A method to create a microwave notch filter through dynamic Brillouin gratings is proposed and numerically demonstrated. It exploits the thumbtack correlation peaks of pseudo random bit sequences.

1. Introduction
Dynamic Brillouin gratings (DBG) in polarization maintaining fibers (PMFs) [1] have been shown to be a powerful technique to enhance the performance of Brillouin fiber sensors [2, 3, 4, 5, 6, 7, 8, 9], and to obtain unconventional signal processing [10] or delay lines [11, 12]. By exploiting stimulated Brillouin scattering (SBS) of two counter-propagating optical waves at frequency \( \nu_{w1} \) and \( \nu_{w2} = \nu_{w1} - \nu_B \) (where \( \nu_B \) is about 11 GHz in silica fibers) an acoustic wave is generated by the electrostriction effect. The acoustic wave longitudinally modulates the fiber refractive index, thus creating a grating. The DBG decays on a time scale of several ns and moves at the sound speed; so, over the short time of its life, it can be considered static. If a new light beam is injected into the fiber it can be scattered back by the DBG. In PMFs, the DBG writing and reading processes can be decoupled by using orthogonal states of polarization, aligned to the birefringence axis [1]. In fact, the acoustic wave equally scatters all light polarizations owing to its longitudinal nature.

Recently, a method to obtain a stable and localized DBG has been introduced by exploiting the thumbtack correlation of chaotic waveforms [8, 9]. In that case the aperiodicity of chaotic waveforms guarantees the creation of a unique DBG within the fiber.

Here, by considering a similar setup, a method to create multiple, stable and well-localized DBG within a fiber is presented. This can be realized by exploiting the multiple thumbtack correlation peaks of pseudo random bit sequences (PRBS), by properly setting the PRBS length, the PRBS bit-rate and the fiber length. In particular, the generation of two DBGs is considered and a possible application in microwave photonics (MWP) is demonstrated: in fact, two DBGs actually realize an interferometer. A continuous wave injected as reading waveform is reflected by both DBGs and the output signal is the superposition of the two reflected waves, so a microwave notch filter can be achieved. The generation of DBGs and the realization of the notch filter are theoretically and numerically investigated.

2. Model
In Fig. 1a, a simplified setup of the proposed scheme is shown. The same PRBS signal drives two optical phase modulators, that modulate a laser signal at frequency \( \nu_{w1} \) and a sideband obtained from the same laser, shifted by \( \nu_B \) from \( \nu_{w1} \). The modulator biases are adjusted such that at the mean value of the driving electrical signal the phase shift is zero. Phase modulators are used to exploit the constructive-destructive correlation operation that is peculiar of the Brillouin interaction. The write setup is very similar to that introduced in refs. [8, 9].

The DBG writing/reading processes in a PMF are governed by the following equations:

\[
\partial_z A_{w1} + \beta_1 s \partial_t A_{w1} = -\eta g_B Q A_{w2},
\]

\[
-\partial_z A_{w2} + \beta_1 s \partial_t A_{w2} = \eta g_B Q^* A_{w1},
\]
The theory for the DBG generation is similar to one introduced for writing through chaotic waveforms [8, 9]. By considering eq. 5, the term $A_{w1}A_{w2}^*$ represents the correlation of the two writing waveforms, both modulated by the PRBS: given that the time envelope of the counterpropagating waveforms is the same, the DBGs are created only at discrete positions within the fiber, $z_i$, $i = 1, 2, \ldots$, at which the writing waveforms are perfectly correlated. The distance between adjacent correlation peaks is determined by the PRBS length. By tuning the PRBS length or the PRBS bit-rate or fiber length, one can obtain one or more DBGs within the fiber.

The generated DBGs are also permanently sustained because the writing signals are countedly written through the fiber. In fact, the DBGs time evolution at $z_i$ ($Q(z_i, t) = Q_i(t)$) is governed by eq. 5 which can be casted in the following form:

$$\partial_\tau Q_i(\tau) = -Q_i(\tau) + C(\tau),$$

where $\tau = t/(2\tau_B)$ and $C(\tau) = A_{w1}(\tau)A_{w2}(\tau)$. As in the case of chaotic waveforms [8, 9], the estimation of the mean value of each $Q_i$ can be obtained by reformulating eq. 6 as:

$$\partial_\tau Q_i(\tau) = -Q_i(\tau) + C_0 + \Delta C(\tau),$$

where $C_0 = \sqrt{P_{w1}P_{w2}/(\eta A_{\text{eff}})}$ is the mean of $C(\tau)$ and $\Delta C(\tau)$ the fluctuation around the mean value. By averaging and solving the equation one obtains the mean value of the DBGs as $\overline{Q_i(\tau)} = C_0 [1 - \exp(-\tau)]$. Therefore, after a transient regime, permanent DBGs are sustained at the locations $z_i$. In the simulation presented here, a $2^{5} - 1$ long PRBS with a rate $R = 10$ GHz is considered, so two DBGs are generated within the fiber ($L = 50$ cm). In fig. 1b,
the numerical solutions of eqs. 1-5 (solid blue and red curves), obtained through a split-step Fourier method, and the analytical solutions $\mathcal{Q}_i$ (black dashed curves) are compared, showing a good agreement. The two created DBGs are also very well localized in space, as shown in Fig. 1c, where a time-space contour diagram is shown.

A double DBG can be exploited to create a microwave photonics notch filter. In fact, let us consider a reading waveform $A_r$, with a state of polarization aligned to the fast axis of the fiber. The reading waveform is scattered by the two DBGs, so the output signal $A_{\text{out}}$ is the superposition of the two delayed reflections from the DBGs. This can be easily demonstrated in the ideal case in which the two DBGs are Dirac functions. By considering a change of variable $z' = z - ct$ such that the reference frame moves with $A_{\text{out}}$, from eq. 4 the output waveform can be straightforwardly determined:

$$A_{\text{out}} = -\eta g_B \int_0^t A_i(z' + 2ct')Q(z' + ct')dt'.$$

(8)

The acoustic wave is then expressed by $Q(z' + ct) = q_1 \exp(j\phi_1) \delta(2z' + 2ct) + q_2 \exp(j\phi_2) \delta(2z' + 2ct + \Delta)$, where $q_1$, $q_2 = kg_1$, $\phi_1$ and $\phi_2 = \phi_1 + \phi$ are respectively the DBGs amplitudes and phases and $\Delta = c(2^3 - 1)/R$ is the distance between the two DBGs. The factor 2 in the argument of Delta functions is due to the fact that the DBG creation also entails a spatial compression [13]. Let us notice that in the ideal case the DBGs represent the transfer function of an ideal interferometer [14]. Let us consider a reading waveform modulated by a sinusoidal continuous wave at a frequency $\nu_M$, by substituting $A_i(z' + 2ct) = A_M \sin(2\pi \nu_M(z' + 2ct)/c)$ in eq. 8, one can get that the output signal is the superposition of the two reflections on the DBGs:

$$A_{\text{out}} = \eta g_B A_M q_1 \exp(j\phi_1) [\sin(2\pi \nu_M(z - ct)/c) + k \exp(j\phi)\sin(2\pi \nu_M(z - ct + \Delta)/c)],$$

(9)

in the reference frame fixed with the fiber. By tuning the modulation frequency $\nu_M$ and by calculating the maximum detected output amplitude for each considered frequency, a microwave notch filter can be achieved, as shown in Fig. 2b. The numerical results (blue curve) are obtained by integrating eqs. 1-5. The theoretical evolution (black curve) is given by the frequency response of the ideal notch filter and expressed by the Fourier transform of $Q$. It can be easily shown that the ratio between the two DBGs amplitude ($k$) modifies the notch depth while the phase difference between the two DBGs ($\phi$) shifts the notch frequency. Here, the coefficients $k$ and $\phi$ are estimated from the numerical simulations, which yield $k \sim 0.994$ and $\phi \sim -0.24 \text{ rad}$. It has been also verified that both coefficients are quite constant with time, thus indicating a strong relative stability of the two DBGs. From Fig. 2b, the agreement is good. The notch depth obtained from simulations is only $-20 \text{ dB}$ stemming from the fact that $k \neq 1$ and that the real notch frequency is not reached exactly in the simulations. Finally, the free spectral range of the filter is given by $FSR = c/\Delta$ and it is freely tunable as it is determined by the PRBS length and rate. Finally, let us remark that this filter, differently from previously realized filters
is operated in the coherent regime. In fact, DBGs are phase conjugating mirrors and so any optical phase shift is self-compensated when the reflected beams interfere.

4. Conclusions

In conclusion, a microwave photonics notch filter is designed by exploiting permanent dynamical Brillouin gratings. The gratings are created in polarization maintaining fibers by exploiting the thumbtack correlation peaks of pseudo random bit sequences. The analytical results well compare with the solutions of the full interaction model, based on Brillouin equations. The free spectral range of the obtained notch filter is tunable by varying the pseudo random bit sequence length or rate.

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References