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AN ELASTO-PLASTIC-DAMAGE MODEL FOR QUASI-BRITTLE SHALES

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ABSTRACT: A constitutive model that couples elastic-plastic and damage theories is developed to predict the mechanical behavior of a shale from the Mont Terri rock laboratory (Opalinus Clay). The framework of continuum damage mechanics allows to predict the degradation of the elastic parameters with strains, while the coupling with plasticity correctly reproduces the irreversible strains typical of hard clayey materials. The yield surfaces (one for damage and one for plasticity) are postulated and the evolution equations of the internal variables are derived throughout the application of normality rule. Thermodynamic consistency of the model is investigated. The plastic behavior is described with a non-linear strain hardening function and is coupled with an isotropic damage model suitable for brittle and quasi-brittle geomaterials. The model is integrated with an implicit scheme that guarantees convergence and accuracy. Numerical simulations carried out with the proposed model in triaxial conditions well reproduced observed behavior from experiments.

1. INTRODUCTION

The disposal of radioactive waste arising from nuclear power plants operations and decommissioning represents a major issue for those economies that adopted nuclear technology for energy production purposes. In Switzerland, as well as in other developed countries, according to international recommendations, deep geological disposal of high level radioactive waste is seen at present as the best solution to protect humans and the environment from the dangers associated with radioactive substances. The concept of engineered barrier system (EBS) is applied and the host rock selected is Opalinus Clay (OPA), of which properties of low hydraulic conductivity will guarantee the isolation from the biosphere [1].

According to Bossart et al. [2] OPA can be viewed as a “stiff, over-consolidated clay with a hydraulic conductivity less than 10^{-12} m/s, a Young’s modulus varying between 4000 and 10,000 MPa (large range due to bedding anisotropy) and a cohesion greater than 2 MPa”. Some typical features that are common to many shales can be clearly distinguished such as intrinsic structural anisotropy, quasi-brittle behavior, strength dependency on mean pressure and degradation of stiffness. Quasi-brittle materials are defined as those materials that exhibit no or negligible plastic strain prior to failure [3]. Concrete, some ceramics and in this case Opalinus Clay [4, 5], usually exhibit this type of behavior. The formation and growth of micro-cracks can be considered to be responsible for both the softening behavior observed in the post-peak stress-strain curve and the development of plastic irreversible strains. To account on a solid base for this behavior the most suited solution is employing a coupled elasto-plastic-damage model. Micro-cracks growth, semi-brittle behavior, inelastic permanent strain development can all be accounted for in this unified formulation. In the following a coupled elasto-plastic-damage model, which is based on the theory of Continuum Damage Mechanics, will be presented in details. Thermodynamic admissibility of the proposed formulation will be investigated to set a strong physical base on which the model is founded. The proposed formulation is integrated numerically with an implicit scheme. Simulations at Gauss point level, which are representatives of simple conditions such as standard triaxial or oedometric tests, are performed. Model predictions are then compared with experimental findings.

2. PROBLEM FORMULATION

The coupling between the theories of Continuum Damage Mechanics and the theory of plasticity is particularly useful for the representation of quasi-brittle materials that exhibit both microcracks formation, inelastic strains and post peak softening behavior. A series of constitutive models coupling plasticity and isotropic damage for concrete, and in general geomaterials, were proposed in literature (e.g. [6-10]). Generally the coupling between the two theories can be realized following different approaches. Models can be built by specifying one single yield surface (either damage or plastic as in [11, 12]), or multiple surfaces [13]. Rate equations can be derived by applying normality rules to the yield surfaces. Plastic-damage models are usually derived from thermodynamic potential to ensure the respect of the first and second laws of thermodynamics. This theory is valid for generalized standard materials. Geomaterials are often considered not belonging to this class of materials, since to predict realistic values of dilatancy they often need non associated flow rule. For this class of materials the authors of [14] have proposed a different approach in which both yield surfaces and rate equations of internal variables are simply postulated and then checked afterwards for thermodynamic compatibility. In this work we follow the latter approach since it generates models that can be readily extended to non-associated frameworks. The constitutive equation that relates stresses σ_{ij} to strains ε_{ij}^{e} can be written as:

$$\sigma_{ij} = D_{ijkl}^{e}\varepsilon_{kl}^{e} \tag{1}$$

where D_{ijkl}^{e} is the elastic stiffness tensor. By introducing the damage internal variable d, the strain decomposition between plastic ε_{ij}^{p} and elastic ε_{ij}^{e}:

$$\varepsilon_{ij} = \varepsilon_{ij}^{e} + \varepsilon_{ij}^{p} \tag{2}$$

and the concept of effective stress $\tilde{\sigma}_{ij}$ as:

$$\sigma_{ij} = \left(1-d\right)\tilde{\sigma}_{ij} \tag{3}$$

the stress strain equation can be rewritten as:

$$\sigma_{ij} = \left(1-d\right)\tilde{\sigma}_{ij} = \left(1-d\right)D_{ijkl}^{e}\left(\varepsilon_{kl} - \varepsilon_{kl}^{p}\right) \tag{4}$$

In this work when referring to "effective stress" it will be intended "damage effective stress". Damage effective stress is the stress that will act on the undamaged part of the material (if damage is seen as the reduction of the area A_s on which stresses act, then by external equilibrium consideration effective stress $\tilde{\sigma}_{ij}$ acting on the reduced area $\tilde{A}_s$ is equal to nominal stress acting on the area considered as $\tilde{\sigma}_{ij}\tilde{A}_s = \sigma_{ij}A_s$). For a complete derivation of the concept of effective stress refer to [3].

2.1. Plasticity

The plastic response of the model is formulated in the effective stress space and includes the definition of the yield surface, the flow rule (associated in this case) and the rate equation of the internal variable that governs the hardening. The plastic yield function f^{p} is:

$$f^{p}\left(\tilde{\sigma}_{ij}, h_r\right) = 0 \tag{5}$$

where f^{p} is formulated in the damage effective stress space and h_r is the variable which controls the hardening of the plastic yield surface. The evolution of the hardening variable is a function of the rate of plastic strain as:

$$\dot{h}_r = g\left(\dot{\varepsilon}_{ij}^{p}\right) \tag{6}$$

The plastic strain rate, whose magnitude is controlled by the plastic multiplier $\dot{\lambda}^{p}$ is given by the plastic flow rule, associated in this case:

$$\dot{\varepsilon}_{ij}^{p} = \dot{\lambda}^{p}\partial_{\tilde{\sigma}_{ij}} f^{p} \tag{7}$$

The loading-unloading problem is represented by the Karush-Kuhn-Tucker conditions:

$$\dot{\lambda}^{p} \geq 0 \qquad \dot{f}^{p} \leq 0 \qquad \dot{\lambda}^{p}\dot{f}^{p} = 0 \tag{8}$$

2.2. Damage

The damage response is formulated in the elastic strain space so that the yield surface, analogously to plasticity, can be expressed in the following way:

$$f^{d}\left(\varepsilon_{ij}, \varepsilon_{ij}^{p}, d\right) = 0 \tag{9}$$

in which f^{d} is the damage yield function formulated in terms of total strain, plastic strain and damage internal variable. The rate of damage internal variable d is normal to the damage yield function through the damage associated variable Y (often called damage thermodynamic force), therefore:

$$\dot{d} = \dot{\lambda}^{d}\partial_{Y} f^{d} \tag{10}$$

in which $\dot{\lambda}^{d}$ is the damage multiplier that controls the magnitude of damage rate. The loading unloading problem is once again represented by the Karush-Kuhn-Tucker conditions applied to the damage formulation:

$$\dot{\lambda}^{d} \geq 0 \qquad \dot{f}^{d} \leq 0 \qquad \dot{\lambda}^{d}\dot{f}^{d} = 0 \tag{11}$$

3. THERMODYNAMIC FRAMEWORK

Thermodynamic compatibility of the proposed formulation ensures the respect at all time of the first and second postulates of thermodynamics. As already mentioned, the model is not derived from a dissipation potential and therefore is clear the necessity to examine its consistency so that no postulates are violated (one of the dangers often encountered is that spurious energy generation might arise during simulations). The framework is based on the work of Lubliner on the thermodynamics of non-linear solids [15]. The local thermodynamic state of a solid body is assumed in this sense to be uniquely determined by the strain tensor ε_{ij}, the entropy per unit volume S and a set of the so called internal variables α_i (in the method of local state the latter represent the history of the material, i.e. are representative of dissipative phenomena [16]). Therefore the internal energy of the solid is a function of the above mentioned variables $E = E\left(\varepsilon_{ij}, S, \alpha_i\right)$ and the local equation of energy conservation is expressed as:

$$\dot{E} = \partial_S E\dot{S} + \partial_{\varepsilon_{ij}} E\dot{\varepsilon}_{ij} + \partial_{\alpha_i} E\dot{\alpha}_i = \sigma_{ij}\dot{\varepsilon}_{ij} - \partial_{x_k} h_k \quad (12)$$

where h_k is the heat flow vector and x_k is the vector of spatial coordinates. The Clausius-Duhem inequality, which is representative of the second law of thermodynamics, can be written as:

$$\dot{S} + \partial_{x_k}\left(\frac{h_k}{T}\right) = \dot{S} + \frac{1}{T}\partial_{x_k} h_k - \frac{h_k}{T^2}\partial_{x_k} T \geq 0 \quad (13)$$

where T is the temperature. Alternatively:

$$T\dot{S} + \partial_{x_k} h_k - \frac{h_k}{T}\partial_{x_k} T = d_m + d_T \geq 0 \quad (14)$$

where $d_m = T\dot{S} + \partial_{x_k} h_k$ is the mechanical dissipation and $d_T = -h_k / T\,\partial_{x_k} T$ is the thermal dissipation. For slow processes the thermal dissipation becomes small if compared with the mechanical dissipation, which implies itself to be non-negative. Under the assumption of isothermal conditions, substitution of Eq. (12) into Eq. (13) while keeping in mind the alternative standard expression for the rate of E, leads to:

$$\left(T - \partial_S E\right)\dot{S} + \left(\sigma_{ij} - \partial_{\varepsilon_{ij}} E\right)\dot{\varepsilon}_{ij} - \partial_{\alpha_i} E\dot{\alpha}_i \geq 0 \quad (15)$$

By definition of the state equations: $T = \partial_S E$ and $\sigma_{ij} = \partial_{\varepsilon_{ij}} E$, it results in the mechanical dissipation:

$$d_m = -\partial_{\alpha_i} E\dot{\alpha}_i \geq 0 \quad (16)$$

For coupled plastic-damage models the internal energy can be expressed as a function of the total strain, the entropy per unit volume, the damage d and plastic ε_{ij}^p internal variables as:

$$E\left(\varepsilon_{ij}, \varepsilon_{ij}^p, S, d\right) = \frac{1}{2}\left(1-d\right) D_{ijkl}^e \left(\varepsilon_{kl} - \varepsilon_{kl}^p\right)^2 + TS \quad (17)$$

which yields to the constitutive equation:

$$\sigma_{ij} = \partial_{\varepsilon_{ij}} E = \left(1-d\right) D_{ijkl}^e \left(\varepsilon_{kl} - \varepsilon_{kl}^p\right) \quad (18)$$

The damage associated variable Y, which represents the elastic energy density inside the material, is:

$$Y = -\partial_d E = \frac{1}{2} D_{ijkl}^e \left(\varepsilon_{kl} - \varepsilon_{kl}^p\right)^2 \quad (19)$$

and the associated variable for plasticity:

$$\sigma_{ij} = -\partial_{\varepsilon_{kl}^p} E = \left(1-d\right) D_{ijkl}^e \left(\varepsilon_{kl} - \varepsilon_{kl}^p\right) \quad (20)$$

The dissipation equation in isothermal conditions for an elasto-plastic-damage model is:

$$-\partial_{\varepsilon_{ij}^p} E\dot{\varepsilon}_{ij}^p - \partial_d E\dot{d} = \sigma_{ij}\dot{\varepsilon}_{ij}^p + Y\dot{d} \geq 0 \quad (21)$$

in which the first term accounts for the plastic dissipation and the second one for damage dissipation.

According to [14] the Clausius-Duhem inequality must be valid globally, although a sufficient but non necessary condition is that each term is non negative. In this case $Y\dot{d} \geq 0$ is always valid since the elastic energy density is a quadratic form and the damage rate is always positive by definition. By applying the normality rule to plastic dissipation mechanism one can write:

$$\sigma_{ij}\dot{\varepsilon}_{ij}^p = \dot{\lambda}^p \left(1-d\right)\tilde{\sigma}_{ij}\partial_{\tilde{\sigma}_{ij}} f^p \geq 0 \quad (22)$$

which means that the scalar product between the damage effective stress $\tilde{\sigma}_{ij}$ and the plastic flow rule must be non-negative. This is always true for associated flow rules and if the hypothesis of coaxiality between stresses and strains is accepted. In the proposed model an associated flow rule is applied. An extension to non-associated flow rules that better describe dilatancy of shales can be made considering the thermodynamics restrictions that follow from the previous considerations.

4. PLASTIC-DAMAGE MODEL

The formulation of the complete elasto-plastic-damage model requires appropriate plastic and damage yield functions as well as rate equations for the respective internal variables. The plastic yield surface implemented is a non-linear extension of the Lade-Duncan model as proposed by [17]. The advantage of such a choice is that Opalinus Clay has shown a strong non-linear dependence of the yield locus on the mean stress, and has often been interpreted with bi-linear Mohr-Coulomb type of models (e.g. [4]). The following formulation in the tridimensional effective stress space is implemented:

$$f^p = \tilde{q} - f_{q-p}^p\left(\tilde{p}\right) f_\pi^p\left(\tilde{\vartheta}\right) \quad (23)$$

which is a combination of functions in the $\tilde{q}-\tilde{p}$ plane and in the octahedral plane $\tilde{\pi}$ to account for a complete tri-dimensional formulation of class C^1 in the stress space that depends on three invariants of the stress tensor defined as:

$$\tilde{p}=\frac{\tilde{I}_1}{3} \quad (24)$$

$$\tilde{q}=\sqrt{3\tilde{J}_2} \quad (25)$$

$$\tilde{\vartheta}=\frac{1}{3}\arcsin\left(-\frac{3\sqrt{3}}{2}\frac{\tilde{J}_3}{\left(\tilde{J}_2\right)^{3/2}}\right) \quad (26)$$

in which $\tilde{I}_1=\tilde{\sigma}_{ii}/3$ is the first invariant of the stress tensor, $\tilde{J}_2=1/2\,\tilde{s}_{ij}\tilde{s}_{ij}$ the second invariant of the deviatoric stress tensor, $\tilde{J}_3=\det\tilde{s}_{ij}$ the third invariant of the deviatoric stress tensor and $\tilde{s}_{ij}=\tilde{\sigma}_{ij}-\tilde{p}\delta_{kk}$ is the deviatoric stress tensor. The complete formulation can be written as:

$$f^p=\tilde{q}-\left[\alpha\frac{\tilde{p}^2}{c}+\beta\tilde{p}+h_r c\right]\frac{\sin\left[\frac{\pi}{3}-\frac{1}{3}\arcsin\left(\sqrt{\frac{k_0+\eta e^{-\left(\tilde{I}_1-3c\right)/t1}-27}{k_0+\eta e^{-\left(\tilde{I}_1-3c\right)/t1}}}\right)\right]}{\sin\left[\frac{\pi}{3}+\frac{1}{3}\arcsin\left(\sqrt{\frac{k_0+\eta e^{-\left(\tilde{I}_1-3c\right)/t1}-27}{k_0+\eta e^{-\left(\tilde{I}_1-3c\right)/t1}}}\sin 3\tilde{\vartheta}\right)\right]} \quad (27)$$

where α , β , c , k_0 , η and t_1 are material parameters. This formulation ensures regularity and convexity of the yield surface and can account for anisotropic strength available in the octahedral plane π , which is a typical feature of geomaterials (the strength in triaxial extension is lower than the strength available in triaxial compression conditions). In this paper only simulations in triaxial compression conditions are considered, therefore from now on only the function in the $\tilde{q}-\tilde{p}$ plane $f^p=\tilde{q}-f^p_{q-p}\left(\tilde{p}\right)$ is retained, since in such conditions $\tilde{\vartheta}=-30°$, and therefore $f^p_{\pi}\left(\tilde{\vartheta}\right)=1$. In the proposed model the parameter c represents the cohesion component of the material in the effective stress space. Fig. 1 illustrates the plastic yield surface for different values of the hardening variable h_r . Negative values of p correspond to traction components.

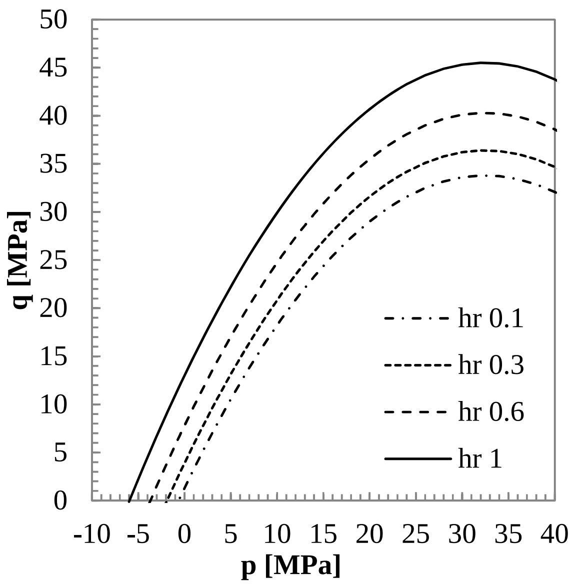

Fig. 1. Plastic yield function in the $\tilde{q}-\tilde{p}$ plane for different values of the hardening parameter h_r .

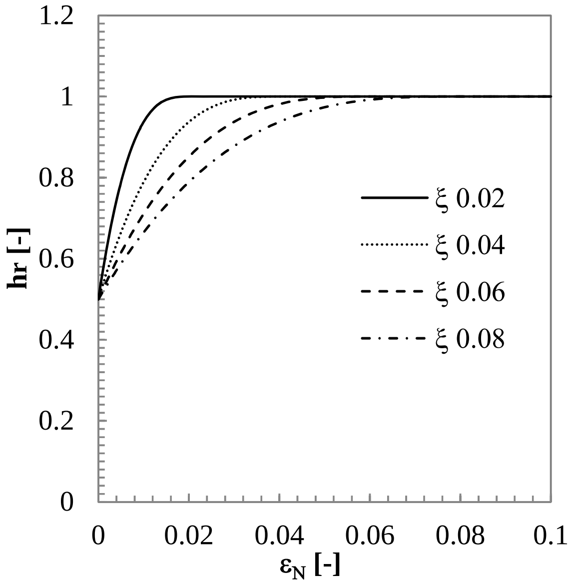

Fig. 2. Evolution of the hardening variable with the plastic strain norm for different values of the parameter ξ .

The hardening of the yield function is governed by the parameter h_r which depends on the norm of the plastic strain tensor $\left\|\varepsilon^p_{ij}\right\|=\sqrt{\varepsilon^p_{ii}\varepsilon^p_{ii}}$ and its expression is taken after [14]:

$$h_r=\begin{cases} h_{r0}+\left(1-h_{r0}\right)\frac{\left\|\varepsilon^p_{ij}\right\|}{\xi}\left[\left(\frac{\left\|\varepsilon^p_{ij}\right\|}{\xi}\right)^2-3\frac{\left\|\varepsilon^p_{ij}\right\|}{\xi}+3\right] & if\ h_r\le 1 \\ 1 & if\ h_r>1 \end{cases} \quad (28)$$

The hardening function dependency on the plastic strain norm is shown in Fig. 2. The material parameters to be determined are α, β, c, h_{r0} and ξ. For the response in damage the model proposed by Marigo [18] has been followed. This model is particularly suitable for brittle materials. The yield surface for damage can therefore be represented in the elastic strain space as:

$$f^d\left(\varepsilon_{ij},\varepsilon_{ij}^p,d\right)=Y_- - k\left(d\right) \tag{29}$$

where:

$$Y_- = \frac{1}{2}D_{ijkl}^e\left\langle \varepsilon_{kl}^e\right\rangle_-^2 \tag{30}$$

is the elastic energy density associated with negative (tensile) elastic strains. No sign distinction is made on the shear strains, for which both positives and negatives ones are responsible for damage initiation, so that finally damage results from the accumulation of tensile and shear fractures. Another advantage of such a choice is the implicit accounting of confining stress dependency. The expression of $k\left(d\right)$ follows from the work of Weibull as reported in [3].

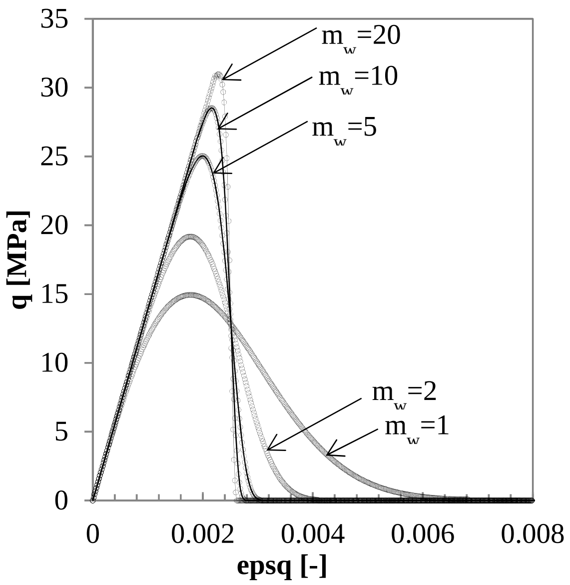

Fig. 3. Stress strain plot for pure damage with different values of the Weibull modulus.

This model originates from a probabilistic approach to the fracture of brittle materials and results in the following expression for $k\left(d\right)$:

$$k\left(d\right)=\alpha_d\left[\log\left(\frac{1}{1-d}\right)\right]^{1/m_w} \tag{31}$$

where α_d and m_w are material parameters. $m_w/2$ is the Weibull modulus and controls the brittleness of the material, while for example in uniaxial conditions:

$$\alpha_d \approx \frac{\sigma_{1,f}^2}{2E} \tag{32}$$

in which $\sigma_{1,f}^2$ is the failure stress (i.e. when damage tends to one, since the limit of $k\left(d\right)$ for damage that tends to one is α_d) in uniaxial conditions and E the Young's modulus in the given direction. In Fig. 3 plots of stress-strain behavior with different values of the Weibull modulus illustrate that an increase in m_w corresponds to an increase in the brittleness of the response.

5. NUMERICAL IMPLEMENTATION

The proposed model has been integrated into an implicit Newton-Raphson scheme to ensure convergence and stability of the solution independently from the step size. The problem can be seen as strain driven, so the objective is to find a right solution of the internal variables of damage d and plasticity ε_{ij}^p and the effective stress $\tilde{\sigma}_{ij}$ for a given value of total strain increment $\dot{\varepsilon}_{ij}$. In order to do so a stress return algorithm for multidissipative materials has been implemented based on the work of [19]. The scheme is based on a minimization of the residuals $R\left(\Xi\right)$ representing the discretized form of the problem at the load step $n+1$ (in this case the engineering notation is assumed instead of tensorial notation):

$$R\left(\Xi\right)=\left\{\begin{array}{c} f_{n+1}^p \\ f_{n+1}^d \\ \varepsilon_{i,n+1}^p-\varepsilon_{i,n}^p-\Delta\lambda_{n+1}^p\partial_{\tilde{\sigma}_i}f_{n+1} \\ d_{n+1}-d_n-\Delta\lambda_{n+1}^d\partial_Y f_{n+1}^d \\ \tilde{\sigma}_{i,n+1}-\tilde{\sigma}_{i,n}-E_{ij}\left(\varepsilon_{j,n+1}-\varepsilon_{j,n}\right)+\Delta\lambda_{n+1}^p E_{ij}\partial_{\tilde{\sigma}_j}f_{n+1} \end{array}\right\} \tag{33}$$

which are functions of the variables that represent the unknowns of the problem:

$$\Xi=\left\{\begin{array}{c} \Delta\lambda_{n+1}^p \\ \Delta\lambda_{n+1}^d \\ \varepsilon_{i,n+1}^p \\ d_{n+1} \\ \tilde{\sigma}_{i,n+1} \end{array}\right\} \tag{34}$$

Writing the rate problem equation of the residuals in a discretized form leads to:

$$R\left(\Xi_{n+1}^{m+1}\right)=R\left(\Xi_{n+1}^m\right)+J_{n+1}^m\left(\Xi_{n+1}^{m+1}-\Xi_{n+1}^m\right) \tag{35}$$

where the matrix J is the Jacobian matrix of the problem and contains the second derivatives of the constitutive model, i.e. the derivatives of the residual vector with respect to the unknowns of the problem. New values of the unknowns are computed by setting $R\left(\Xi_{n+1}^{m+1}\right)=0$ which leads to:

$$\Xi_{n+1}^{m+1}=\Xi_{n+1}^{m}-\left(J_{n+1}^{m}\right)^{-1}R\left(\Xi_{n+1}^{m}\right) \quad (36)$$

The process is iterated through the m index until the norm of the residuals is less than a given value of tolerance $\left\|R\left(\Xi_{n+1}^{m+1}\right)\right\|=\sqrt{\sum_{i=1}^{nres}\left[R_i\left(\Xi_{n+1}^{m+1}\right)\right]^2}\leq\theta_{tol}$ and the solution Ξ_{n+1}^{m+1} is obtained.

6. NUMERICAL SIMULATIONS

The proposed constitutive model is used to numerically simulate experimental finding reported in [4]. A series of undrained triaxial compression tests on Opalinus Clay were performed to investigate the strength resistance of the material. Three different tests at 1, 3 and 6 MPa of confining pressure were used to validate the proposed elasto-plastic-damage formulation. The parameters were determined based on literature values, triaxial tests interpretation and numerical calibration against the experimental findings. The model parameters determination procedure starts with the determination of the elastic parameters. The parameters to be determined are Young's modulus E_e and Poisson's ratio ν. Parameters are taken from literature [20] and Young's modulus is adjusted to the experimental evidence considered in this work [4] and their values are shown in Tab. 1.

A rigorous procedure to determine the values of the parameters involved in the damage model will require the probabilistic analysis of a consistent number of tests (between 10 and 20 according to [3]). Therefore, it is hardly applicable unless such a number of tests is available. Alternatively the Weibull modulus m_w can be calibrated numerically on uniaxial or low confinement tests so that the brittle response is correctly predicted. In this case its value was taken to reproduce the behavior observed at 1 MPa of confinement. In the absence of a good number of tests a rough estimate of α_d can be obtained as:

$$\alpha_d\approx\frac{q_{1f}^2}{6G}\approx\frac{16^2}{6\times3769}=0.0113 \quad (37)$$

Where G is the shear elastic modulus and q_{1f}^2 is the square of the failure stress (residual stress) taken as medium value between the three tests. For what concerns plasticity, since it governs in the present model the values of stress at steady states, the points at this state can be plotted in the $q-p$ plane as in fig. 4.

The data can be interpolated with a second degree polynomial function which gives the values of the three coefficient a_{pol}, b_{pol} and c_{pol} that are related to plastic parameters via the following equations:

$$\begin{cases}a_{pol}=\alpha/c\\ b_{pol}=\beta\\ c_{pol}=c\end{cases} \quad (38)$$

so that plastic parameters can be obtained by solving this simple system.

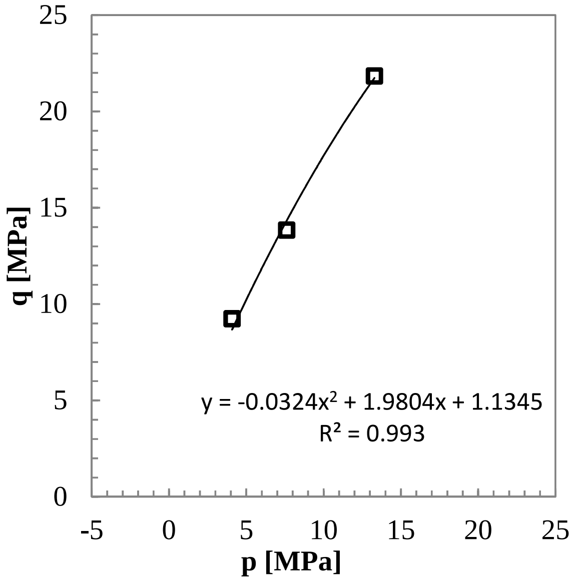

Fig. 4. Calibration of plastic parameters from experimental results.

Keeping in mind that the plasticity is formulated in terms of damage effective stress, the surface should be translated to account for those values of stress (it is not possible to predict the damage effective stress a-priori from analyses given the high non-linearities). Therefore the cohesion c is increased to the value of 13.2 MPa to better represent the experimental behavior, which gives then $\alpha=a_{pol}\times c=-0.0324\times13.2=-0.428$. The set of parameters used to simulate triaxial behavior of Opalinus Clay is illustrated in tab. 1. The value of h_{r0} is quite difficult to be measured from experimental data. Hence in this case the assigned value has been calibrated to well reproduce low values of irreversible strains shown before the peak of stress is reached. To calibrate ξ one would require the whole measurements of all the plastic strain components, which is practically impossible to perform, so that the parameter has to be adjusted in numerical simulations.

Table 1. Set of parameters used for the simulation of triaxial tests on Opalinus Clay.

Parameter	Values	Units
E_e	9800	MPa
ν	0.3	-
α	-0.428	-
β	1.9804	-
c	13.2	MPa
h_{r0}	0.35	-
ξ	0.01	-
m_w	10	-
α_d	0.016	MPa

The simulation of triaxial tests has been carried out by first applying an isotropic pressure to reach the values of confinement, and then vertical displacement has been applied to simulate the shearing phase. The samples are so called P samples, in which the bedding planes that confer anisotropy to Opalinus Clay are perpendicular to the direction of the maximum principal stress applied [5]. Therefore the results can be interpreted as being representative of the shale matrix. The comparison between the numerical results and experimental findings is shown in Figs. 5, 6 and 7. It is shown how the proposed model can correctly predict the main characteristics of the investigated material. The damage formulation is responsible for the post peak softening that represents the brittleness of the material. One of the hypotheses is that before the peak of stress is reached, the response shows very low values of irreversible strains. In this phase the microcracks inside the material are growing and this is well represented by the damage part.

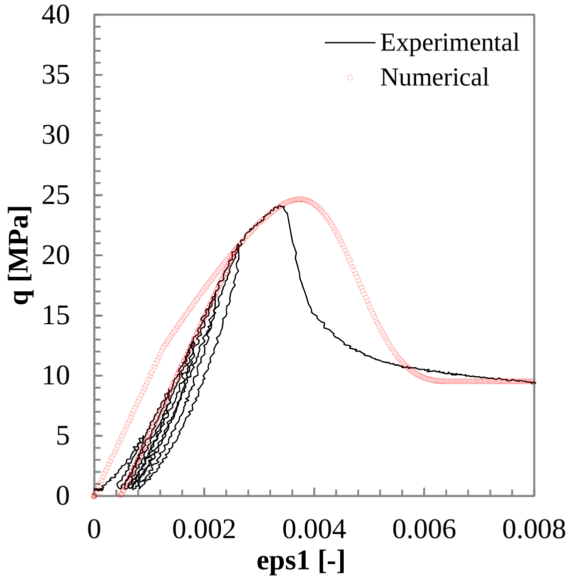

Fig. 5. Deviatoric stress vs axial strain plot for comparison of experimental results against numerical simulations at 1 MPa of confining pressure.

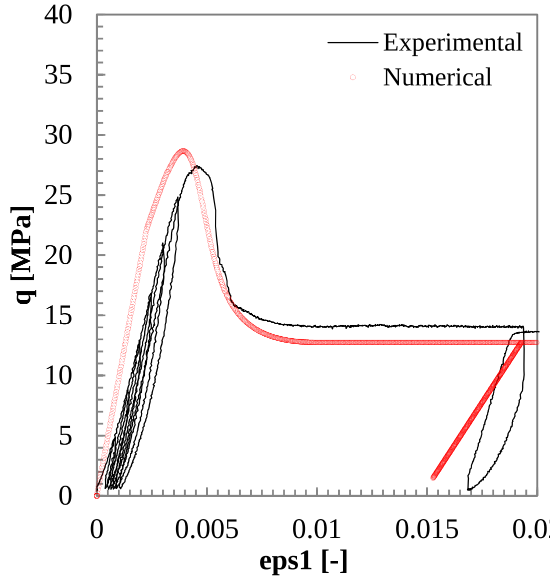

Fig. 6. Deviatoric stress vs axial strain plot for comparison of experimental results against numerical simulations at 3 MPa of confining pressure.

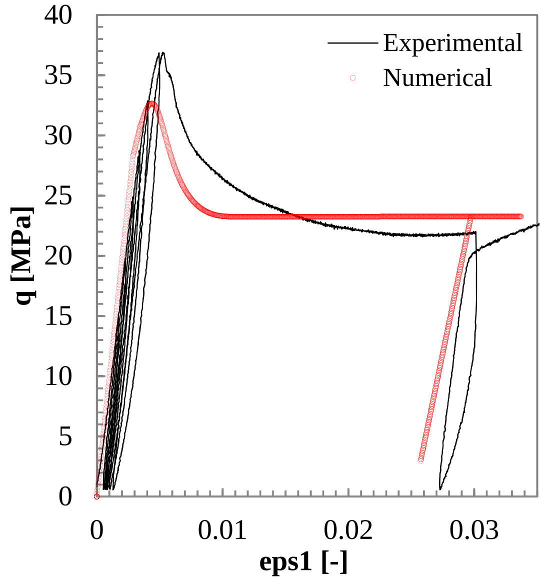

Fig. 7. Deviatoric stress vs axial strain plot for comparison of experimental results against numerical simulations at 6 MPa of confining pressure.

Low values of irreversible strain before the stress peak is reached can be predicted by the model. After the softening response the material exhibit constant values of the deviatoric stress: it is possible to assert that the material is plastically flowing in this phase as confirmed by the unloading carried out. In the proposed model the steady conditions for plasticity reached in the last phase are responsible for this behavior. The elastic unloading is well predicted both in the pre peak and in the post peak phase (slight discrepancy can be observed for the test at 3 MPa of confinement). An overall good agreement between the proposed model and experimental evidences is shown, which validates the hypothesis made in the formulation.

7. CONCLUSIONS

An elasto-plastic-damage model for quasi-brittle shales was presented. The aim was to develop a model that well represents the main mechanisms of the stress-strain feature of Opalinus Clay. The physical interpretation of the mechanical response of the material has set the starting point of the modeling process. The proposed framework includes coupling between the theories of plasticity and continuum damage mechanics. The plastic and damage yield functions as well as the rate equations of internal variables have been postulated with respect to thermodynamic restrictions. The stress return algorithm implemented to integrate the model is presented and results were compared with experimental findings. The good agreement between model simulation and experimental data lead to the validation of the proposed formulation.

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