

# Energy Efficient Locomotion with Adaptive Natural Oscillator

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## Motivation

We are presenting a novel adaptive oscillator, called Adaptive Natural Oscillator (ANO), to exploit natural dynamics in robotic application. This tool is built upon Adaptive Frequency Oscillator (AFO), and can be used as pattern generator. Like AFO that adapts to the frequency of an external signal, ANO adapts to the natural dynamics of a system. In this work, we prove that for linear systems, ANO adapts to the natural frequency. We show that this tool exploits the natural dynamics for energy efficiency through minimization of actuator effort. Proposed tool can be used in cyclic robotic applications, especially legged locomotion systems.

## Lyapunov Stability for Linear case

Having the dynamics of the system, applied force for a certain trajectory can be calculated as:

$$F = m\ddot{x}_r + kx_r$$

Considering the ANO dynamics, and with the assumption of perfect tracking ( $x_r = x_d$ ), we can calculate the applied force as follows:

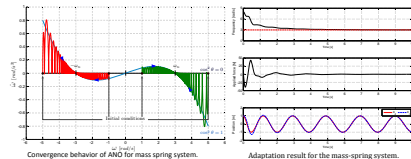
$$F = (k - m\omega^2) \cos(\theta) - \dot{\omega} \sin(\theta)$$

By defining  $\omega_n = \sqrt{k/m}$  and substituting in adaptation equation, we have the following dynamical system for the frequency.

$$\dot{\omega} = -\epsilon\omega(\omega - \omega_n)(\omega + \omega_n)\cos^2(\theta)$$

where  $p = 1 + \epsilon \omega \sin(\theta) \cos(\theta)$ . With the assumption of slow adaptation, we can make sure that  $p > 0$ , and the stability can be studied using following Lyapunov functions:

$$V = \frac{1}{2}(\omega \pm \omega_n)^2$$



## Hopper Leg

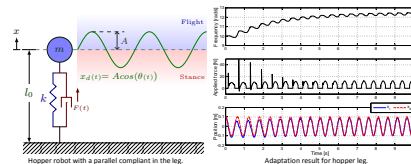
Consider a simple vertical hopper leg illustrated in Fig. 4. Hopper leg consists of a prismatic joint (actuator) and a parallel spring. Despite its simplicity, this system has hybrid dynamics..

- mass-spring dynamics during contact
- ballistic dynamics during flight

Reaching the spring rest length determines the transition between these dynamics.

### Task:

The task for this hopper is to move on a sinusoidal trajectory with amplitude of 0.1m and the offset equal to spring rest length ( $l_0 = 1m$ ). However, frequency of this oscillation, which is unknown, is left to ANO to be adapted.

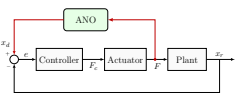


### Results:

- Frequency, in average, converges to 12.4rad.
- It is not easy to find this frequency analytically ( $\omega_n = \sqrt{k/m}$ ).
- Small ripples insinuate that sinusoidal trajectory is not perfectly consistent with natural dynamics of this system.
- The magnitude of applied force is reduced significantly.
- system quickly learns not to exert negative force.
- Tracking performance improves with adaptation.

## Adaptive Natural Oscillator

In some robotic application, such as locomotion, we try to reshape the desired trajectory according to the system dynamics; i.e. natural dynamics exploitation. By trying so, while performing a task, we expect to improve the energy efficiency. In this work, we only focus on the phase and the frequency of the desired trajectory, and we propose the following adaptive oscillator which uses applied force as the sensory feedback for adaptation as depicted in this figure.



Adaptive natural oscillator (ANO), is built upon adaptive frequency oscillator (AFO).

### AFO:

Adapting the frequency of the oscillator to the frequency of a target signal along with phase locking are the ultimate goals of AFO.

$$\begin{cases} \dot{\theta} = \omega + \dot{\omega} \\ \dot{\omega} = -\epsilon(\Gamma - x)\sin(\theta) \\ \dot{x} = A\cos(\theta) \end{cases}$$

$\Gamma$  is the external trajectory.

### ANO:

Adapting the frequency of the oscillator to the natural frequency of the system is the ultimate goal of ANO.

$$\begin{cases} \dot{\theta} = \omega \\ \dot{\omega} = \epsilon F \cos(\theta) \\ \dot{x} = A\cos(\theta) \end{cases}$$

$F$  is the applied force in the control loop.

## Force Minimization

It can also be shown that proposed adaptive oscillator tries to minimize the following cost function.

$$J = \frac{1}{2}F^2$$

Using gradient descent, and using ANO equations, we have:

$$\dot{\omega} = -\lambda \nabla_{\omega} J = -\lambda \epsilon \left( \frac{\partial F}{\partial \omega} \right) = 2m\lambda \omega F \cos(\theta)$$

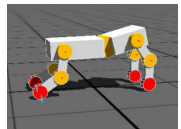
By defining  $\epsilon = 2m\lambda$ , we reach the adaptation rule.

Given a predefined trajectory,  $F^2$  and  $|F|$  minimization are roughly equivalent. Thus, it is true to be said that ANO minimizes the instantaneous power consumption.

## Quadruped robot with flexible spine

A bounding quadruped robot with flexible spine is an interesting case for this study.

Despite its contribution to forward velocity, adding an additional actuator, the spinal joint, will increase the complexity of the controller. Using adaptive natural oscillator to tune the frequency and phase lag of the spine can lead to exploitation of spinal joint natural dynamics, and subsequently result in faster locomotion.



### Gait:

A CPG-network is used to generate motions for bounding gait as follows:

	Front hip	Mid hip	Spine
Frequency [Hz]	2.4	2.4	2.4
Amplitude [rad]	0.7	0.7	0.3
Offset [rad]	0.0	0.0	0.0
Phase lag [rad]	0	2.7	-1.0

### ANO:

An adaptive oscillator in Cartesian space is used to adapt the motion of the spine.

$$\begin{cases} \dot{x}(t) = y \cdot (x^2 + y^2) \cdot x - \omega y \\ \dot{y}(t) = y \cdot (x^2 + y^2) \cdot y + \omega x + \epsilon F(t) \\ \dot{\omega}(t) = \frac{\epsilon F(t) y}{\sqrt{x^2 + y^2}} \end{cases}$$

Parameter	value
Initial frequency	$f_0$ 2.4 Hz
Amplitude	$\mu$ 0.3 rad
Limit cycle convergence rate	$\gamma$ 1
Frequency adaptation rate	$\epsilon$ .007

### Results:

- Frequency is slightly adjusted round its initial condition.
- Phase lag is evolved drastically
- Average velocity increased significantly

Adapted gait is faster, energy efficient and more natural

### References:

- [1] Ludovic Righetti, Jonas Buehl, and Auke Jan Ijspeert. "Adaptive frequency oscillators and applications". In: Open Cybernetics & Systems Journal 3 (2009), pp. 44–69.
- [2] M. Khoramshahi, Alexander Sprowitz, Alexandre Tuleu, M. N. Ahmadabadi, and Auke Ijspeert. "Benefits of an Active Spine Supported Bounding Locomotion With a Small Compliant Quadruped Robot". ICRA 2013.
- [3] M. Khoramshahi, Rezvan Nasiri, Auke Ijspeert, and M. N. Ahmadabadi. "Adaptive Natural Oscillator: A Novel Tool to Exploit Natural Dynamics for Energy Efficiency". IMDS 2014.