Energy Efficient Locomotion with Adaptive Natural Oscillator

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1 Introduction

For robotic systems, energy efficiency is one the most crucial goals. Many studies have been done to accomplish this goal from design and control point of view. In the second view, one of the preferred method is to design the desired trajectory in harmony with the dynamics of the system; i.e. natural dynamics exploitation. Assuming a structure for the desired trajectory, such as sinusoidal trajectories, we can have a parameterized control system as in CPG-Network. Therefore, having an adaptation method for those parameters to reach energy efficiency can be beneficial to control of robotic systems.

Here, we present the Adaptive Natural Oscillator (ANO); an adaptive pattern generator which exploits the natural dynamics of the system using instantaneous applied force. This tool is structurally similar to Adaptive Frequency Oscillator (AFO); see [1] and [2]. Preliminary studies have been done to use AFO in the close-loop fashion to exploit the natural dynamics of the system; see [3] and [4]. In those studies, position or velocity is used as the sensory feedback for the oscillator and no convergence proof has been presented. Here, we take advantage of applied force signal and we derive our adaption law from an optimization. Stability of the adaptation could also be shown by Lyapunov Theory.

2 Adaptive Natural Oscillator

Equation of a simple ANO in polar coordinate are

$$\begin{cases} \dot{\theta} &= \omega \\ \dot{\omega} &= \varepsilon \omega F(t)\cos(\theta) \\ x_d &= A\cos(\theta) \end{cases} \tag{1}$$

Phase of the oscillator is shown by ϕ, and the frequency (ω) is begin adapted using applied force of the system (F). Output of the oscillator x_d is being used as the desired trajectory for the system. Fig. 1 shows how ANO is used as a pattern generator in the close-loop fashion.

It can be shown that in a linear (mass-spring) system, ANO tries to minimize the instantaneous applied force. Dynamics of the mass-spring system are $F = m\ddot{x} + kx$ where m, k, and x are mass, spring constant, and position respectively. With assumption of perfect tracking when we use ANO as the pattern generator, we have $x = x_d$ which helps us to calculate the

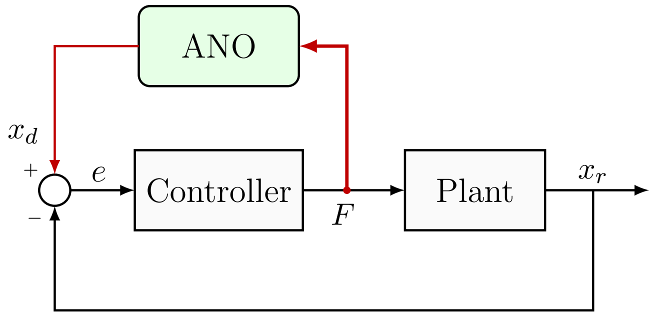

Fig. 1: Using adaptive natural oscillator to exploit natural dynamics. Oscillator is used as a pattern generator, and applied force is used as feedback.

applied force as:

$$F = A(k - m\omega^2)\cos(\theta) - A\dot{\omega}\sin(\theta) \tag{2}$$

To show that ANO minimize the instantaneous applied force, we consider the following cost function.

$$J(t) = \frac{1}{2}F^2(t) \tag{3}$$

Using the gradient descent method, we can derive an adaptation law for the oscillator frequency.

$$\dot{\omega} = -\lambda \nabla_\omega J(t) = -\lambda F(t)(\frac{\partial F}{\partial \omega}) \tag{4}$$

By substituting the derivative of Eq. 2, we have

$$\dot{\omega} = 2Am\lambda\omega F(t)\cos(\theta) \tag{5}$$

By taking $\varepsilon = 2Am\lambda$, we derive the proposed adaptation method.

Moreover, using Eq. 2 and to cancel out the applied force (F) in the adaptation law, we reach the dynamics of the adaptation as:

$$(1 + \varepsilon\omega\sin(\theta))\omega = -\varepsilon\omega(\omega - \omega_n)(\omega + \omega_n)\cos^2(\theta) \tag{6}$$

where $\omega_n = \sqrt{k/m}$. With the assumption of slow adaptation ($\varepsilon < 2/|\omega|$), stability of adaption for linear system can be studied. Using Lyapunov stability theory ($V = (\omega \mp \omega_n)^2$) and Barballat Lemma, stability of ω_n ($-\omega_n$) for positive (negative) frequencies can be proved.

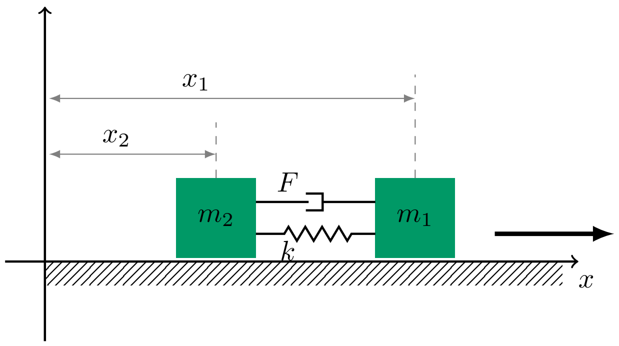

Fig. 2: Simple, adaptive toy simulation ($m_1 = m_2 = 1kg$, $k = 8N/m$)

3 Adaptive Toy

In this section, we use ANO to exploit the natural dynamics of the simple adaptive toy system presented in [4]. This model consists of two mass connected with an prismatic actuator and a parallel spring. As in [4], to have a locomotory behavior, ground applies asymmetric viscous friction to each mass as follows:

$$F_f(\dot{x}) = \begin{cases} 0.1 & \text{if} \quad \dot{x} > 0 \\ -0.2 & \text{if} \quad \dot{x} < 0 \end{cases} \tag{7}$$

Dynamics of the system are

$$\begin{cases} m_1\ddot{x}_1 + k\Delta x = \quad F - F_f(\dot{x}_1) \\ m_2\ddot{x}_2 - k\Delta x = -F - F_f(\dot{x}_2) \end{cases} \tag{8}$$

where $\Delta x = x_1 - x_2$. Prismatic force (F) is controlled by a PID-controller ($k_p = 100$, $k_i = 1$, and $k_d = 10$) where Δx tries to follow ANO output (x_d). Prismatic force (F) is used as the sensory input for ANO as illustrated in Fig. 1.

Fig. 3 shows the results of adaption to the natural dynamics for this simple model. First subplot shows that frequency of $4rad/s$ ($-4rad/s$) is stable for positive (negative) frequencies. The rest of subplots are corresponding to $\omega_0 = 0.1$.

Concurring our F^2 optimization in previous section, the second subplot shows that applied force converges to zero. Small ripples, however, exist to compensate the friction. The third subplot shows the locomotion of the system, while the fourth subplot shows the tracking performance over Δx.

To justify the adaptation equilibrium points, we can rewrite system dynamics in Δx coordinate as follows:

$$\frac{m}{2}\Delta\ddot{x} + k\Delta x = F + \underbrace{(F_f(\dot{x}_1) - F_f(\dot{x}_2))/2}_{Ds} \tag{9}$$

Eq. 9 can be seen as a disturbed mass spring system, where its natural frequency is $\sqrt{2k/m}$. Despite this disturbance (Ds), we see that ANO converges to the this natural frequency. Moreover, we observe that after convergence, ANO minimizes the applied force to the point where actuator only compensates the disturbance/friction. This simulation shows the efficiency and the robustness of our proposed tool.

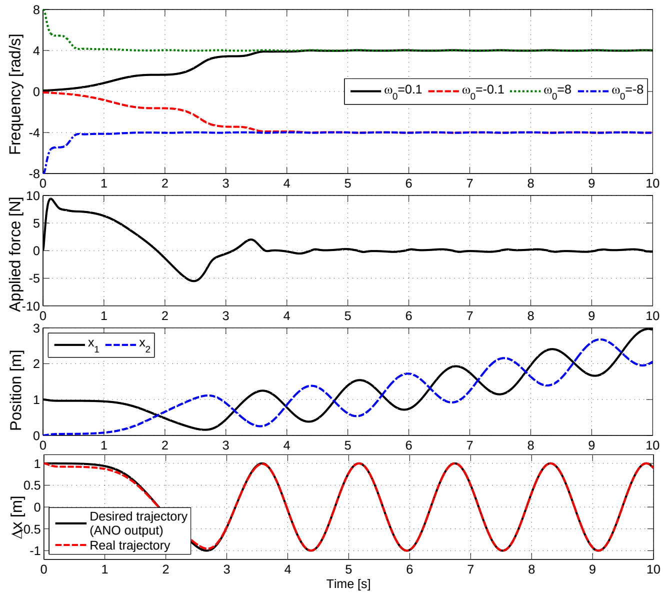

Fig. 3: Results of natural dynamics adaptation using ANO ($\varepsilon = 0.2$)

4 Conclusion

We presented a tool for natural dynamics exploitation. This tool, ANO, uses the applied force as the external signal for the oscillator. By exploiting natural frequency of the applied force, ANO is able to exploit the natural dynamics of the system. For the linear case, We have shown that ANO can be derived from a F^2 minimization problem, and its stability can be proved by Lyapunov Theorem. We have tested this tool to exploit the natural dynamics of the adaptive toy system, where we observed the efficiency and the robustness of ANO.

5 Future Works

Here we derived our proposed method for the linear case. We will extend this method to more general cases. Learning the limit cycle consistent with the natural dynamics is the next step of this research. We will test this method in real-world experiments, as well as in more advanced simulations such as legged systems.

References

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