First-principle theory-based scaling of the SOL width in limited tokamak plasmas and comparison with experiments

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How can we develop a first-principle scaling of the SOL width?

The first step: simulations capturing SOL key features

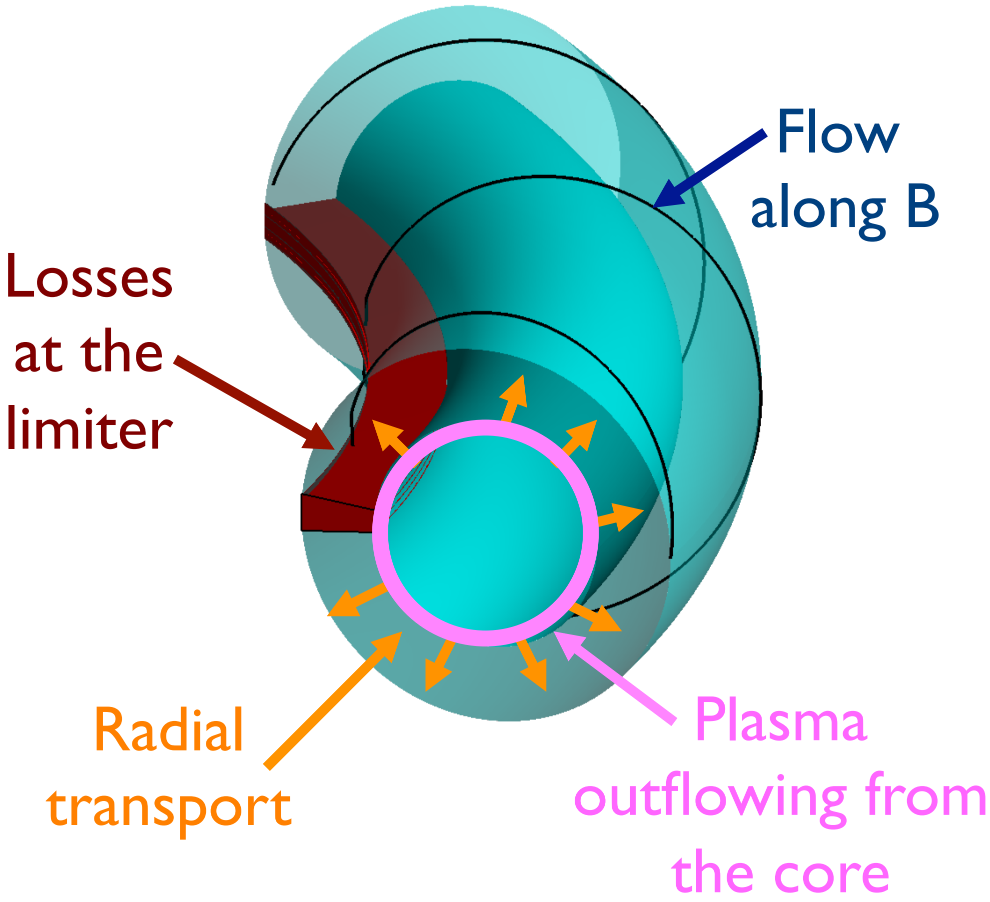
Interpretation of the simulation results to get the SOL width scaling

How do our theoretical estimates agree with experimental data?

EPFL

CRPP

First-principle full-scale 3D SOL simulations



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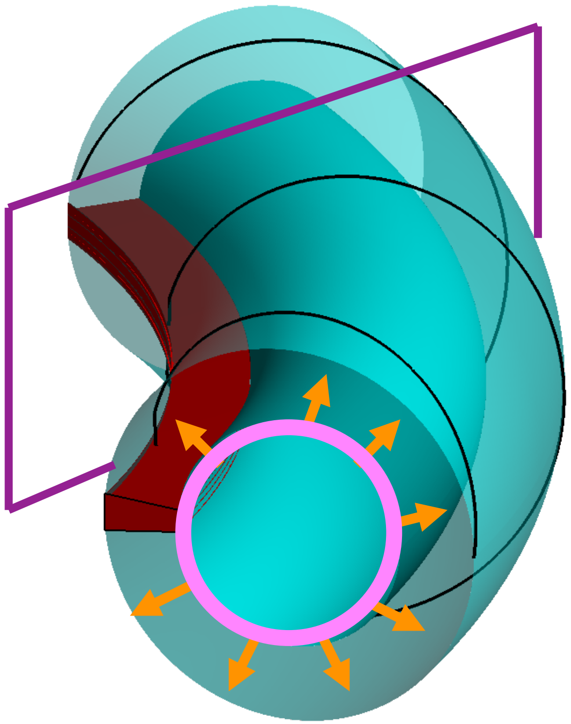

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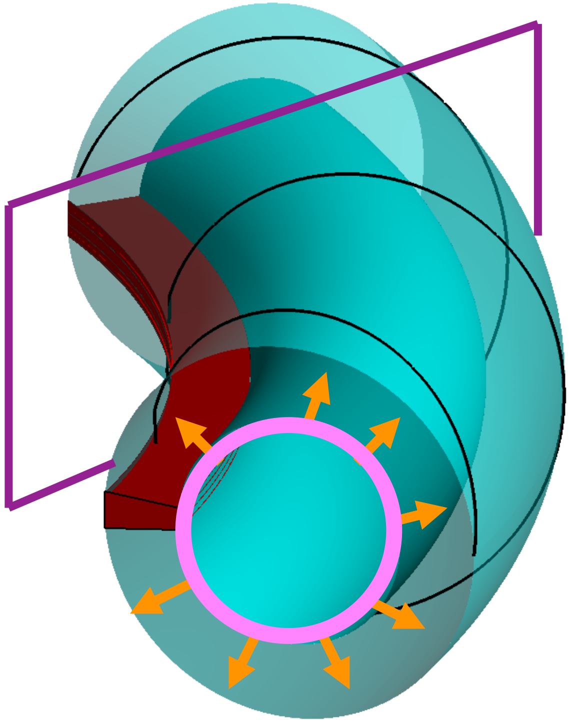
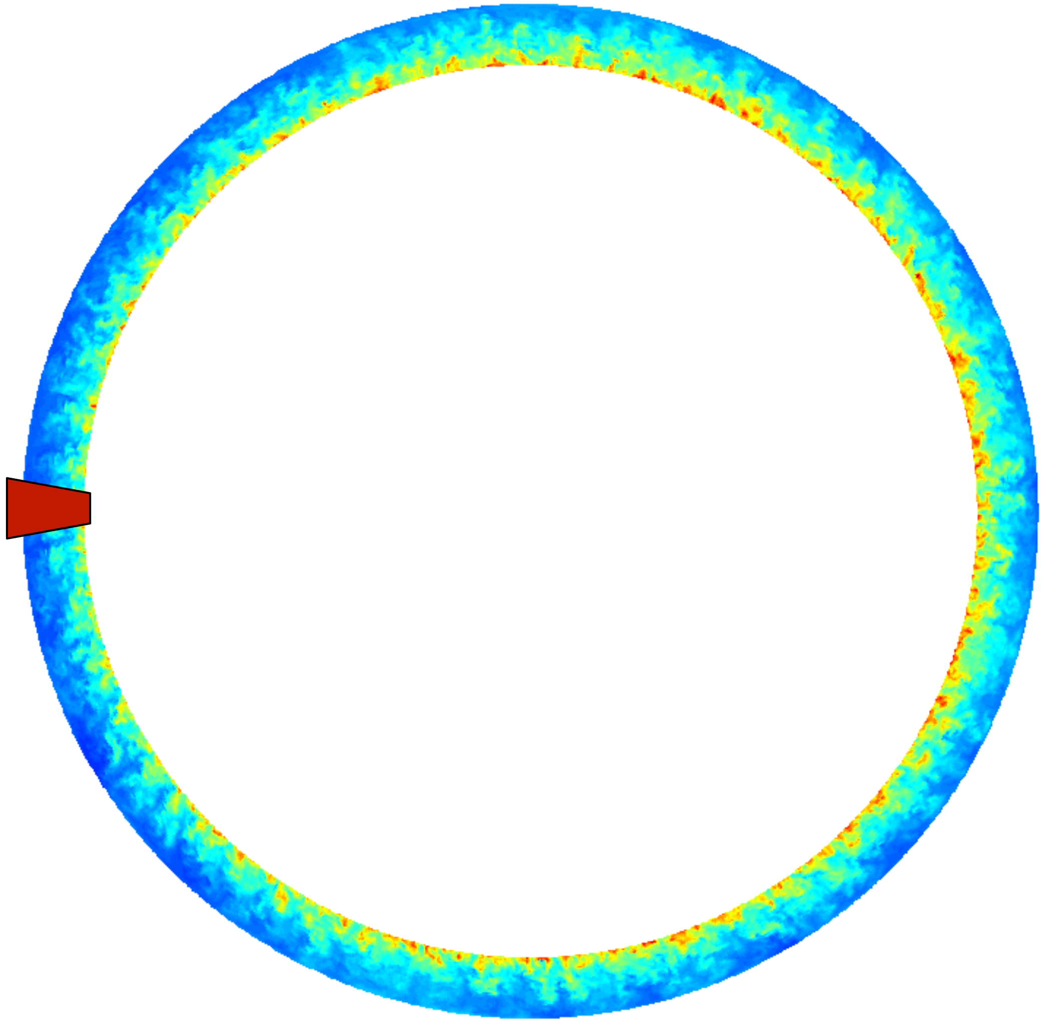
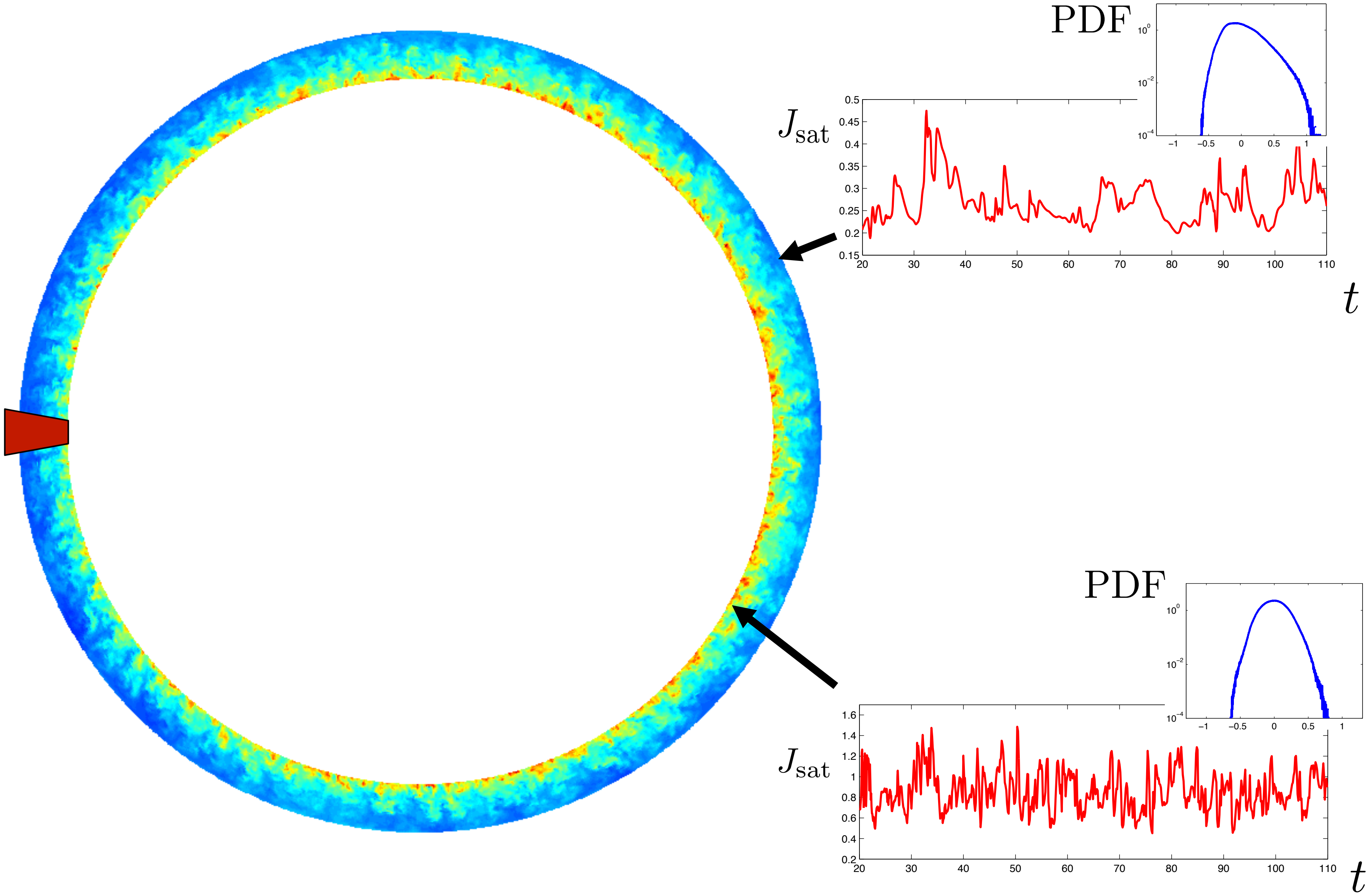
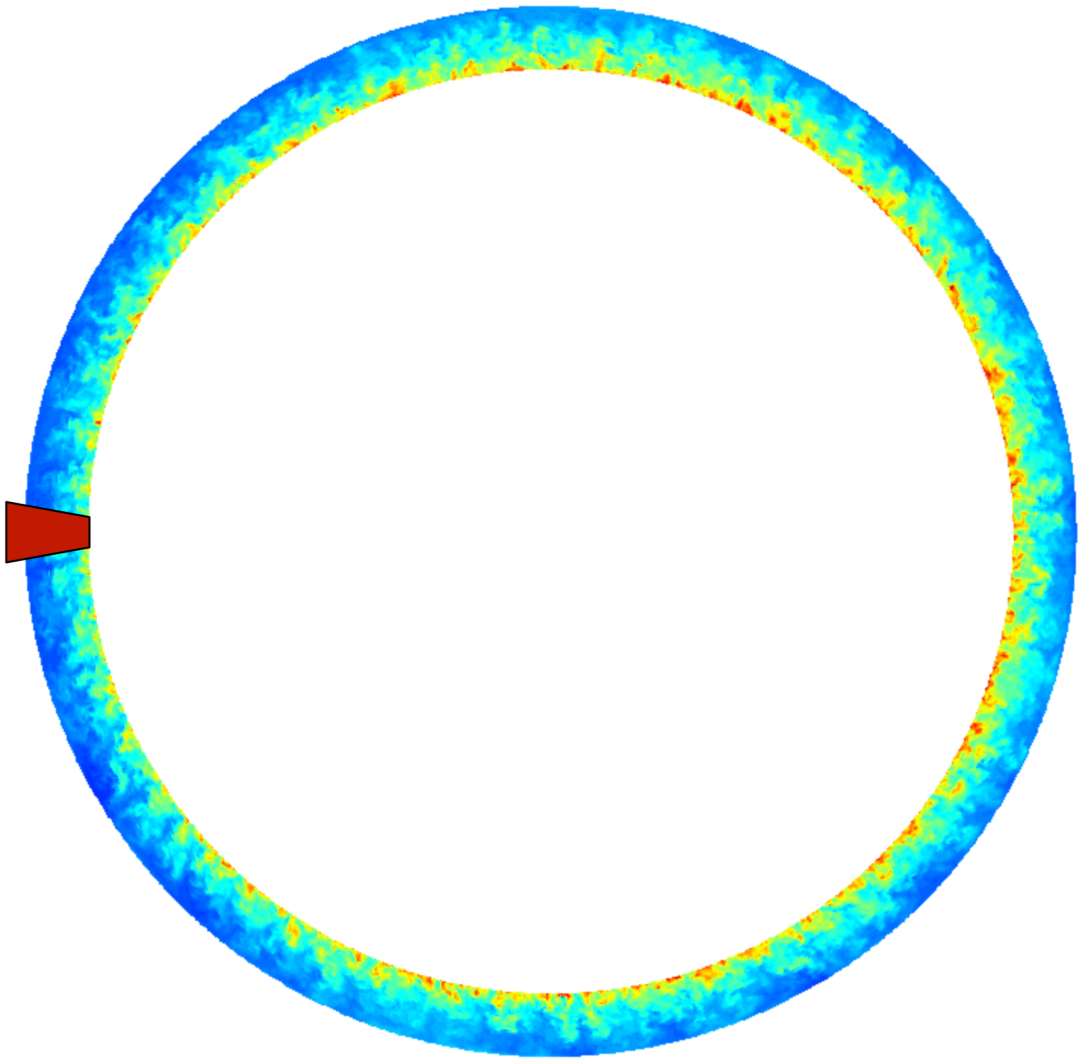


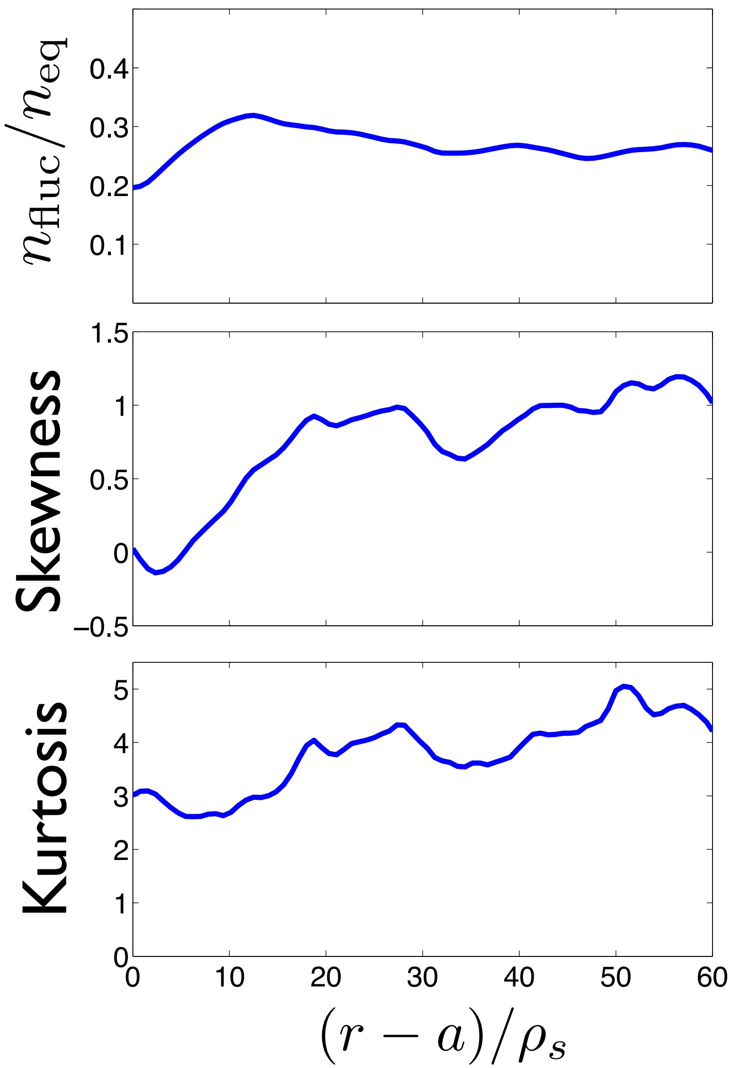


Intermittency in far SOL, Gaussian in near SOL

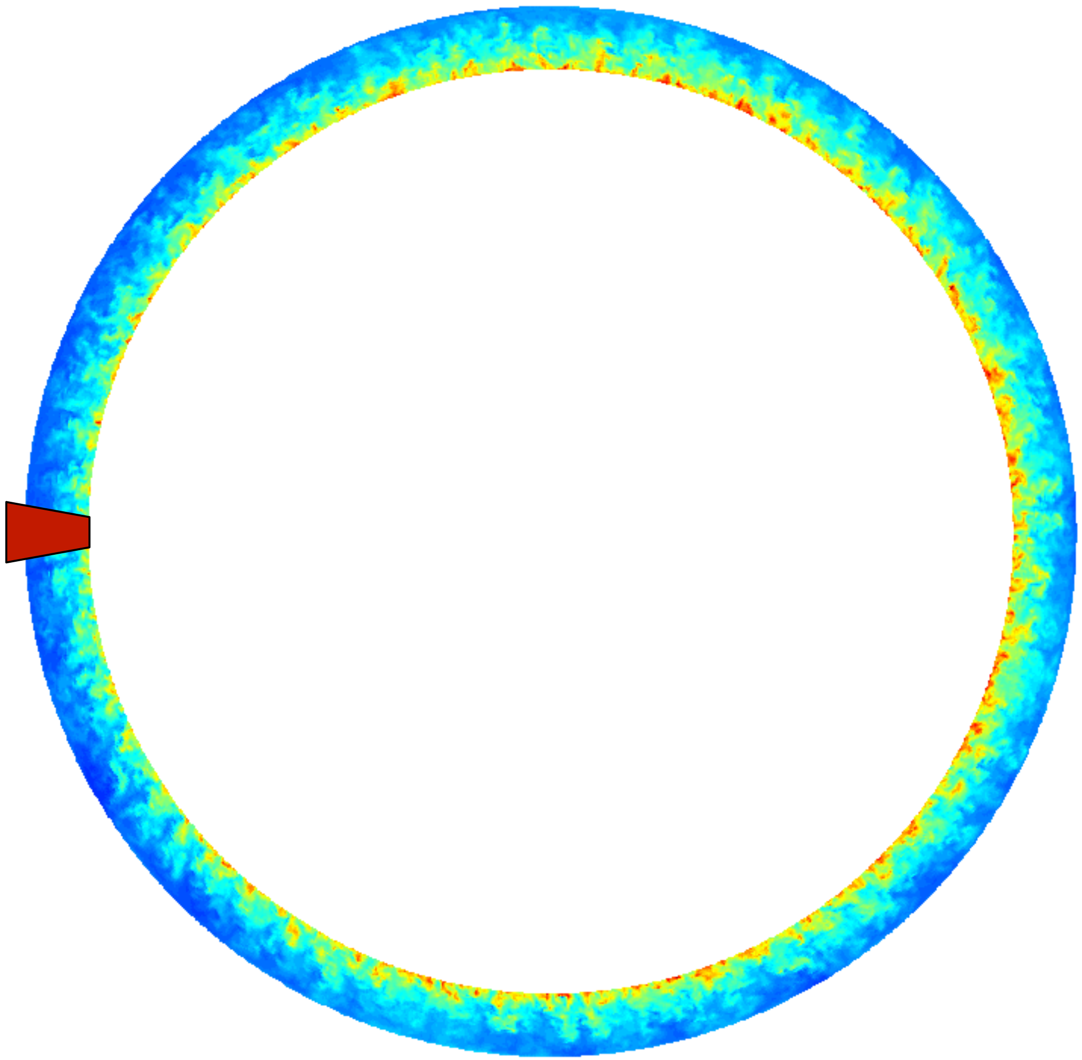


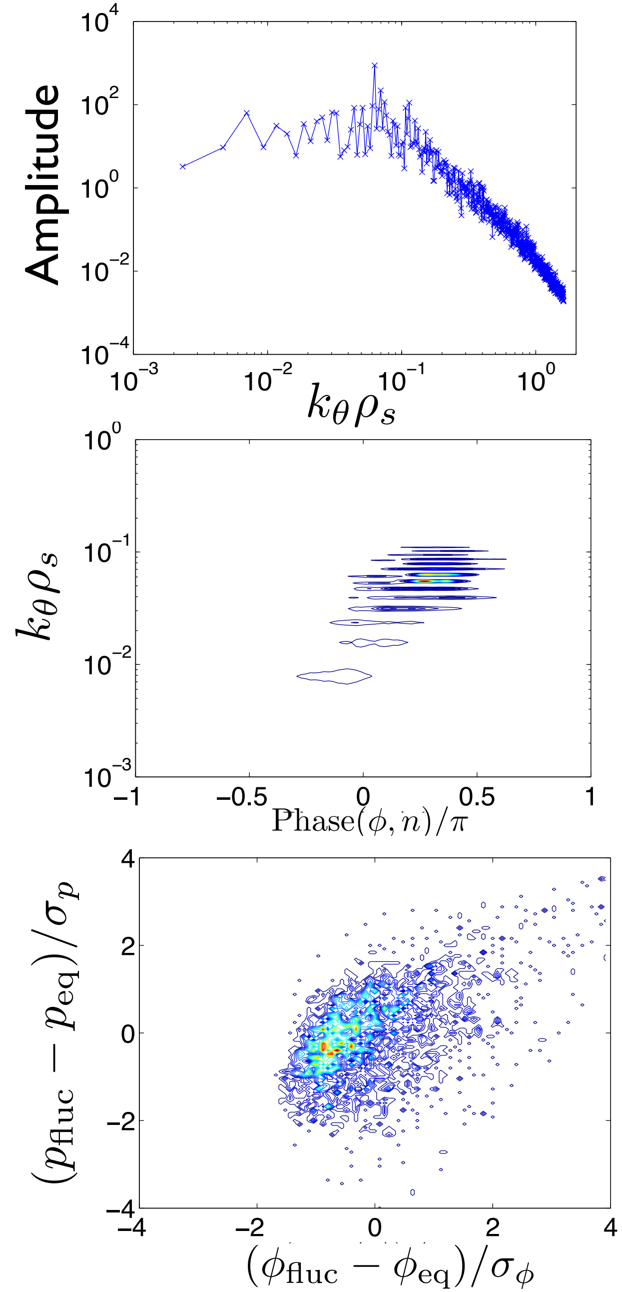
High fluctuation level, skewed PDF



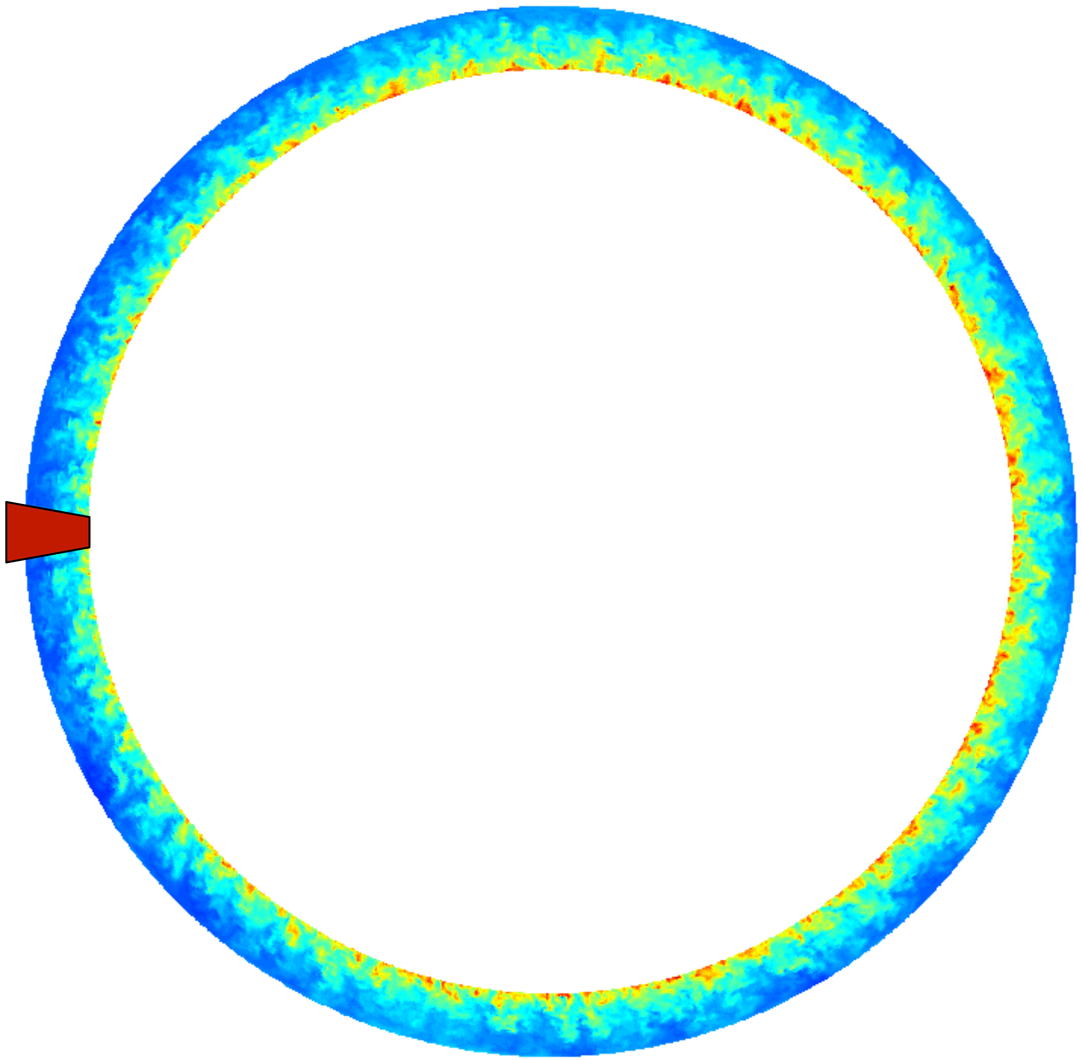


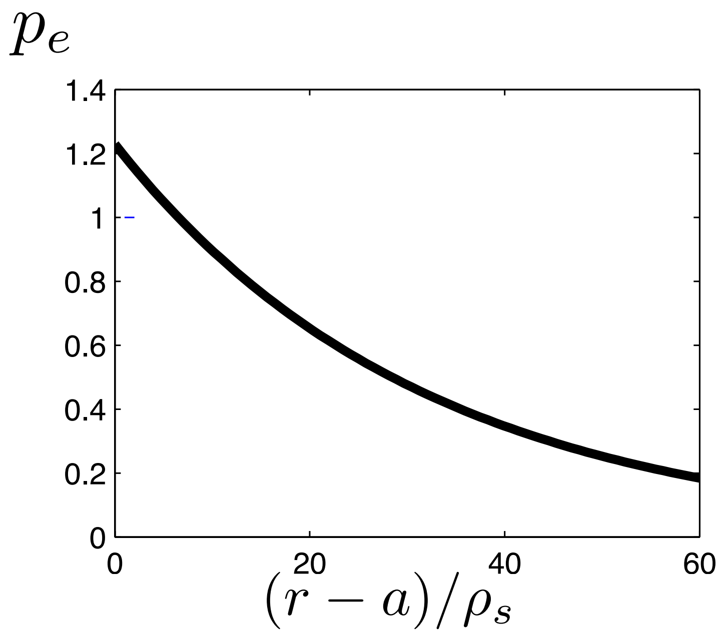
Macroscopic ballooning turbulence





Pressure profile fitted with an exponential





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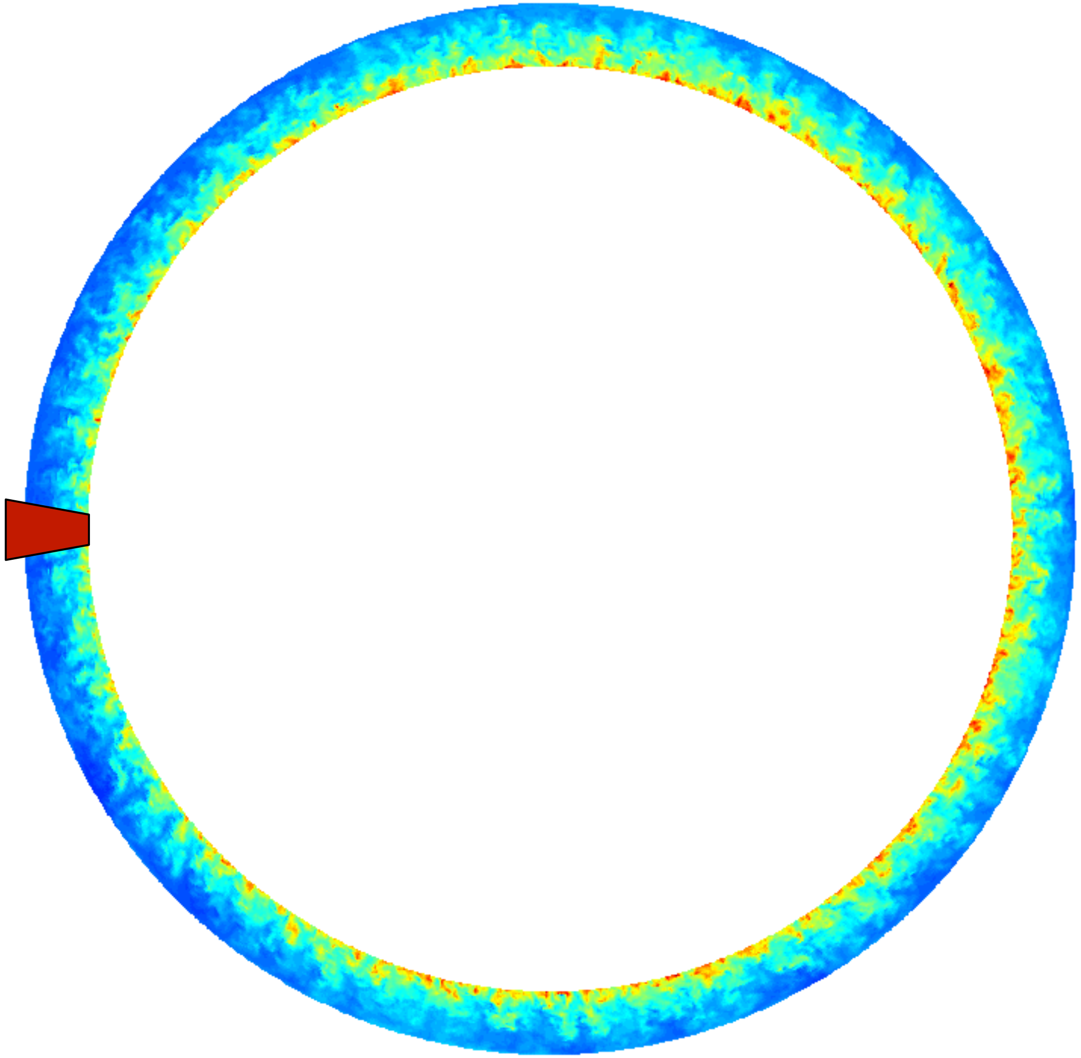
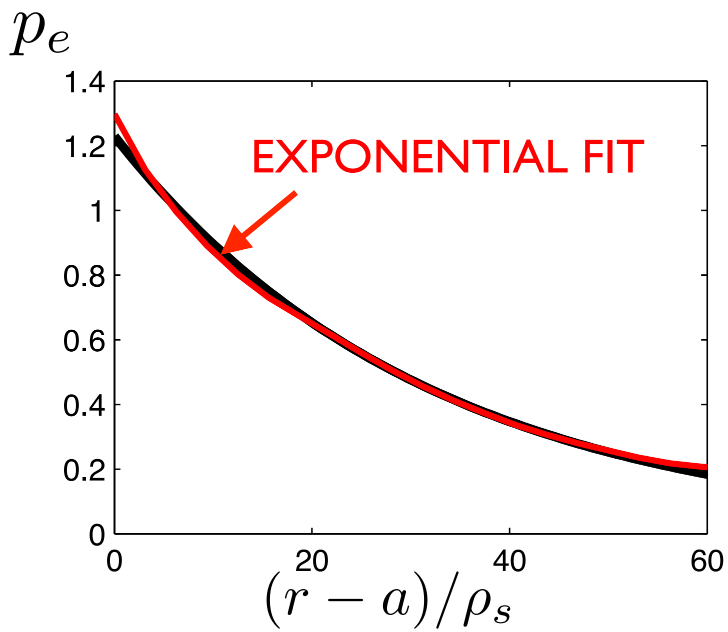




The GBS code, a tool to simulate SOL turbulence

Collisional Plasma → **Braginskii model** → $\rho_i << L$, $\omega << \Omega_{ci}$ → **Drift-reduced Braginskii equations**

E×B Convection · Magnetic curvature · Parallel dynamics · Outflow from core

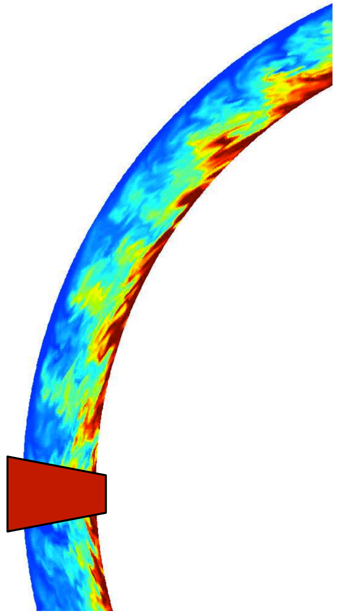
$$\frac{\partial n}{\partial t} + [\phi, n] = \hat{C}(nT_e) - n\hat{C}(\phi) - \nabla_{\parallel}(nV_{\parallel e}) + S$$

T_e, T_i, Ω (vorticity) → similar equations

$V_{\parallel e}$, $V_{\parallel i}$ → parallel momentum balance

$$\nabla_{\perp}^2 \phi = \Omega$$

We derived a new, first-principle, set of boundary conditions, generalizing Bohm-Chodura



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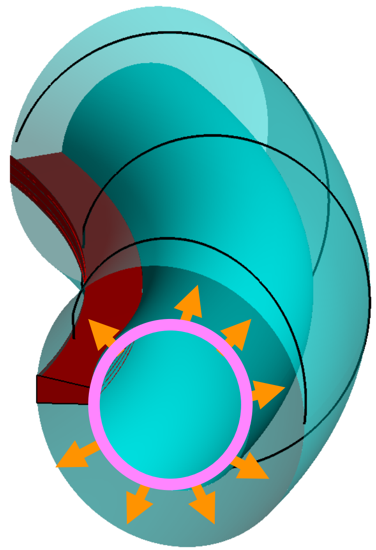
Solved in 3D, dynamics resulting from: plasma outflow, turbulent transport, and parallel losses



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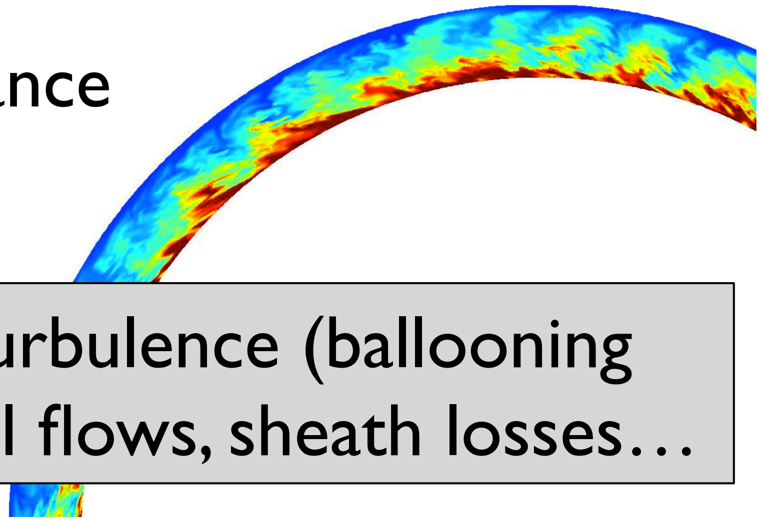

Simulations contain drift physics, turbulence (ballooning modes, drift waves, …), blobs, parallel flows, sheath losses…

The key questions

- How is the SOL width established?
- The differences between LFS and HFS limited configurations?
- What determines the SOL electrostatic potential?
- Are there mechanisms to generate toroidal rotation in the SOL?

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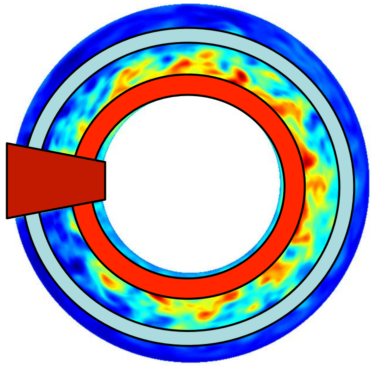
Turbulence saturated by removing its drive (gradient removal mechanism)



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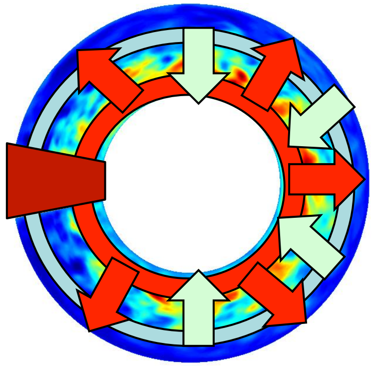

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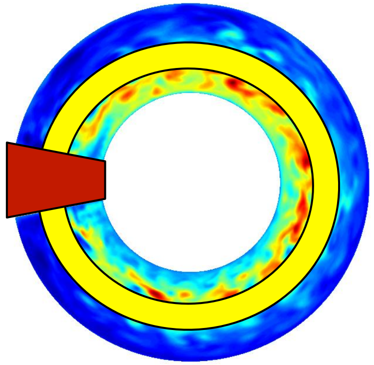

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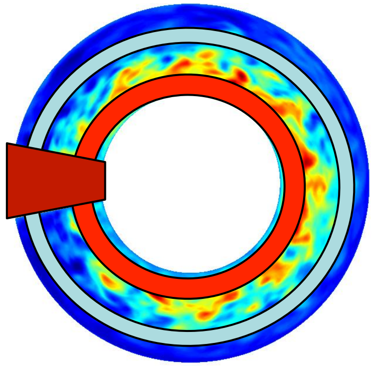

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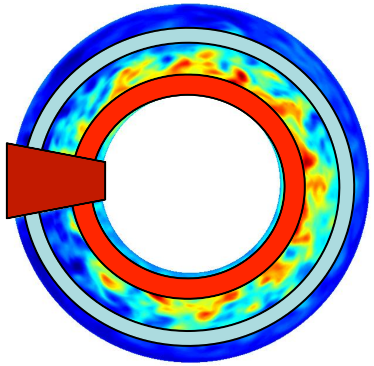

Turbulence saturation: $\frac{\partial \tilde{p}}{\partial r} \sim \frac{\partial \overline{p}}{\partial r} \rightarrow k_r \tilde{p} \sim \overline{p}/L_p$

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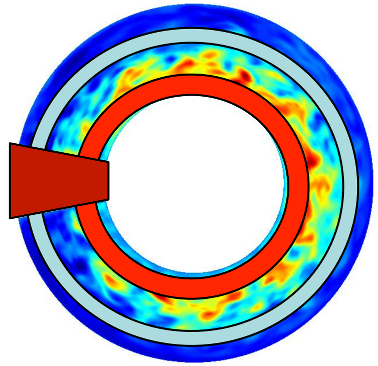

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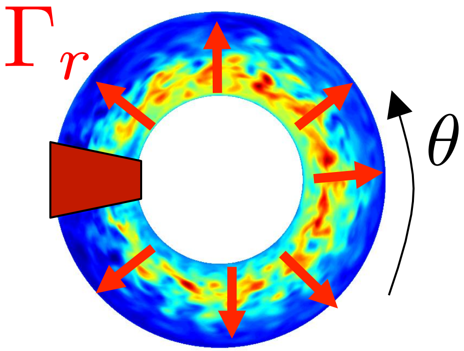

$$\Gamma_r = \left\langle \tilde{p} \frac{\partial \tilde{\phi}}{\partial \theta} \right\rangle_t$$

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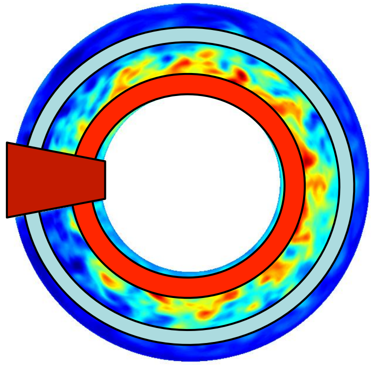

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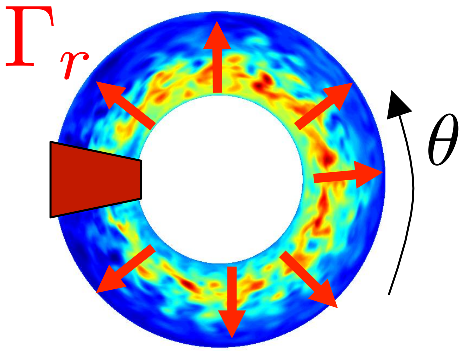

$$\frac{\partial p}{\partial t} \simeq [p, \phi]$$

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GR hypothesis

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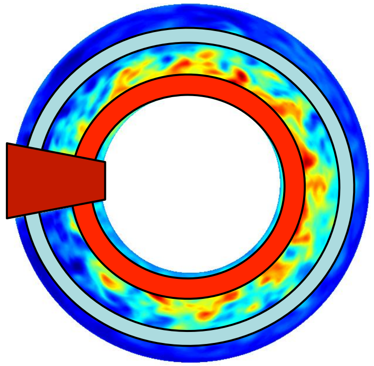

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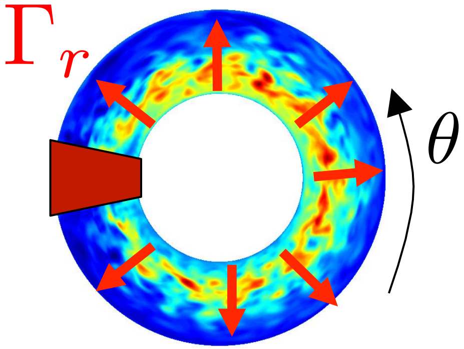

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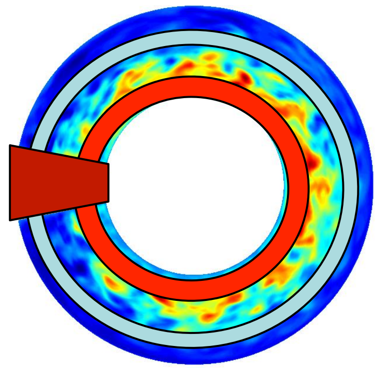

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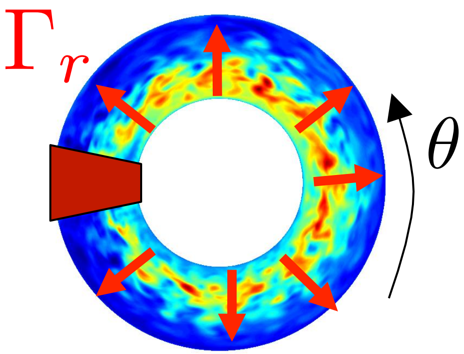

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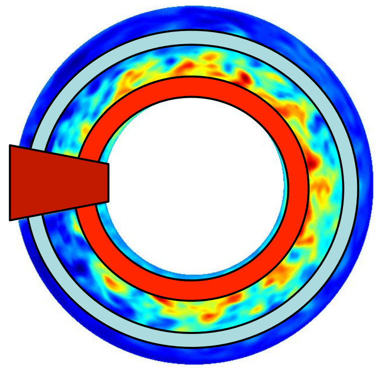

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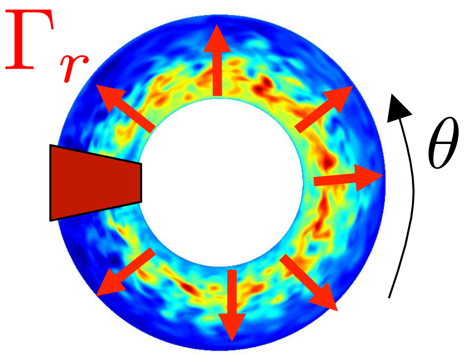

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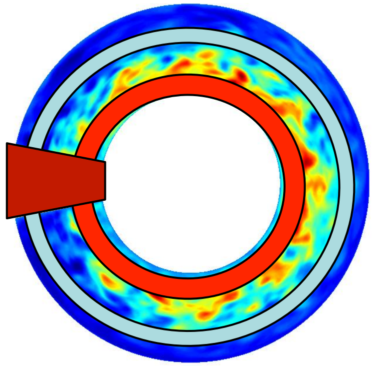

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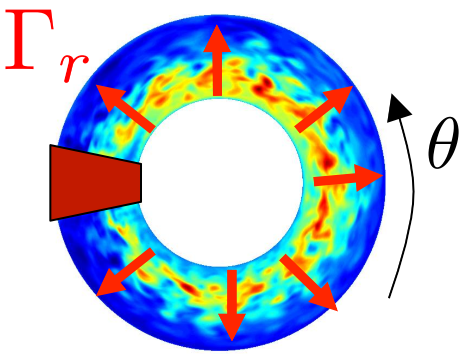

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$$\Rightarrow \quad D_{GR} = \frac{\Gamma_r}{\overline{p}/L_p} \sim \frac{\gamma L_p}{k_\theta}$$

SOL width – operational parameter estimate

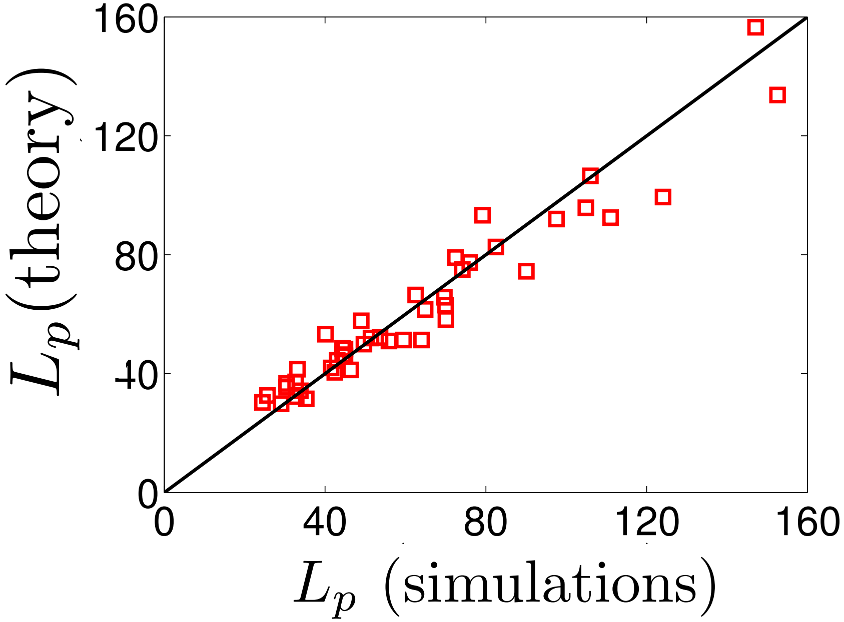
Balance of perpendicular transport and parallel losses $\rightarrow \dfrac{d\Gamma_r}{dr} \sim L_\parallel \sim \dfrac{nc_s}{qR}$

Bohm's

$$L_p \simeq q\left(\frac{\gamma}{k_\theta}\right)_{\max}$$

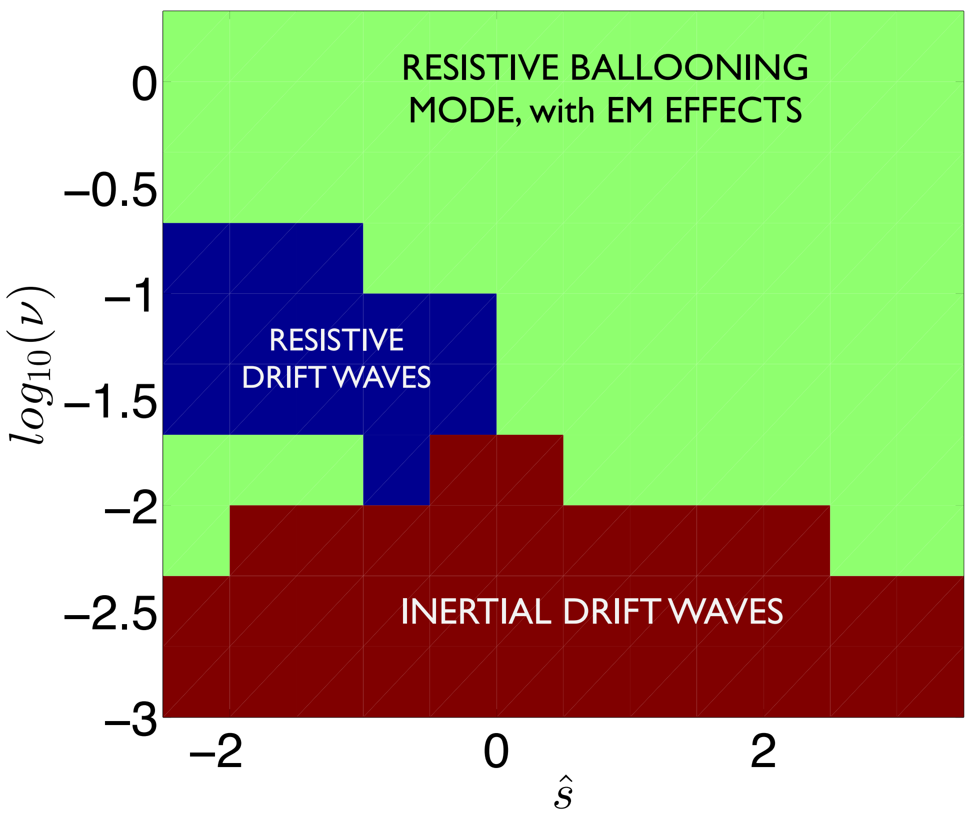
$$L_p = L_p(\rho_*, q, \hat{s}, \beta, \nu)$$

Simulations show expected scaling



SOL turbulent regimes

Instabilities driving turbulence depends mainly on $q, \nu, \hat{s}$.



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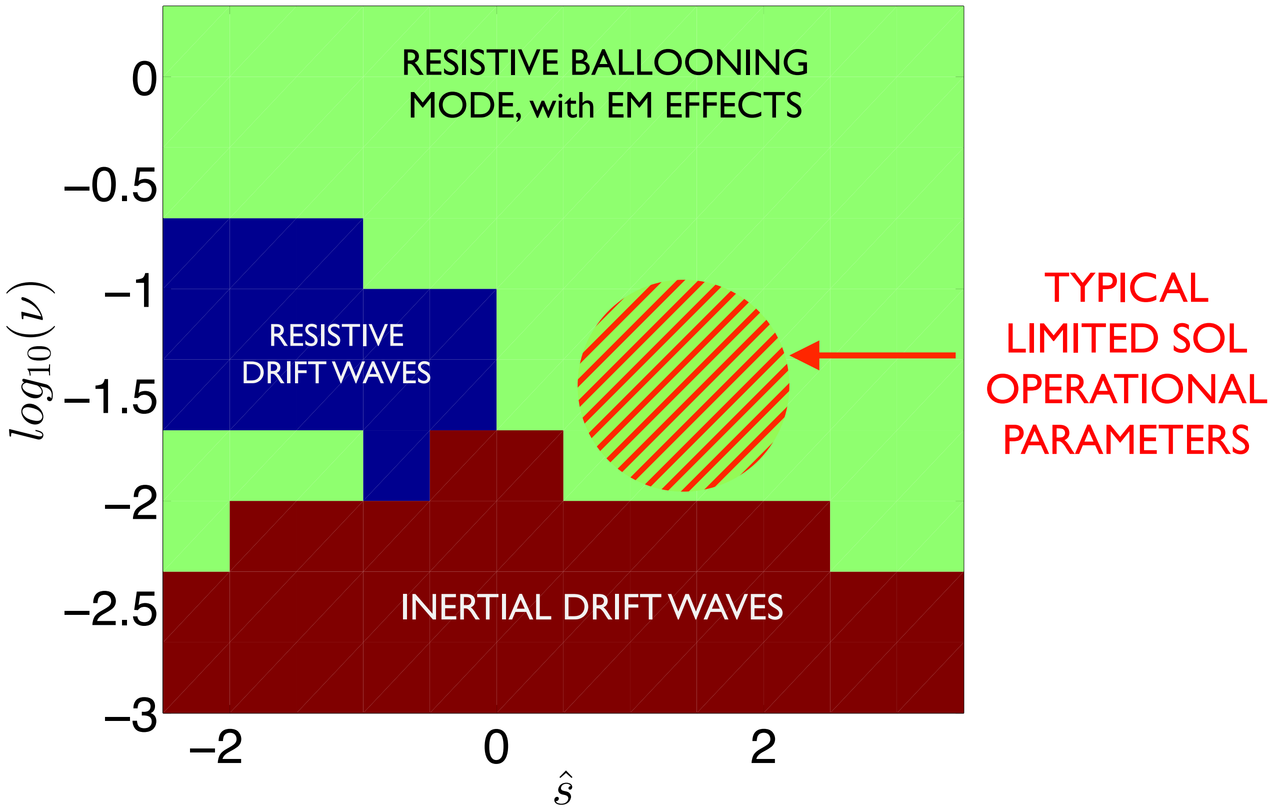

SOL width in ballooning regime

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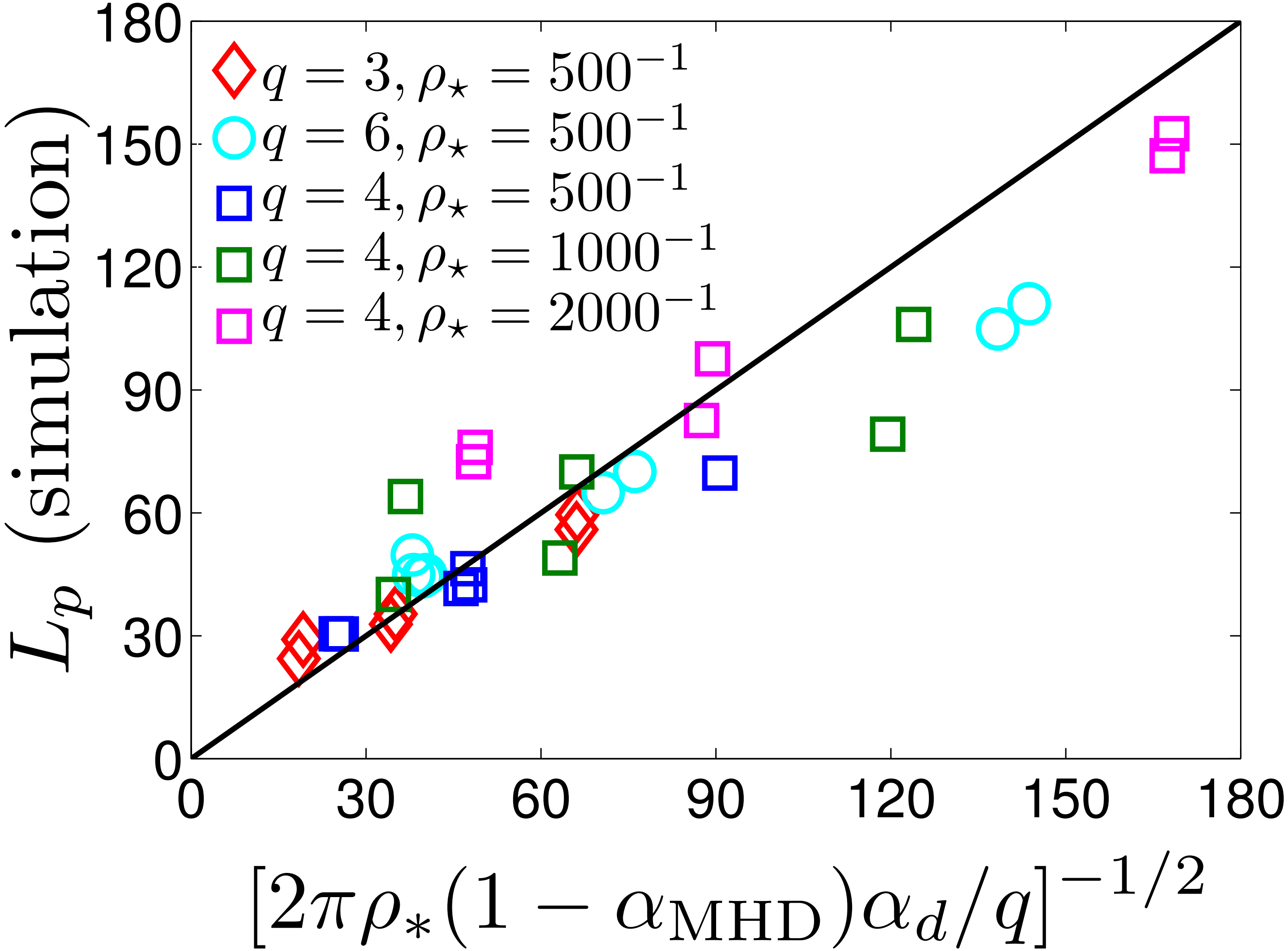
$$\Rightarrow \quad L_p = \left[2\pi \rho_* (1 - \alpha_{\text{MHD}}) \alpha_d / q\right]^{-1/2}$$

TOKAMAK SIZE (ρ_*)

$\alpha_{MHD} \sim q^2 \beta R / L_p$

$\alpha_d \sim (R/L_p)^{1/4} \nu^{-1/2} / q$

Simulations agree with ballooning estimates



Good agreement with multi-machine measurements

The ballooning scaling, in SI units:

$$L_p \simeq 7.22 \times 10^{-8} q^{8/7} R^{5/7} B_\phi^{-4/7} T_{e,\mathrm{LCFS}}^{-2/7} n_{e,\mathrm{LCFS}}^{2/7} \left(1 + \frac{T_{i,\mathrm{LCFS}}}{T_{e,\mathrm{LCFS}}}\right)^{1/7}$$

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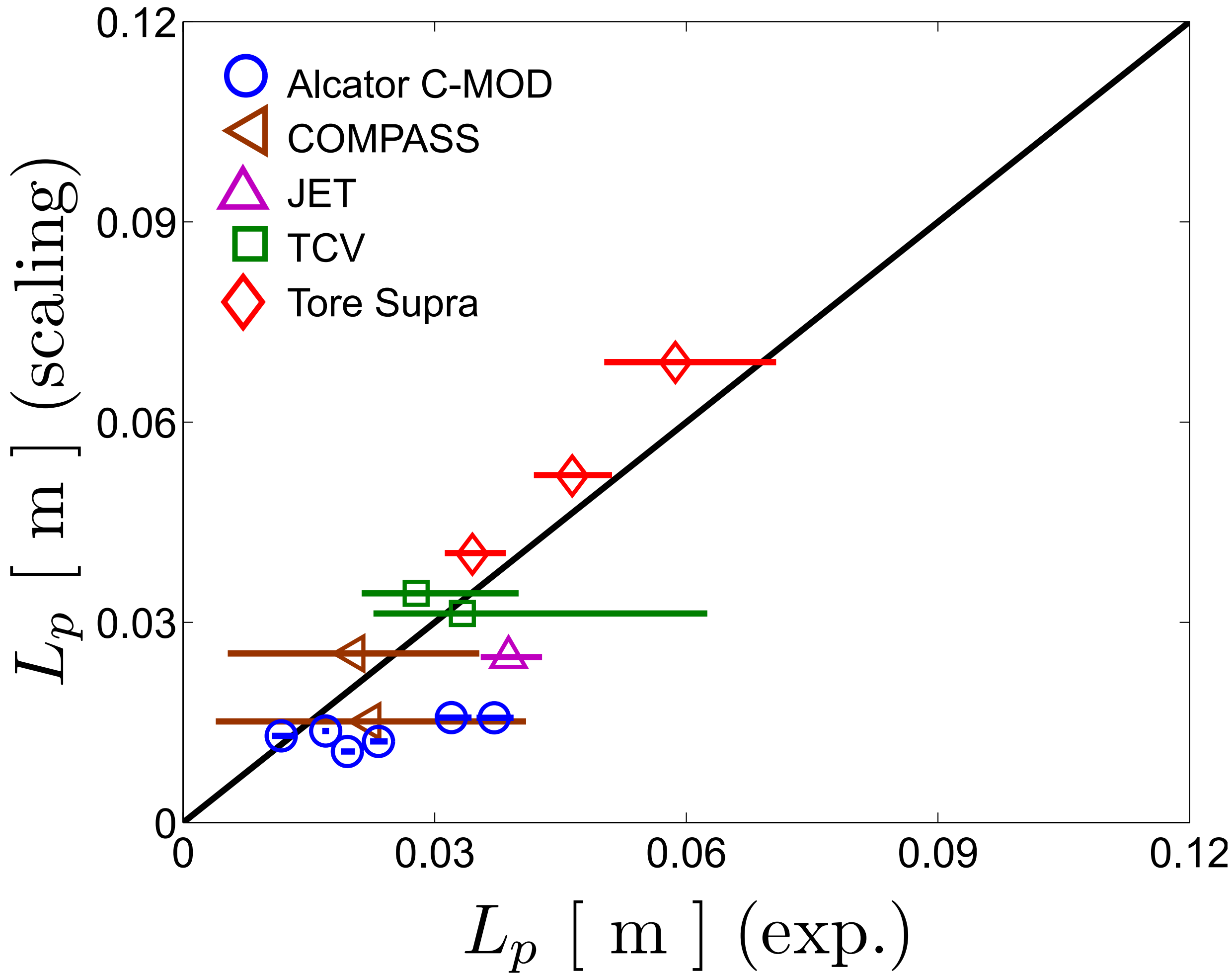

Experimental results provided by G. Arnoux, I. Furno, J.P. Gunn, J. Horacek, M. Kocan, B. LaBombard, B. Labit, and C. Silva.

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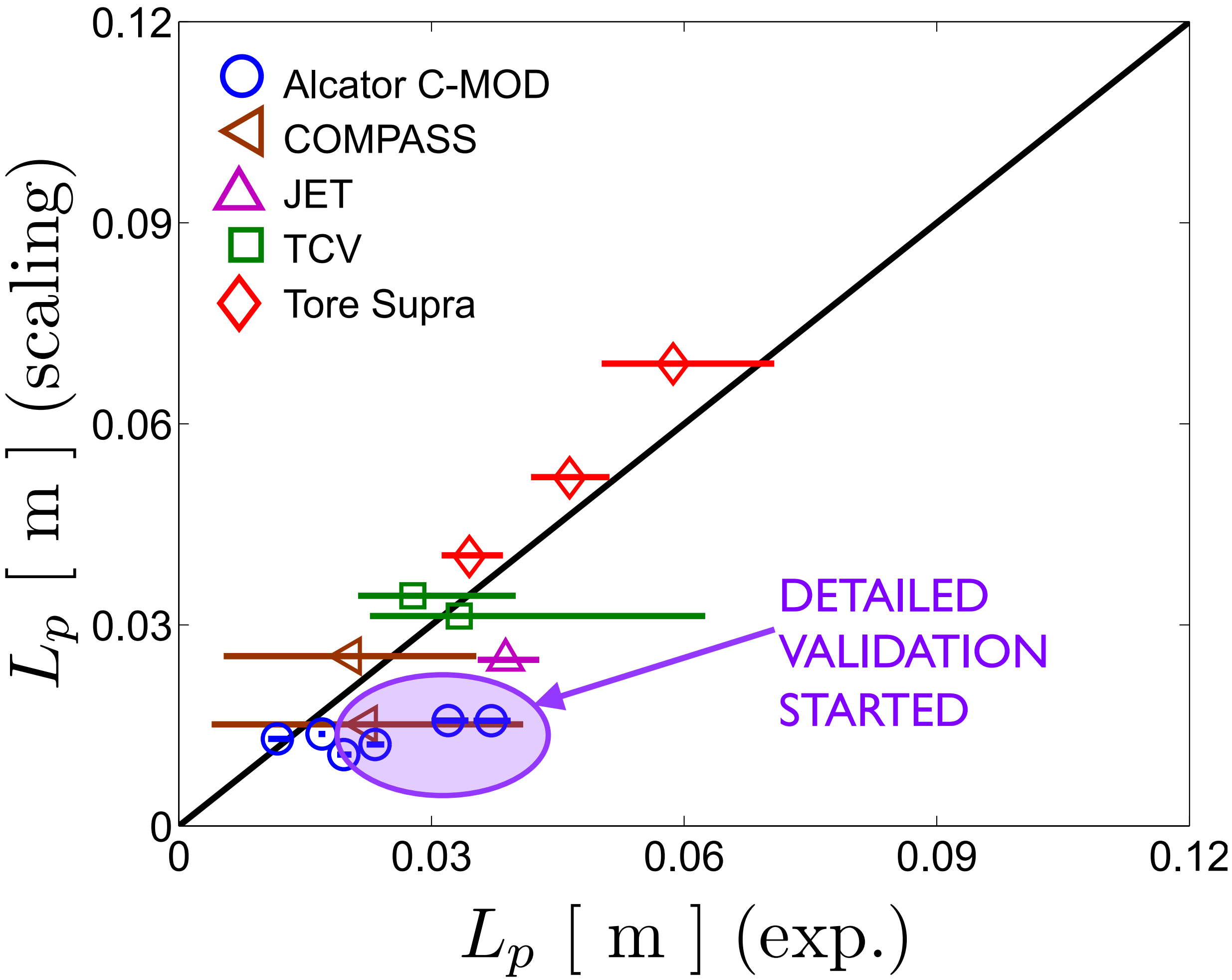

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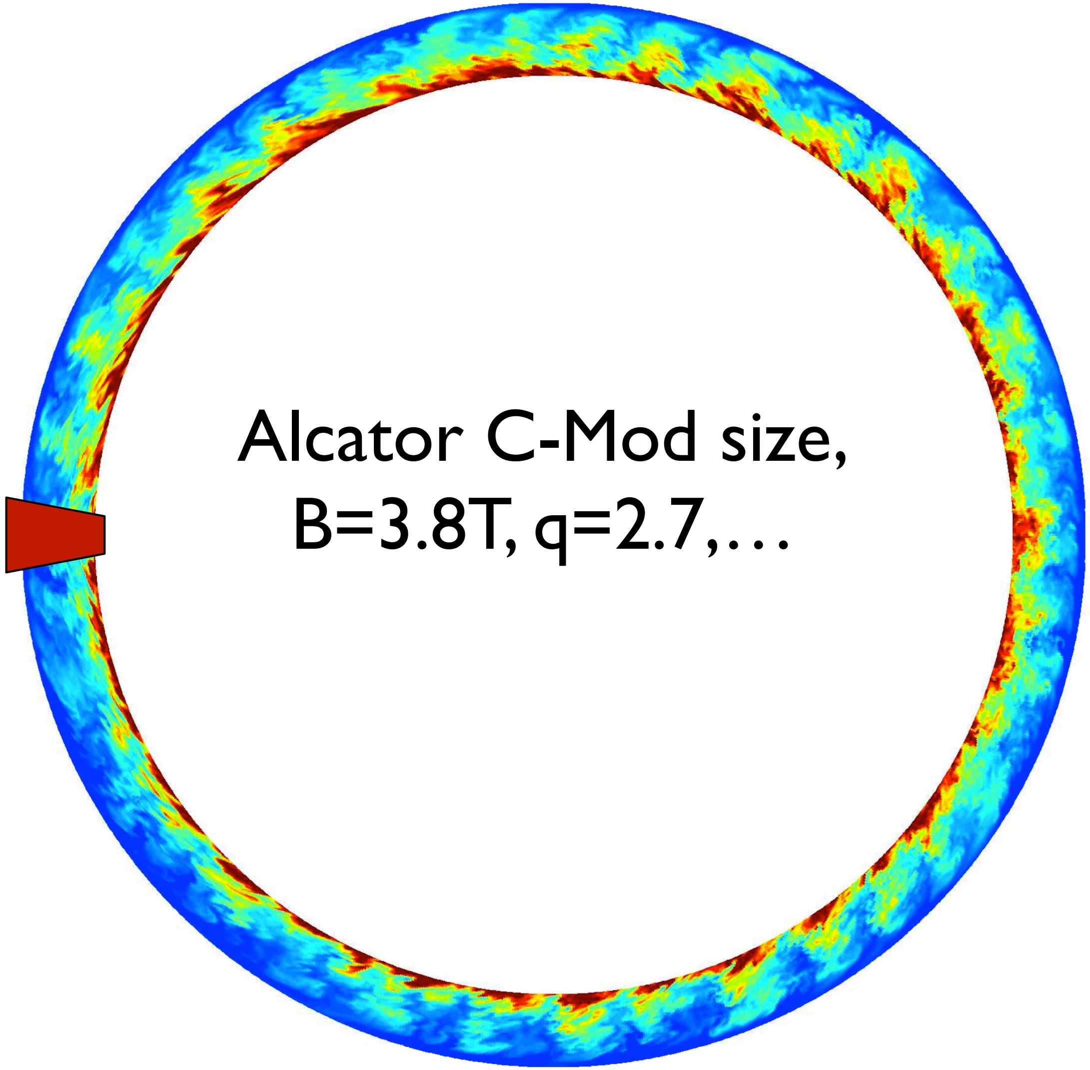
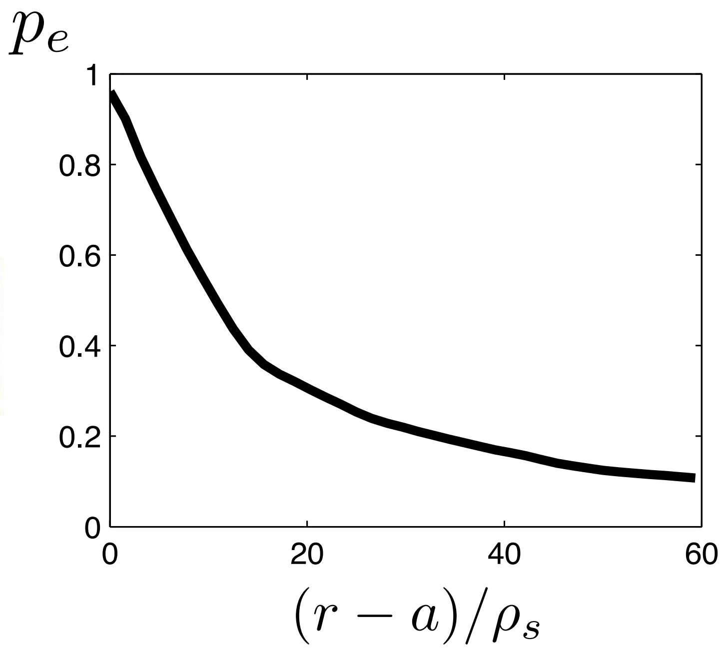
C-Mod simulations: 2 pressure scale length





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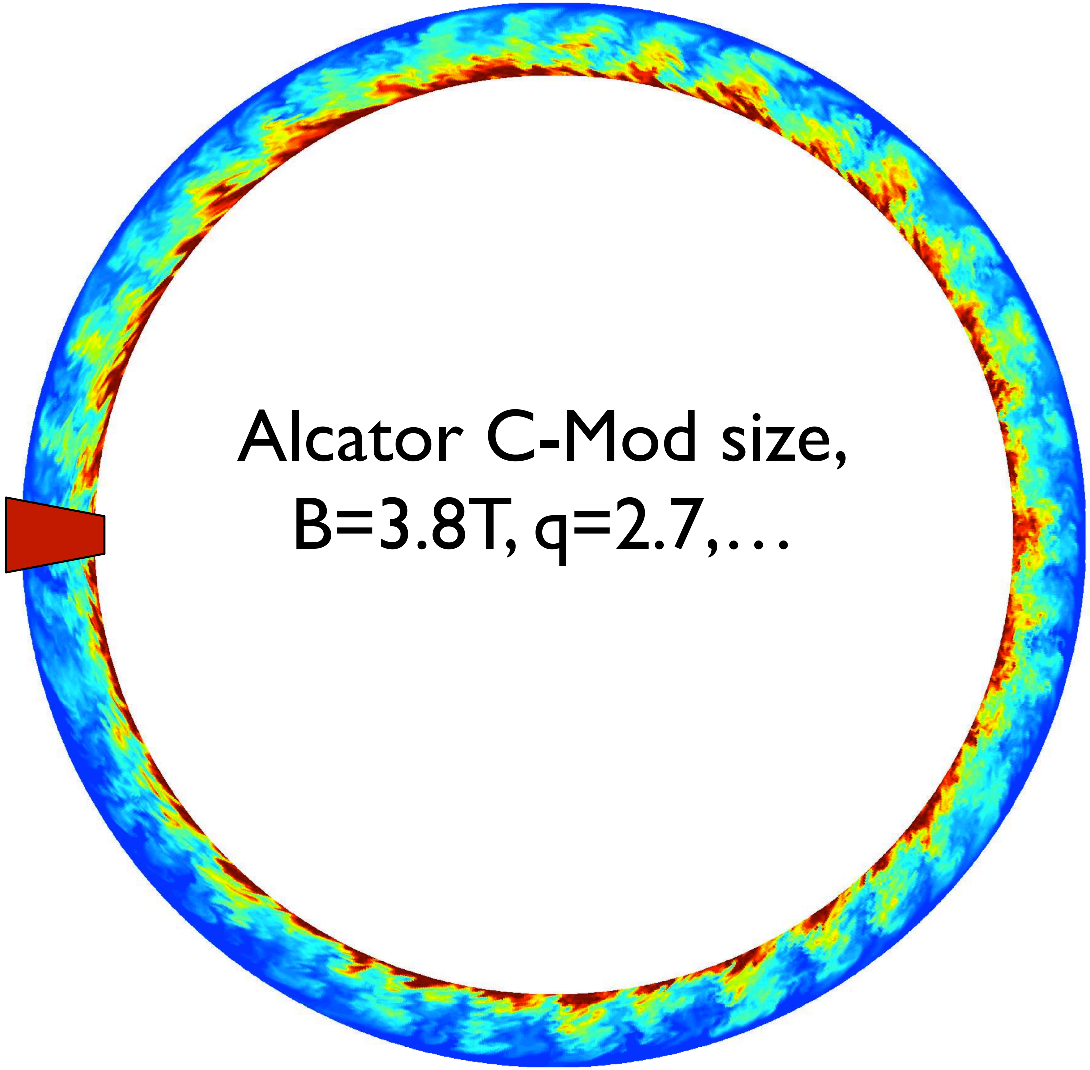

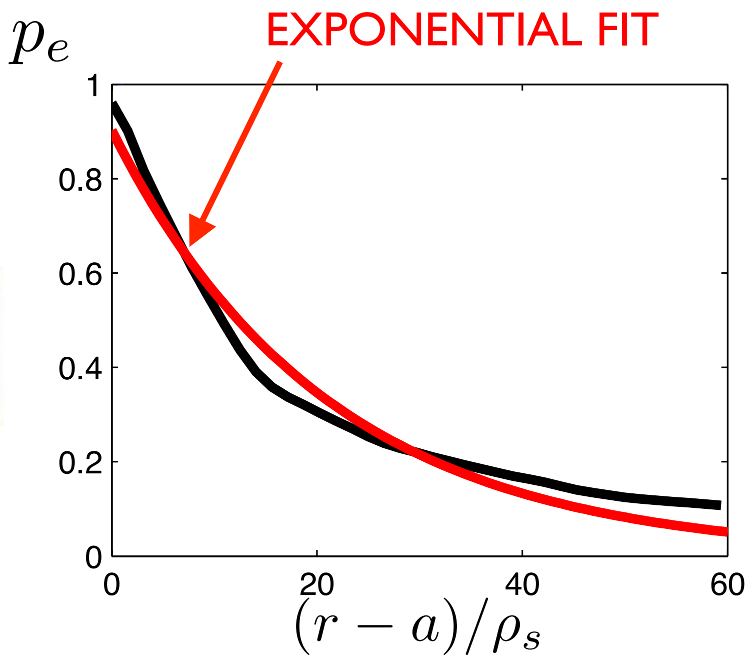

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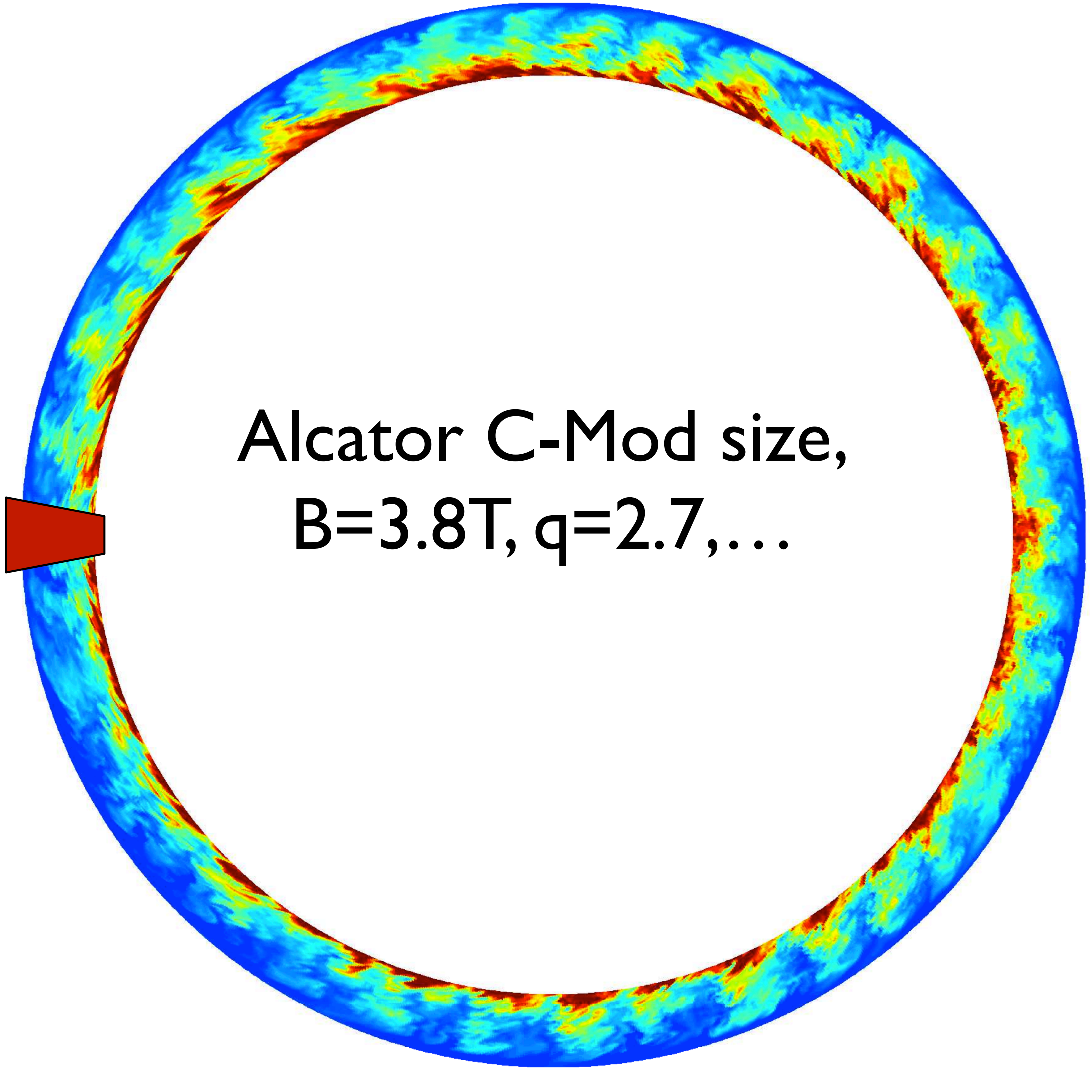

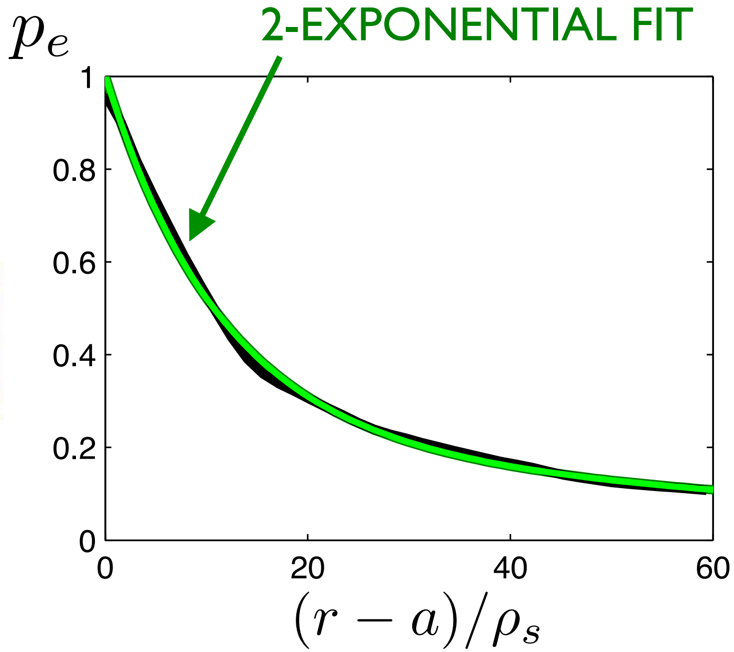

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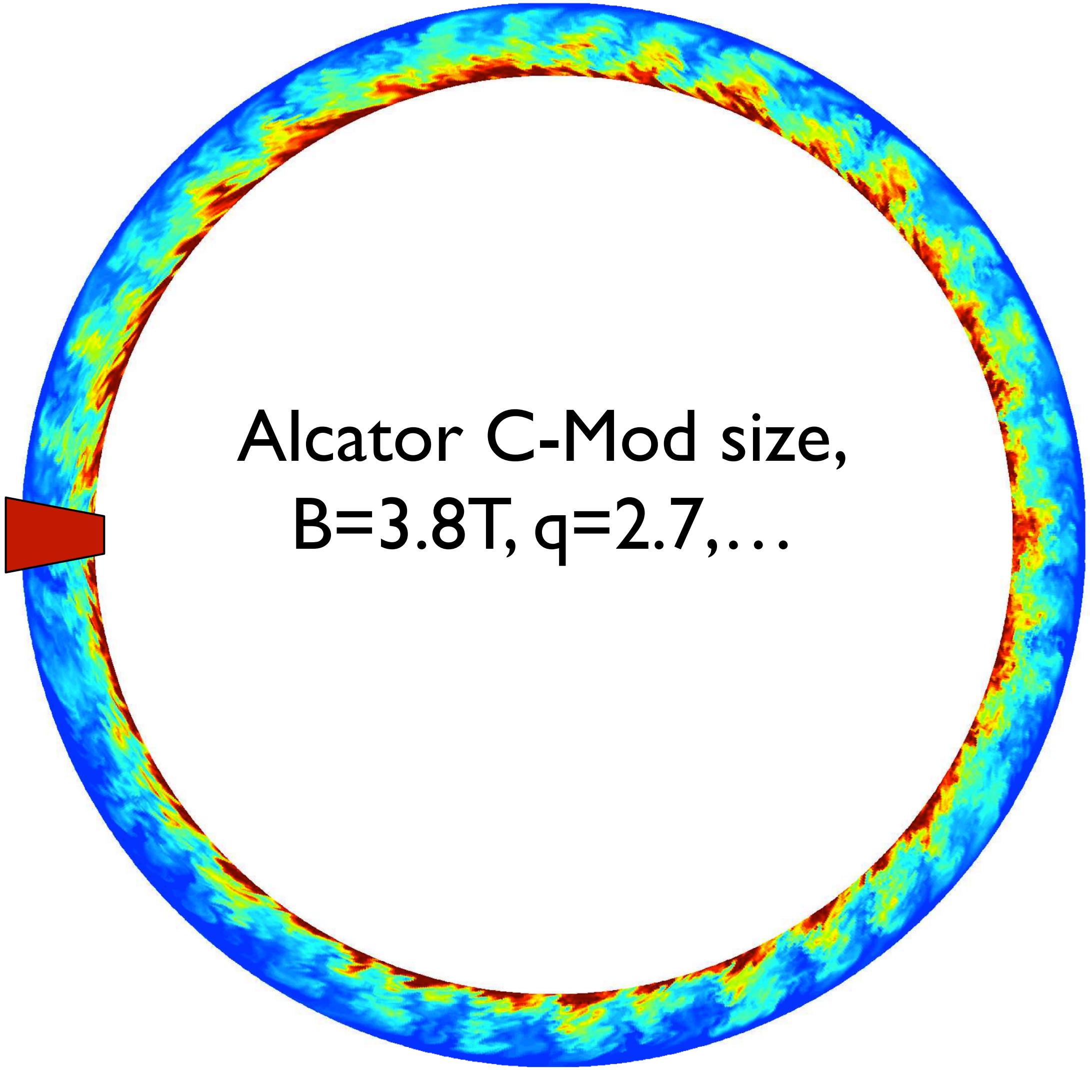
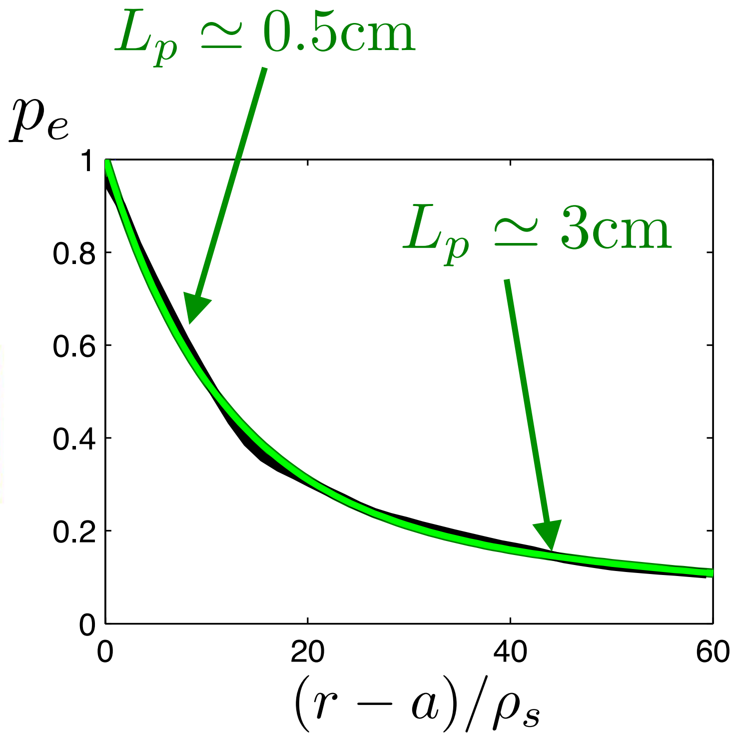




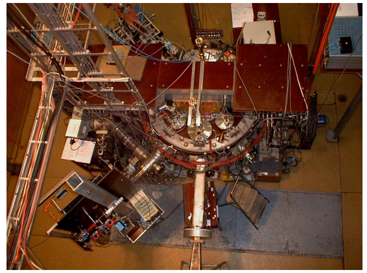
How can we approach the SOL width scaling?

- We can derive a first-principle scaling of the SOL width
- A drift-reduced model is able to represent the main features observed experimentally in the SOL
- Full-size simulations show large fluctuations, intermittent events, large scale ballooning turbulence
- SOL width established from the balance of parallel losses and perpendicular transport, driven by the ballooning instability and saturated by the gradient removal mechanism
- Experimental observations generally in agreement with theoretical observations

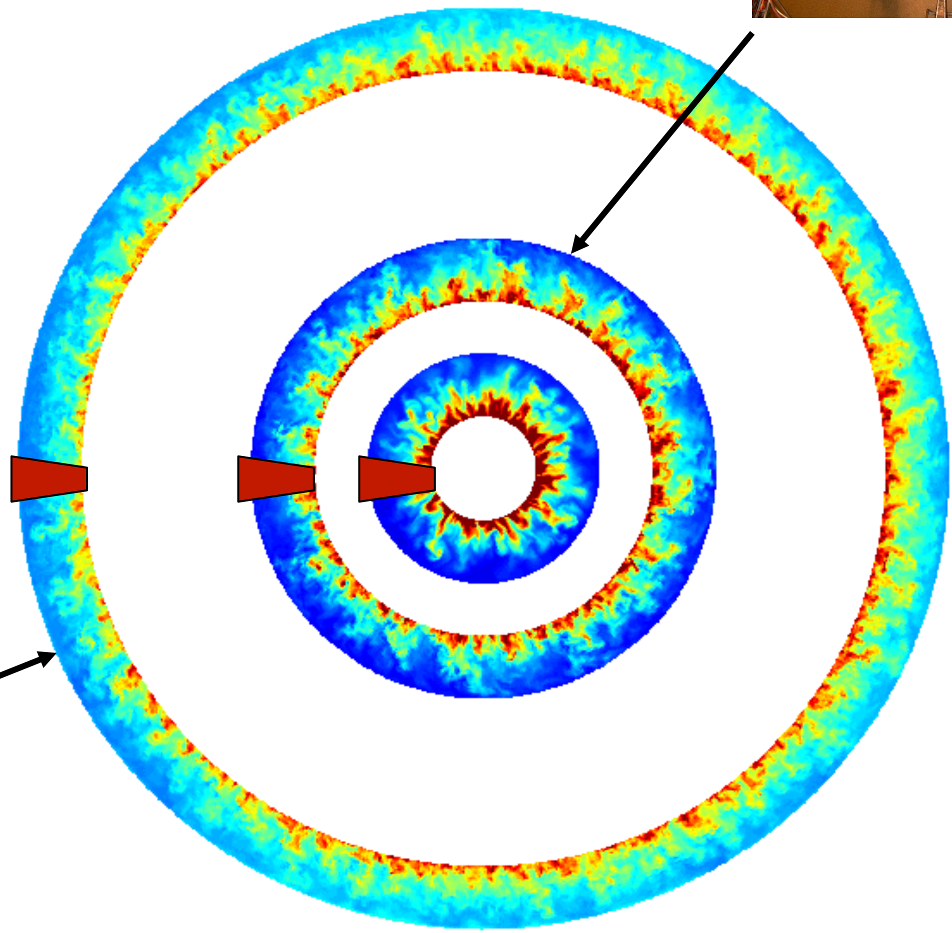
http://people.epfl.ch/paolo.ricci

Limited SOL width widens with R

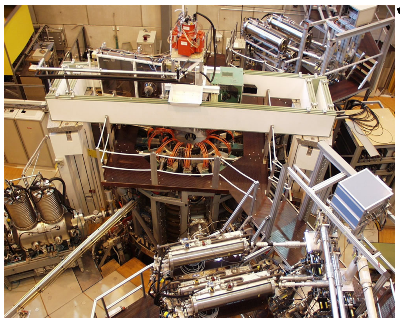
$$L_p = R^{1/2}[2\pi(1-\alpha_{\mathrm{MHD}})\alpha_d/q]^{-1/2}$$



CASTOR



TCV



Limited SOL transport increases with β and ν

$$L_p = R^{1/2}[2\pi(1-\alpha_{\mathrm{MHD}})\alpha_d/q]^{-1/2}$$