Robust optimization to deal with uncertainty

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ROADEF 2014 - Bordeaux

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Stochastic optimization

- Static Problems
  - Formulation based on decision theory: utility and expected utility. (Hypothesis: uncertainty is not influenced by the decisions.)
  - Shortcomings. Computation of expectation as multidimensional integrals
  - Incomplete information on the distributions
- Dynamic problems
  - Recourse decisions as functions of past history
  - Explosion of the complexity.

Contents

Optimization problem with uncertain coefficients

For simplicity we limit the presentation to linear programming with uncertain coefficients.

$$\min \quad \sum_j c_j x_j$$
$$\sum_j a_{ij} x_j \leq 0, \forall i$$

The coefficients *a* are uncertain.
The focus is on maintaining feasibility in the constraints. Always? Most of the time? What if we know little about the uncertainties?

Model for the uncertain constraint

Model for a_j (we drop the index i of the constraint in the LP).

$$a_j = a_j^0 + \sum_k a_j^k \xi_k$$

a_j^0 is the *nominal value* and ξ is the uncertainty factor acting through a_j^k.
The LP constraint is

$$\underbrace{\sum_j a_j^0 x_j}_{\text{certain}} + \underbrace{\sum_k (\sum_j a_j^k x_j)\xi_k}_{\text{uncertain}} \leq 0. \qquad (1)$$

From now on we focus on the uncertain term and on its possible large values.

Capturing the knowledge on uncertainty

Suppose the decision-maker can only provide a range of variation for each a_j

$$\underline{a}_j \leq a_j \leq \bar{a}_j.$$

Define

$$a_j^0 = \frac{\bar{a}_j + \underline{a}_j}{2}$$

$$a_j^k = \frac{\bar{a}_j - \underline{a}_j}{2} \quad \text{for } k = 1,\ 0 \text{ otherwise}$$

$$\xi_k \in [-1, 1]$$

Worst possible situation according to the D-M

The worst value of the uncertain term when

- $\xi_k = 1$ if $\sum_j a_j^k x_j > 0$
- $\xi_k = -1$ if $\sum_j a_j^k x_j \leq 0$

ξ_k is such that

$$\left(\sum_j a_j^k x_j\right) \xi^k = |\sum_j a_j^k x_j|.$$

$$\underbrace{\sum_j a_j^0 x_j}_{\text{certain}} \quad + \quad \underbrace{\sum_k |\sum_j a_j^k x_j|}_{\text{immunization}} \quad \leq \quad 0. \tag{2}$$

Summary

- Model for the uncertain parameters
- Knowledge on uncertainty is captured in an uncertainty set

$$\mathcal{U} = \{\xi \mid -1 \leq \xi_k \leq 1, \forall k\}$$

- The robust counterpart of the uncertain constraint is

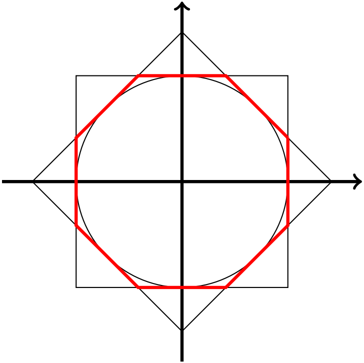
$$\sum_j a_j^0 x_j + \sum_k (\sum_j a_j^k x_j)\xi_k \leq 0, \forall \xi \in \mathcal{U} \tag{3}$$

- A solution of (3) is called robust for the uncertainty set $\mathcal{U}$
- Derive the equivalent of the robust counterpart

$$\sum_j a_j^0 x_j + \sum_k |\sum_j a_j^k x_j| \leq b$$

More sophisticated uncertainty sets

In addition to the range information, the D-M believes that not all uncertain factors ξ_j can achieves simultaneously large absolute values.



More sophisticated uncertainty sets

1. Ellipsoidal uncertainty set

$$\mathcal{U} = \{\xi : (\sum_i \xi_i^2)^{\frac{1}{2}} \leq \kappa, \ -1 \leq \xi_i \leq 1\}$$

2. Polyhhedral uncertainty set

$$\mathcal{U} = \{\xi : \sum_i |\xi_i| \leq \kappa, \ -1 \leq \xi_i \leq 1\}$$

κ is an immunization factor. The larger κ, the larger the uncertainty set, and the larger the worst case value of the uncertain component of the constraint.

Equivalent robust counterpart with an ellipsoidal set

- Set $z_k = \sum_i a_i^k x_i$. The robust counterpart of the uncertain constraint is

$$\sum_i a_i^0 x_i + \max_\xi \{\sum_k z_k \xi_k : ||\xi||_2 \leq \kappa, \ -1 \leq \xi_k \leq 1\} \leq 0.$$

- The dual of the inner maximization problem is

$$\min_u \left(||u||_1 + \kappa ||z - u||_2\right).$$

- The *equivalent robust counterpart* of the uncertain constraint is

$$\sum_i a_i^0 x_i + \min_u \left(||u||_1 + \kappa ||z - u||_2\right) \leq 0.$$

- If the constraint is embedded in an optimization problem, we can drop the min operator and let the overall optimization scheme manage the auxiliary variable u

$$\sum_i a_i^0 x_i + ||u||_1 + \kappa ||z - u||_2 \leq 0.$$

Equivalent robust counterpart with a polyhedral set

- Set $z_k = \sum_i a_i^k x_i$. The robust counterpart of the uncertain constraint is

$$\sum_i a_i^0 x_i + \max_{\xi}\{\sum_k z_k \xi_k : ||\xi||_\infty \leq \kappa,\ -1 \leq \xi_k \leq 1\} \leq 0.$$

- The dual of the inner maximization problem is

$$\min_u \left(||u||_1 + \kappa||z - u||_\infty\right).$$

- The *equivalent robust counterpart* of the uncertain constraint is

$$\sum_i a_i^0 x_i + \min_u \left(||u||_1 + \kappa||z - u||_\infty\right) \leq 0.$$

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$$\sum_i a_i^0 x_i + ||u||_1 + \kappa||z - u||_\infty \leq 0.$$

More complex uncertainty sets

A priori information on P	P satisfies P.2 with parameters μ^-	μ^+	σ	Remark
$\text{supp}(P) \subset [-1,1]$	-1	1	0	
$\text{supp}(P) \subset [-1,1]$ P is unimodal w.r.t. 0	$-\frac{1}{2}$	$\frac{1}{2}$	$\sqrt{\frac{1}{12}}$	
$\text{supp}(P) \subset [-1,1]$ P is unimodal w.r.t. 0 P is symmetric w.r.t. 0	0	0	$\sqrt{\frac{1}{3}}$	
$\text{supp}(P) \subset [-1,1]$ $[-1 <]\ \mu^- \leq \text{Mean}[P] \leq \mu^+\ [< 1]$	μ^-	μ^+	$\Sigma_{(1)}(\mu^-, \mu^+)$	(2.4.29)
$\text{supp}(P) \subset [-1,1]$ $[-\nu \leq]\ \mu^- \leq \text{Mean}[P] \leq \mu^+\ [\leq \nu]$ $\text{Var}[P] \leq \nu^2 \leq 1$	μ^-	μ^+	$\Sigma_{(2)}(\mu^-, \mu^+, \nu)$	(2.4.32)
$\text{supp}(P) \subset [-1,1]$ P is symmetric w.r.t. 0 $\text{Var}[P] \leq \nu^2 \leq 1$	0	0	$\Sigma_{(3)}(\nu)$	(2.4.34)
$\text{supp}(P) \subset [-1,1]$ P is symmetric w.r.t. 0 P is unimodal w.r.t. 0 $\text{Var}[P] \leq \nu^2 \leq 1/3$	0	0	$\Sigma_{(4)}(\nu)$	(2.4.36)

Probabilistic justification of the RO scheme

Theorem

Let $\xi_k \in [-1, 1]$, $k = 1, \ldots, p$, *be independent random variables with:* $E(\xi_k) = 0$*. For any deterministic* z_k, $k = 1, \ldots, p$

$$Prob\left\{\xi \mid \sum_{k=1}^{p} z_k \xi_k > \kappa ||z||_2\right\} \leq \exp(-\frac{\kappa^2}{2}).$$

and

$$Prob\left\{\xi \mid \sum_{k=1}^{p} z_k \xi_k > \kappa ||z||_1\right\} \leq \exp(-\frac{\kappa^2}{2p}).$$

Probabilistic justification of the RO scheme

Let $z^k = \sum_j a_j^k x_j$, $k = 0, 1, \ldots, p$.
The robust counterpart of the uncertain constraint

$$z^0 + \sum_k z_k \xi_k \leq b, \ \forall ||\xi||_2 \leq \kappa$$

is equivalent to

$$z^0 + \kappa ||z||_2 \leq b.$$

Property of the robust solution

The robust solution x satisfies $z^0(x) + \kappa ||z(x)||_2 \leq 0$. It follows

$$\mathrm{Prob}(\sum_j a_j^0 x_j + \sum_k (\sum_j a_j^k x_j) \xi_k) > 0) \leq \epsilon = \exp(-\frac{\kappa^2}{2}).$$

By appropriate choice of the immunization factor $\kappa = \sqrt{2 \ln(1/\epsilon)}$, the robust solution guarantees the satisfaction of the chance-constrained formulation of the uncertain constraint.

$\kappa = 2.5$ guarantees $\epsilon \leq 0.05$.

The same with the 1-norm (Linear Programming)

Let $z^k = \sum_j a_j^k x_j$, $k = 0, 1, \ldots, p$.
The robust counterpart of the uncertain constraint

$$z^0 + \sum_k z_k \xi_k \leq b, \ \forall ||\xi||_1 \leq \kappa$$

is equivalent to

$$z^0 + \kappa ||z||_\infty \leq 0.$$

Property of the robust solution

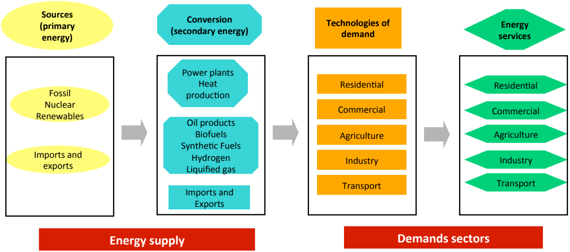
The robust solution x satisfies $z^0(x) + \kappa ||z(x)||_\infty \leq 0$. It follows

$$\text{Prob}(\sum_j a_j^0 x_j + \sum_k (\sum_j a_j^k x_j) \xi_k) > 0) \leq \epsilon = \exp(-\frac{\kappa^2}{2p}).$$

By appropriate choice of the immunization factor $\kappa = \sqrt{2p \ln(1/\epsilon)}$, the robust solution guarantees the satisfaction of the chance-constrained formulation with p uncertain factors

$\kappa = 2.5\sqrt{p}$ guarantees $\epsilon \leq 0.05$.

Long-Term Energy Planning



- Long-Term Energy Planning models (LTEP) are global technology models that represents the entire energy system of a region. It has generally a detailed representation of technologies, energy sources, energy trade, and demand sectors (residential, industry, agriculture, electrical, ...).
- LTEP represents the electric sector (power plants, renewables, demand services, etc).
- LTEP are used to assess the impact of regional and global energy and climate policies. They simulate the evolution of the energy system under those policies.

Long-Term Energy Planning

$$
\begin{aligned}
\min \quad & f(\mathbf{COM}, \mathbf{ICAP}, \mathbf{IMP}, \mathbf{EXP}) \\
& \sum_{k:i \in Prod[k]} \mathbf{COM}_{ikt} + \mathbf{IMP}_{it} = \sum_{k:i \in Cons[k]} \mathbf{COM}_{ikt} + \mathbf{EXP}_{it} + d_{it}, \quad \forall i \, \forall t \\
& \sum_{i \in Prod[k]} \mathbf{COM}_{ikt} \leq f_{kt}(c_{kt} + \sum_{r=t-life...t} \mathbf{ICAP}_{kr}), \quad \forall k \, \forall t \\
& \mathbf{COM}_{ikt} \ (= \leq or \geq) \ \alpha_{ijk} \mathbf{COM}_{jkt}, \quad \forall j \, \forall k \, \forall i \, \forall t \\
& l^{COM} \leq \mathbf{COM} \leq u^{COM} \\
& l^{ICAP} \leq \mathbf{ICAP} \leq u^{CAP} \\
& l^{IMP} \leq \mathbf{IMP} \leq u^{IMP} \\
& l^{EXP} \leq \mathbf{EXP} \leq u^{EXP}
\end{aligned}
$$

- LTEP are large scale linear flow problems (or nonlinear models).
- Database includes techno parameters (input/output, efficiency and availability for all day periods and seasons, life duration, costs, etc), demands, etc
- Examples of models:
  - TIAM is a worldwide model divided in 16 regions,
  - ETEM models are regional/national models used to analyse smart-grids penetration, regional climate change impacts.

Uncertainty in Long-Term Energy Planning

- Uncertainty is everywhere: renewable intermittency, energy prices, future technology costs and efficiencies, energy reserves, energy supplies, impacts of climate change on power generation, etc.
- Uncertainty modeling is challenging: large model size, demand elasticities, etc.
- Literature: Stochastic programming and minimax approaches on small event trees, scenario analysis, etc
- Robust optimization: Applications at Ordecsys
  - European energy supply in the EU FP7 project Ermitage,
  - Climate change impacts in the French Midi-Pyrénées region (French energy agency),
  - Smart-grids and renewables in the Swiss arc Lémanique region (Swiss Federal Office of Energy).

Robust Optimization applied to the EU Energy Security problem

European Energy Security

- EU is strongly dependent on energy imports
- Some foreign energy sources are prone to interruptions, cost fluctuations, and other random events
- Increasing energy security might include:
  - Selection of less risky energy suppliers
  - Diversification of sources, for each energy form: oil, gas, uranium, biomass, electricity
  - Diversification of energy forms (e.g. smaller dependence on oil)
  - Reduction of energy imports
  - Reduction of total energy consumption

Questions: Which measures to implement ? and how ?

Trade route constraints in TIAM

- EU+ is one region, linked to the other 15 regions via 67 trade routes
- Each trade route is a technology, endowed with an investment variable, an activity variable, and several technical and economic parameters

Import via a single corridor

$$ACT_{k,t} \leq AF_{k,t} \times CAP_{k,t}$$

- k is an import corridor (from ROW to EU)
- ACT is the activity of the corridor (decision variable)
- CAP is the capacity of the corridor (decision variable)
- $AF \in [0, 1]$ is the availability factor (random)

A global view for EU energy security

- We are not interested in a particular corridor but in the total energy import.
- We add new constraints representing total EU energy import for each period t.

Total EU energy imports at period t

$$\sum_k (ACT_{k,t} - AF_{k,t} \times CAP_{k,t}) \leq 0$$

$\Rightarrow$ **We robustify those new constraints.**

Uncertainty model

Uncertain availability factors

We assume that AF_k is random

$$AF_k = 1 - d_k \xi_k$$

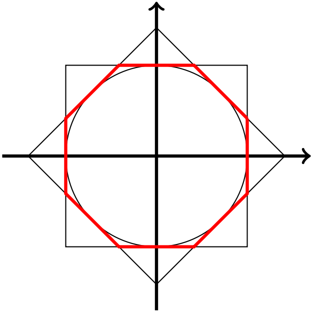
- $0 \leq d_k \leq 1$ is a measure of the severity of the risk of corridor k
- ξ is the set of independent random variables with support $[0, 1]$ and mean μ
- $[1 - d_k, 1]$ is the range of uncertainty of the factor AF_k
- A small d_k means that the corridor has little variability, and conversely when $d_k = 1$, there is the possibility of a complete corridor shutdown

Uncertain total EU energy import constraint

$$\underbrace{\sum_k (ACT_k - CAP_k)}_{\text{certain}} + \underbrace{\sum_k d_k \cdot CAP_k \cdot \xi_k}_{\text{uncertain}} \leq 0. \tag{4}$$

Uncertainty set

Recall that RO looks for solutions that remain feasible for all events in the uncertainty set. We consider an uncertainty set that is the intersection of balls l_1 and l_{inf}



With this definition,

1. all uncertainties can take their worst value but not simultaneously,
2. we remain in the realm of linear programming.

Robust energy constraints

Proposition

Skipping all technical details, the equivalent of the robust counterpart

$$\sum_k (ACT_k - CAP_k) + \sum_k d_k \cdot CAP_k \cdot \xi_k \leq 0, \quad \forall \xi \in \mathcal{U}$$

is given by deterministic system of inequalities

$$\sum_k (ACT_k - CAP_k) + d_k \mu_k CAP_k + \sum_k (1 - \mu_k) u_k + \sqrt{\frac{K}{2} \ln \frac{1}{\epsilon}} \cdot v \leq 0 \tag{5a}$$

$$u_k + v - CAP_k \cdot d_k \geq 0, \; k = 1, \ldots, K \tag{5b}$$

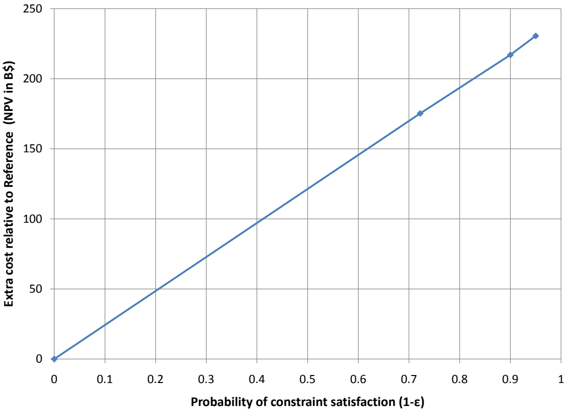
$$u_k \geq 0, \; v \geq 0, \; k = 1, \ldots, K \tag{5c}$$

The solution of (5) satisfies the energy constraint with probability at least $(1 - \epsilon)$.

Application to EU via TIAM model

- In TIAM, there are 67 import corridors, for 4 energy forms (Oil, Gas, Coal, Electricity).
- We assumed that all corridors can be totally closed (they have the same domain of uncertainty with $d_k = 1$)
- We also assumed that all corridors have the same average availability factor $\bar{AF} = 0.6$ (i.e. $\mu_k = 0.4$) that is quite pessimistic
- Five robust constraints were created, for the 5 periods 2020, 2025, 2035, 2045, 2055.
- We tested three satisfaction probability levels 0.72, 0.90, and 0.95 and the reference scenario.

Cost-reliability trade-offs



The extra costs for improving reliability range from 175 B$ to 230 B$, i.e. from 0.52% to 0.68% of total EU cost.

Conclusions on energy security

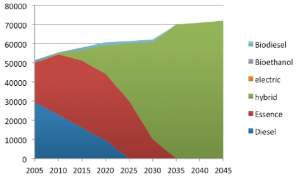
1. The supply of energy can be guaranteed with a known probability, under a very mild assumption.
2. Such reliability is achieved at a moderate extra cost (not exceeding 0.7% of the total EU energy cost).
3. The results show a significant reduction of the concentration of supply sources, a feature that is desirable in itself. RO favors combination of several actions
   - Decrease imports selectively
   - Build extra corridor capacity (again in selective manner)
   - Equalize the market shares

Other applications of Robust Optimization for Energy planning problems

Other recent analysis

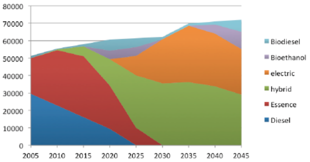
Analyzing smart grids, electricity storage and nuclear phase-out for the Swiss Federal Office of Energy

- What roles for smart-grids and electricity storage combined to nuclear phase-out decision in Switzerland? What evolution for the energy system?
- ETEM model of the Swiss Arc-Lémanique region
- Uncertainties: energy prices, costs and efficiencies of future technologies in the transport and electricity sectors, capacity of the network to integrate renewables production with and without smart-grids, acceptability of smart-grids for demand responses.

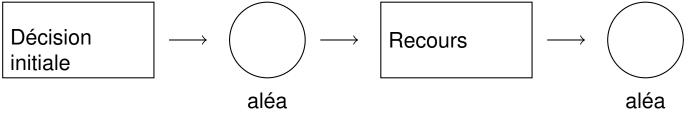


Analyzing impacts of climate change for the French Energy Agency (ADEME)

- What impacts of climate change on power generation, heating and cooling demands? What evolution for the energy system?
- ETEM model of the French Midi-Pyrénées region
- Uncertainties: energy prices, renewables acceptability, impacts of CC on availability of nuclear and hydro power plants, impacts of CC on heating and cooling demands



Decision based on revealed information



In a multistage decision problem under uncertainty

- L'incertitude est révélée graduellement.
- La décision de recours peut et doit exploiter la connaissance acquise de la réalisation du premier aléa
- Le recours est par essence une fonction incertaine
- La décision initiale doit anticiper la nature du recours.

Affine decision rules

A two-stage constraint

$$a_1^T x_1 + a_2^T x_2 \leq b$$

with the uncertain parameters

$$\begin{aligned} a_1 &= a_1^0 + A_1 \xi_1 \\ b &= b^0 + B_1 \xi_1 + B_2 \xi_2. \end{aligned}$$

a_2 is certain (fixed recourse)
Decision x_2 is a recourse, and thus adjustable. We restrict the set of possible recourses to the affine function

$$x_2 = x_2^0 + X_2 \xi_1$$

$$(a_1^0)^T x_1 + a_2^T x_2^0 + (A_1^T x_1 + X_2{}^T a_2 - B_1)^T \xi_1 - B_2 \xi_2 \leq 0$$

Deterministic path-flow formulation

$$\min_{f \geq 0, c \geq 0} \quad \sum_{a \in \mathcal{A}} r_a c_a$$

$$\sum_{k \in \mathcal{K}} \sum_{i \in \mathcal{I}_k} f_{ik} \pi_{ik}^a \leq c_a, \quad a \in \mathcal{A}$$

$$\sum_{i \in \mathcal{I}_k} f_{ik} = d_k, \quad k \in \mathcal{K}$$

- $\mathcal{A}$: set of arcs,
- $\mathcal{K}$: set of commodities.
- $\mathcal{I}_k$: set of available paths for the commodity k.
- **f**: vector of flows
- **c**: vector of capacities.
- **r**: unit installation cost
- **d**: demand.

MIP contraints

We introduce binary variables $y \in \{0, 1\}$ to limit the number of active paths per commodity and we add the contraints :

$$\sum_{i \in \mathcal{I}_k} y_{ik} \leq l_k, \quad k \in \mathcal{K}$$

$$f_{ik} \leq M_k y_{ik}, \quad k \in \mathcal{K}, \ i \in \mathcal{I}_k.$$

The objective is now twofold:

- Compute capacity expansions.
- Propose a reduced list of active paths.

Demand uncertainty

We assume the demand to be independent with a symmetric distribution such that

$$d_k = \bar{d}_k + \xi_k \hat{d}_k,$$

where $\xi_k \in [-1, 1]$ represents the random factor and $\hat{d}_k$ a demand dispersion.

Two-stage problem with recourse

1. Select capacities
2. Observe demand and select routing on the paths (recourse action).

Linear decision rules (LDR)

Using LDR, the second decision variables are defined as linear functions of the revealed uncertainty. We define

$$f_{ik} = \alpha_{0ik} + \sum_{k \in \mathcal{K}} \alpha_{ik} \xi_k .$$

LDR converts 2-stage into static

1. Select capacities and the LDR coefficients
2. Observe demand.

Drawback

Large number of additional variables α

Simplified LDR

Goal: decrease problem size
Technique: use restricted LDR (with neighborhoods)

$$f_{ik} = \alpha_{0ik} + \alpha_{1ik}\xi_k + \alpha_{2ik} \sum_{k' \in V_k} \xi_{k'} + \alpha_{3ik} \sum_{k' \in R_k} \xi_{k'}.$$

A neighborhood for a commodity *k* is set of all commodities whose the shortest path has at least a common arc with the shortest path of the commodity *k*.

Capacity expansion problem with LDR

Replacing the flow variables by their LDR leads to

$$\min_{\alpha,\xi,c\geq 0,y} \quad \sum_{a\in\mathcal{A}} r_a c_a$$

$$\sum_{k\in\mathcal{K}}\sum_{i\in\mathcal{I}_k} \pi^a_{ik}(\alpha_{0ik} + \alpha_{1ik}\xi_k + \alpha_{2ik}\sum_{k'\in V_k}\xi_{k'} + \alpha_{3ik}\sum_{k'\in R_k}\xi_{k'}) \leq c_a, \quad a\in\mathcal{A}$$

$$\sum_{i\in\mathcal{I}_k}(\alpha_{0ik} + \alpha_{1ik}\xi_k + \alpha_{2ik}\xi^v_k + \alpha_{3ik}\xi^r_k) = \bar{d}_k + \xi_k\hat{d}_k, \quad k\in\mathcal{K}$$

$$y_{ik}M_k \geq \alpha_{0ik} + \alpha_{1ik}\xi_k + \alpha_{2ik}\xi^v_k + \alpha_{3ik}\xi^r_k \geq 0, \quad k\in\mathcal{K}, i\in\mathcal{I}_k$$

$$\sum_{i\in\mathcal{I}_k} y_{ik} \leq l_k, \quad k\in\mathcal{K}$$

$$y_{ik}\in\{0,1\}, \quad k\in\mathcal{K}, i\in\mathcal{I}_k.$$

with $\xi^v_k = \sum_{k'\in V_k}\xi_{k'}$ et $\xi^r_k = \sum_{k'\in R_k}\xi_{k'}$.

LDR and demand satisfaction

To meet the demand $k \in \mathcal{K}$ in all circumstances, the following identity must hold

$$\sum_{i \in \mathcal{I}_k} (\alpha_{0ik} + \alpha_{1ik}\xi_k + \alpha_{2ik}\xi_k^v + \alpha_{3ik}\xi_k^r) \equiv \bar{d}_k + \xi_k \hat{d}_k, \quad \forall \xi_k,\ \xi_k^v \text{ and } \xi_k^r.$$

If the random components $\xi_k,\ \xi_k^v, \xi_k^r$ belong to open sets, the coefficients must satisfy the equations

$$\sum_{i \in \mathcal{I}_k} \alpha_{0ik} = \bar{d}_k, \qquad \sum_{i \in \mathcal{I}_k} \alpha_{1ik} = \hat{d}_k$$
$$\sum_{i \in \mathcal{I}_k} \alpha_{2ik} = 0, \qquad \sum_{i \in \mathcal{I}_k} \alpha_{3ik} = 0.$$

Applying RO to capacity and bounds constraints

Constraints to be robustified

Bound constraints Replace the non-negativity and upper bound robust constraints with respect to $\mathcal{U} = B_1(0, \sqrt{n}) \cap B_\infty(0, 1)$ by the appropriate inequalities.

Capacity constraints Replace the robust constraint with respect to $\mathcal{U} = B_1(0, \kappa\sqrt{n}) \cap B_\infty(0, 1)$ by the appropriate inequalities. Study the impact of different immunization factors k_{cap}.

Size of the robust equivalent problem:

- $n_a(2n_k + 1) + 7n_p + 4n_k$ continuous contraints instead of $n_a + n_k$
- $8n_p + n_a(n_k + 2)$ continuous variables instead of $n_a + n_p$

Optimization and validation

Optimization

A LP whose output is a set of capacities and a selection of active paths (plus a LDR on those paths).

Validation

- Assume independent demands distributed according to a triangular distribution with mode $\bar{d}$ and on the support $[\bar{d} - \hat{d}; \bar{d} + \hat{d}]$
- Generate a set of 100 scenarios of demands.
- For each scenario, solve a concurrent flow problem on active paths to minimize the demand violation.
- Output: number of scenarios with at least one violation and relative conditional expected value of violation.

Test problems

`http://sndlib.zib.de/home.action`

Problem	#nodes	#arcs	#commodities	#paths	Version robuste #const.	Version robuste #var.
di-yuan	11	42	22	81	2545	1656
polska	12	36	66	342	7446	5184
nobel-us	14	42	91	627	12439	8922
france	25	90	300	1889	68513	42292

Demand dispersion : 50%

Results without integrality constraints

	Total Protection	Robust solutions with $1-\epsilon$ for capacity constraints 85%	50%	10%	5%	Deterministic solution
			di-yuan			
Solutions	5.95E+006	5.81E+006	5.59E+006	5.07E+006	4.74E+006	3.97E+006
% of violations	0.00%	0.00%	0.50%	24.20%	75.10%	100.00%
Rel. cond. viol.	0.00%	0.00%	1.54%	0.64%	1.04%	7.30%
			polska			
Solutions	7.00E+006	6.56E+006	6.10E+006	5.44E+006	5.27E+006	4.69E+006
% of violations	0.00%	0.00%	0.10%	16.40%	34.90%	100.00%
Rel. cond. viol.	0.00%	0.00%	0.48%	0.51%	0.58%	3.97%
			nobel-us			
Solutions	1.28E+008	1.22E+008	1.16E+008	1.03E+008	9.92E+007	8.60E+007
% of violations	0.00%	0.00%	0.00%	1.60%	12.30%	100.00%
Rel. cond. viol.	0.00%	0.00%	0.00%	0.25%	0.27%	4.39%
			france			
Solutions	6.18E+008	5.86E+008	5.56E+008	5.08E+008	4.92E+008	4.44E+008
% of violations	0.00%	0.00%	0.00%	0.90%	9.80%	100.00%
Rel. cond. viol.	0.00%	0.00%	0.00%	0.16%	0.14%	3.10%

Results with integrality constraints (2 paths)

	Total Protection	Robust solutions with $1-\epsilon$ for capacity constraints 85%	50%	10%	5%	Deterministic solution
			`di-yuan`			
Solutions	5.95E+006	5.81E+006	5.59E+006	5.07E+006	4.74E+006	3.97E+006
% of violations	0.00%	0.00%	0.10%	30.70%	74.90%	100.00%
Rel. cond. viol.	0.00%	0.00%	0.84%	0.69%	1.06%	7.32%
			`polska`			
Solutions	7.01E+006	6.58E+006	6.13E+006	5.45E+000	5.27E+006	4.69E+006
% of violations	0.00%	0.00%	0.00%	14.80%	30.20%	100.00%
Rel. cond. viol.	0.00%	0.00%	0.00%	0.50%	0.60%	4.39%
			`nobel-us`			
Solutions	1.28E+008	>5h	>5h	1.03E+008	9.94E+007	8.60E+007
% of violations	0.00%	-	-	2.40%	7.90%	100.00%
Rel. cond. viol.	0.00%	-	-	0.22%	0.29%	4.55%

Robust optimization vs classical approaches

Classical approaches

1. Stochastic programming, chance constrained programming, etc
2. Posit the existence and the knowledge of a probability distribution.
3. Approximate the distribution to generate a tractable model (Event tree of moderate size for stochastic programming, Computable probabilities and expectations for chance constrained programming, ...).

Robust Optimization

1. Use a simplified, non probabilistic model, of the uncertainty (Uncertainty set).
2. Look for solutions that remain feasible for all events within the uncertainty set.
   - The optimization model is tractable (linear or conic-quadratic).
   - No probability assumption but it exists strong results on lower bounds on the probability of constraint satisfaction.
3. Dynamic problems: Decision rules

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