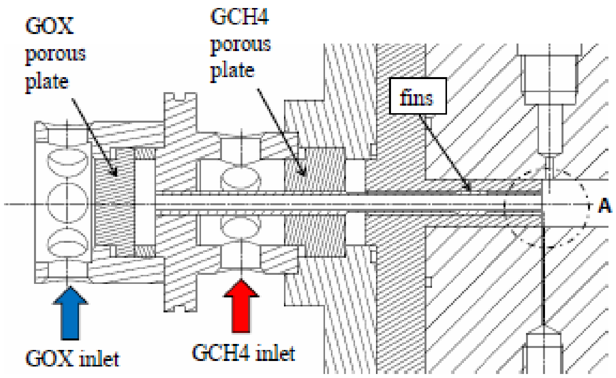


Figure 2: RBM model of flow interacting with the Lagrangian computation of a spray



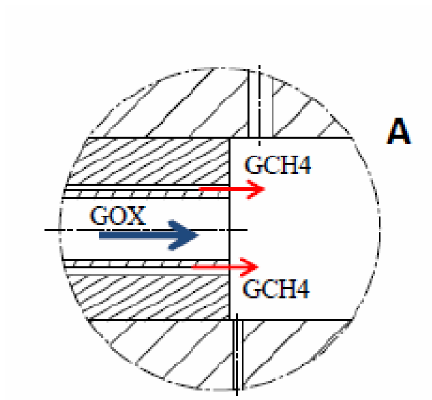
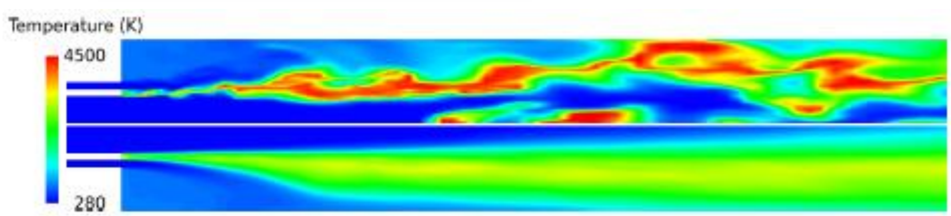


Figure 3: Schematic of the single element injector and computed temperature contours (from ref [6])



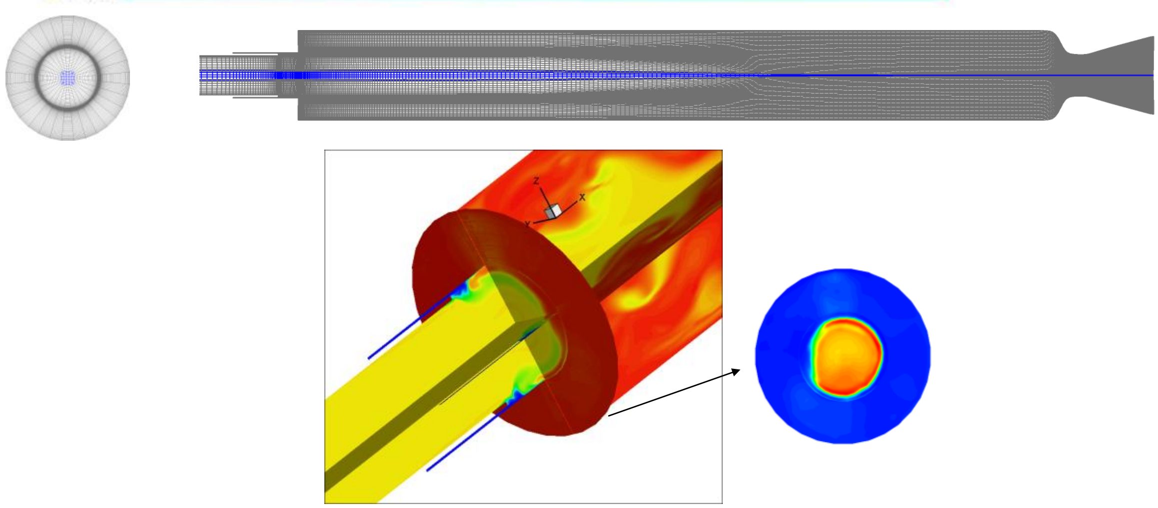


Figure 4: The CVRC geometry (Yu et al.[18]) showing the computed flow detail near the injector

IV. Summary

We will present mathematical procedures and outcomes from ongoing investigations in the computational model reduction for LES calculations of turbulent combustion dynamics for a few representative combustors. These results will be of an exploratory nature in the application of RBM techniques in detailed CFD, and will prepare the ground work towards a formal mathematical treatment of the many computational complexities in general purpose nonlinear model reduction.

Acknowledgments

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