Coupled Neural Associative Memories

Amir Hesam Salavati, Amin Karbasi, Amin Shokrollahi

Puzzle!

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Memorize the following images

Puzzle!

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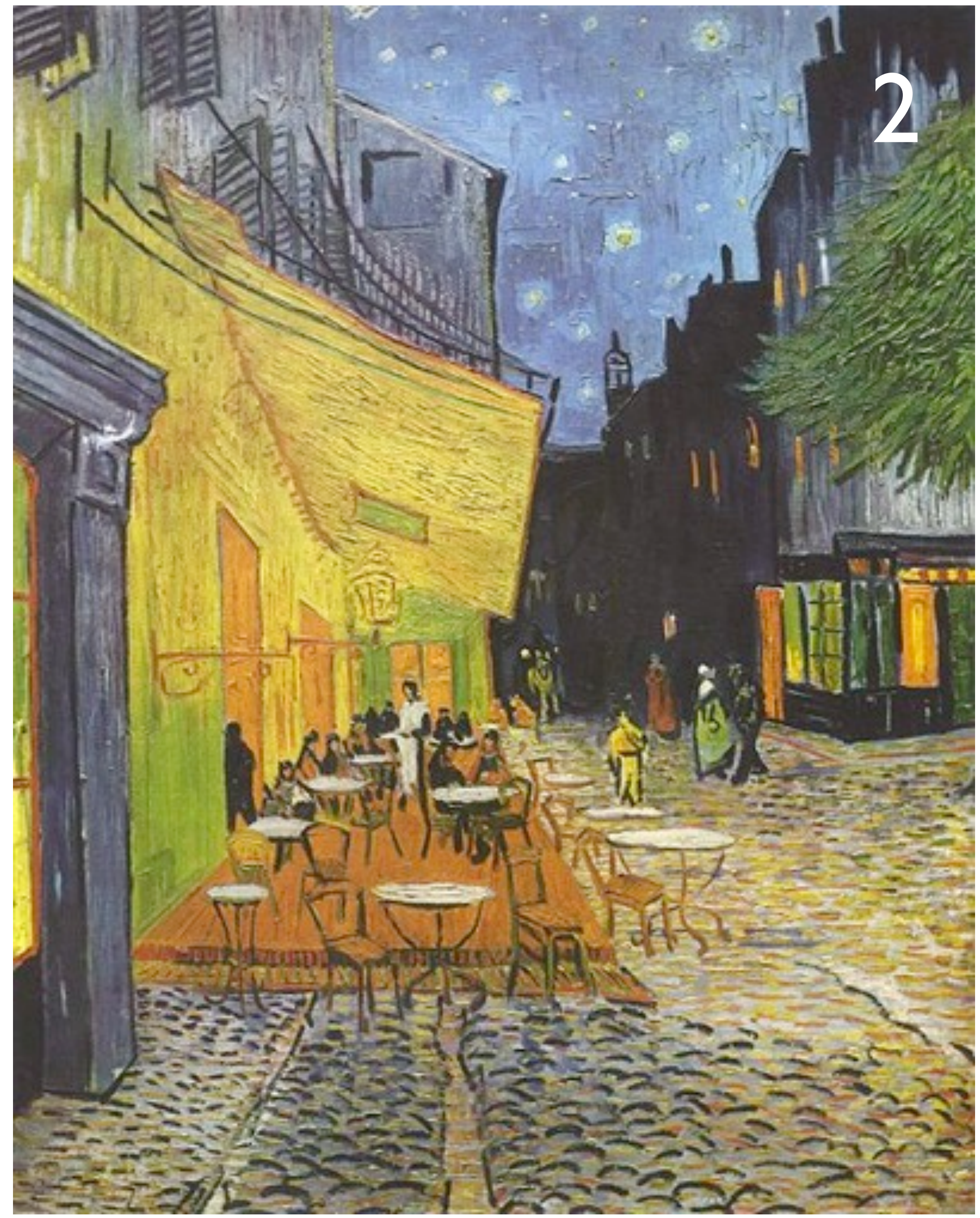

Now answer!

Now answer!

What was the most similar painting to this one?

Now answer!

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Neural Associative Memory

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- Recall in presence of noise

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Problem: versatility causes **low capacity**

Out of 2^n possible binary vectors of length n, only $O(n)$ can be memorized

Puzzle, Again!

Now memorize these images:

Puzzle, Again!

Now memorize these images:

Now Answer!

Now Answer!

What was the most similar painting to this one?

Structured Patterns

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- Structured patterns increase the capacity
    - To $O(n^2)$ [Gripon & Berrou 2011]
    - To $O(a^n)$ with $a > 1$ [Kumar et al. 2011]

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Good noise tolerance

Large capacity

In This Talk...

In This Talk...

- Improve the noise tolerance
  - Some history
  - New perspective: from *convolutional* to *coupled*
- Simulation results
- Ongoing work

The Model
&
Some History

Neural Model

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- Patterns
  - Vectors of length n
  - Integer values and non-negative (firing rate)
    - e.g. quantized grey level values

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  - Integer values and non-negative (firing rate)
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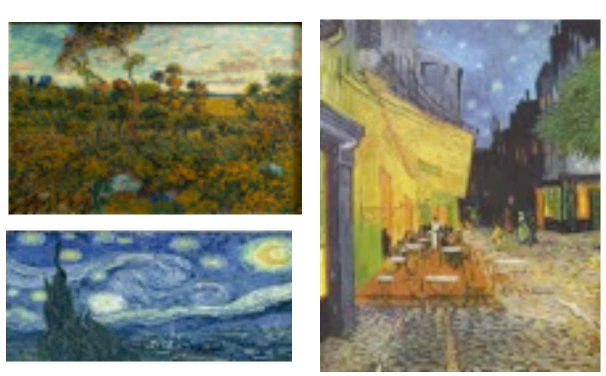
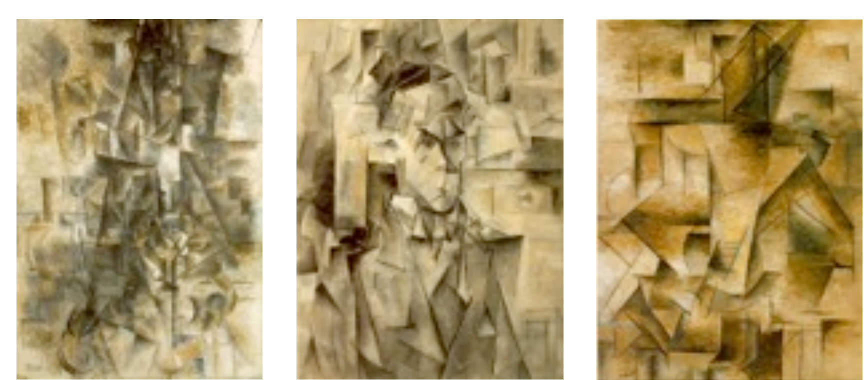
  - Strong local correlations: each sub-pattern comes from a subspace



vs.



The Learning Process

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- Learn the dual vector(s) orthogonal to the sub-patterns
- Look for sparse orthogonal vectors

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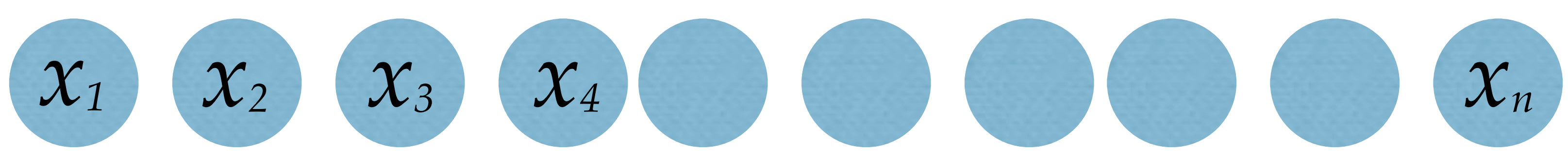

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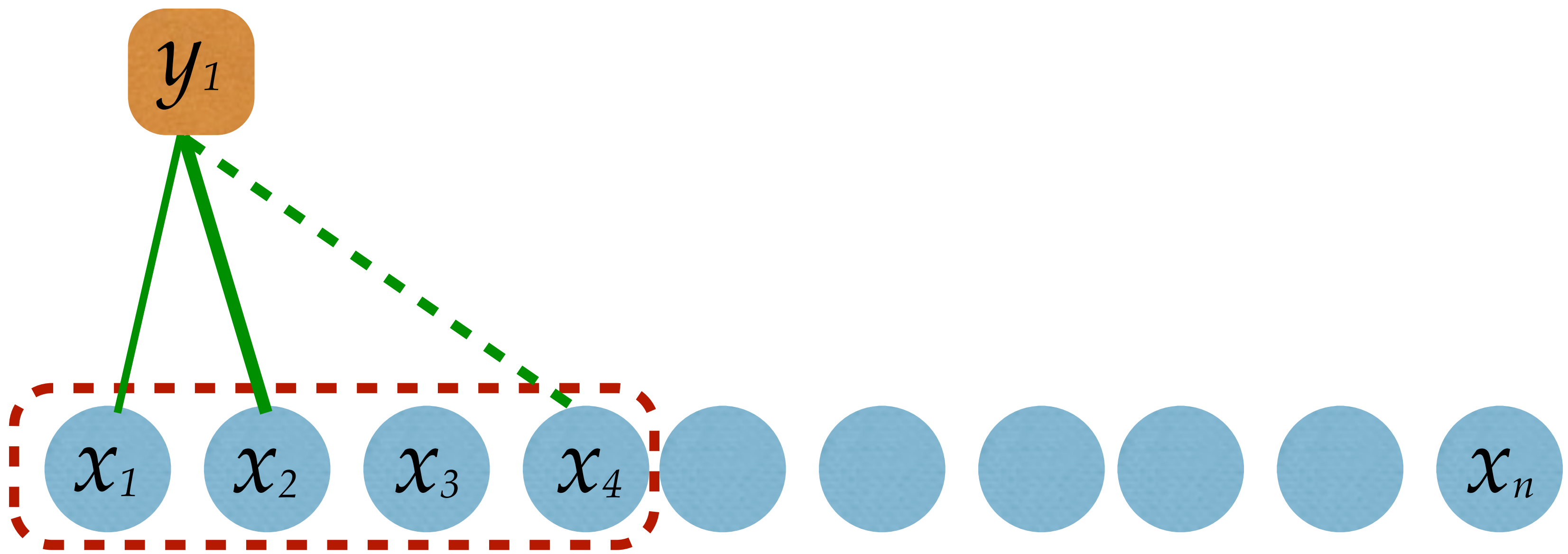

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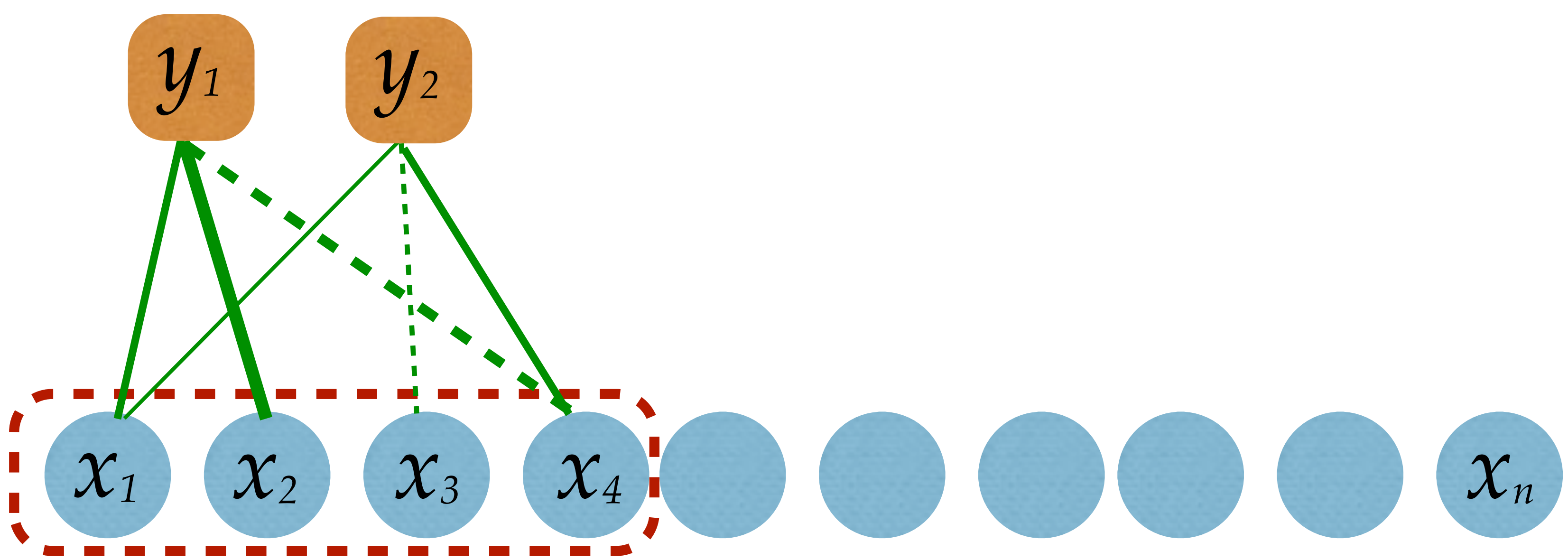

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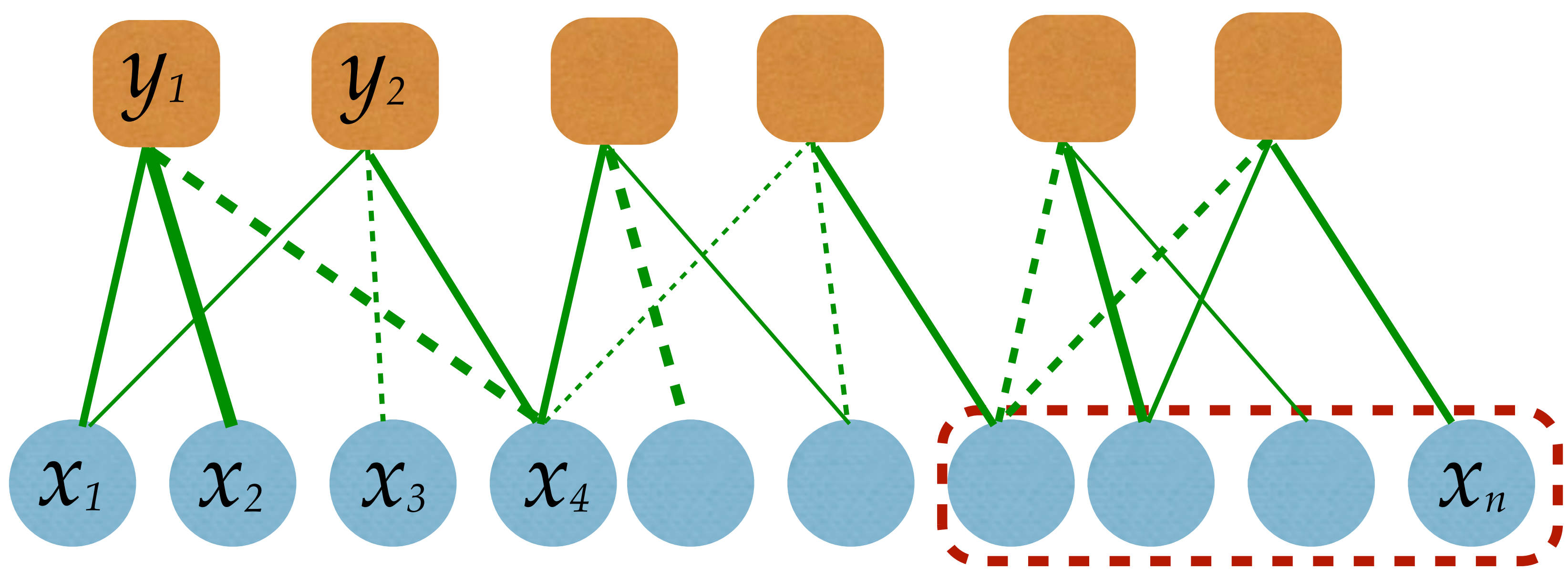

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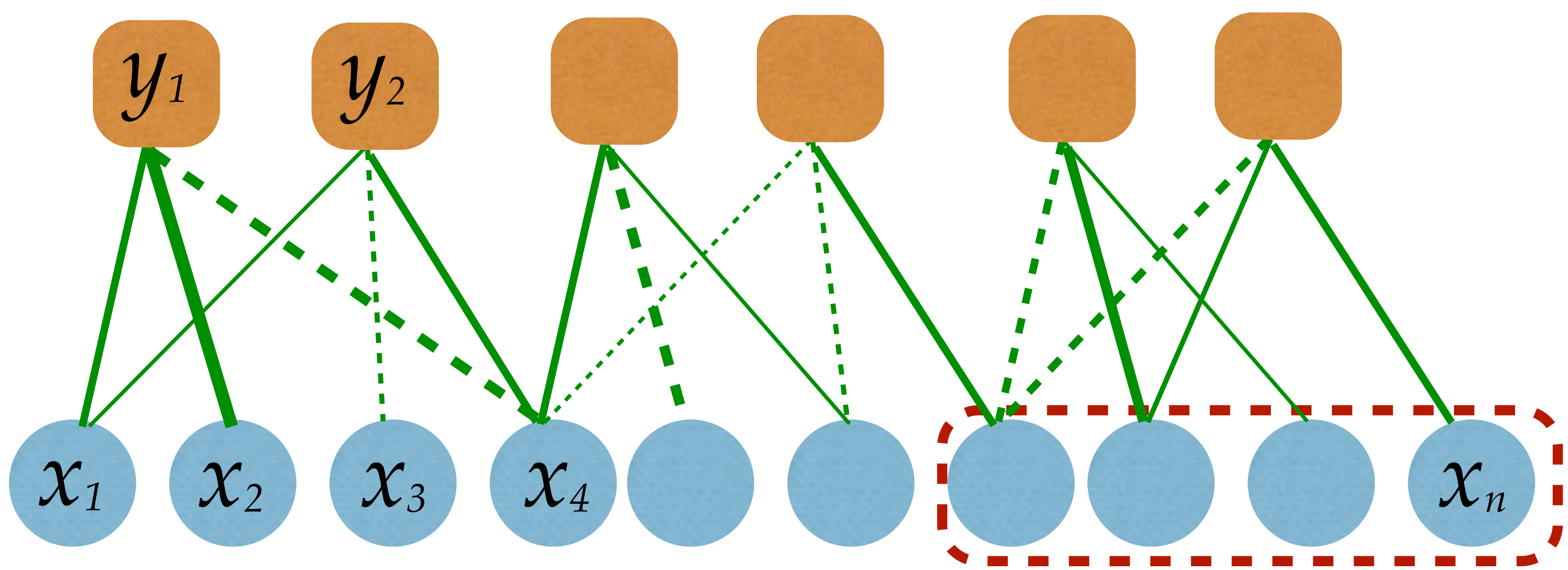

All in all, we have a parity-check graph!

The Recall Phase

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- **Theorem** [1]: Each block can correct *1* error reliably.

[1] *Iterative learning and denoising in convolutional neural associative memories*
A. Karbasi, A. H. Salavati, A. Shokrollahi, ICML 2013

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- **Theorem** [1]: Each block can correct *1* error reliably.
- However, the overlapping blocks help correct a linear fraction of errors [1].

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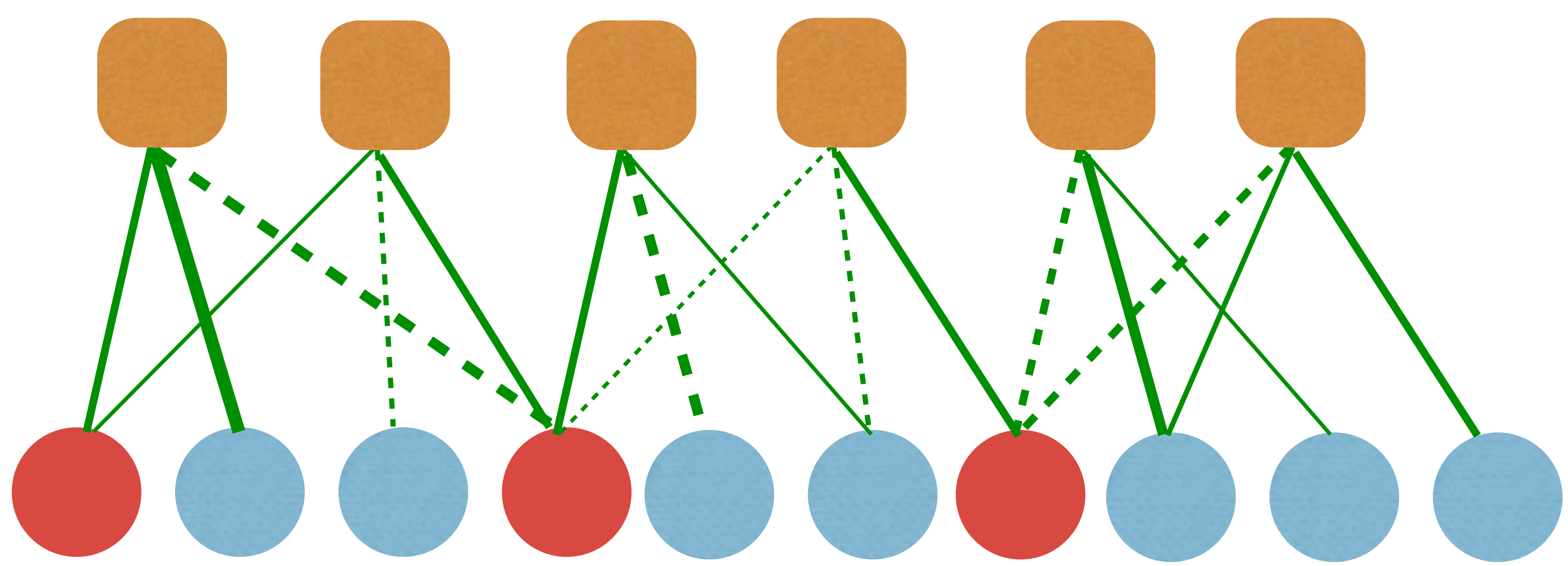

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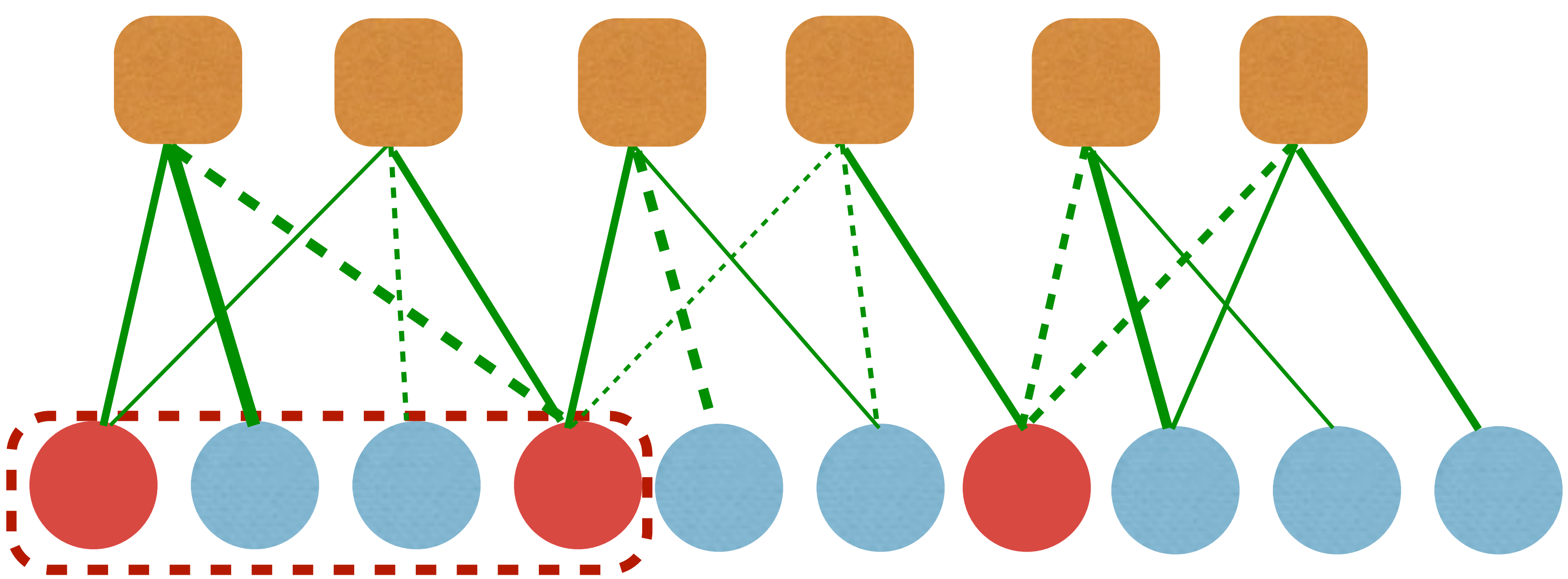

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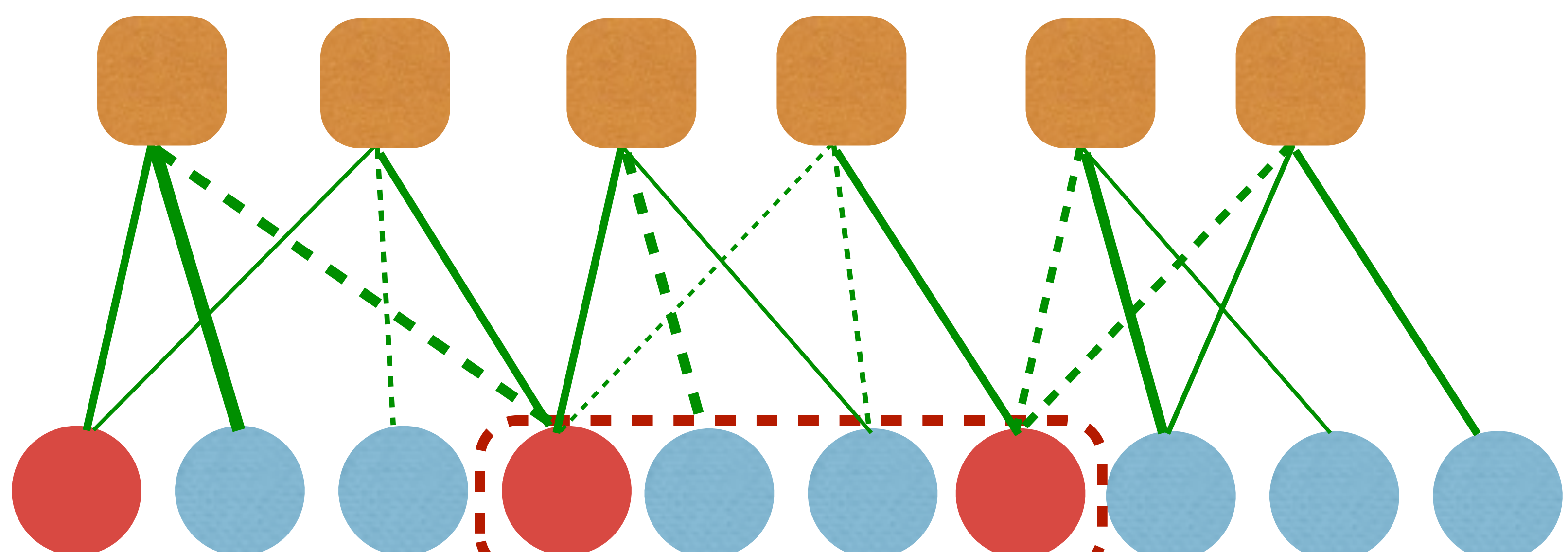

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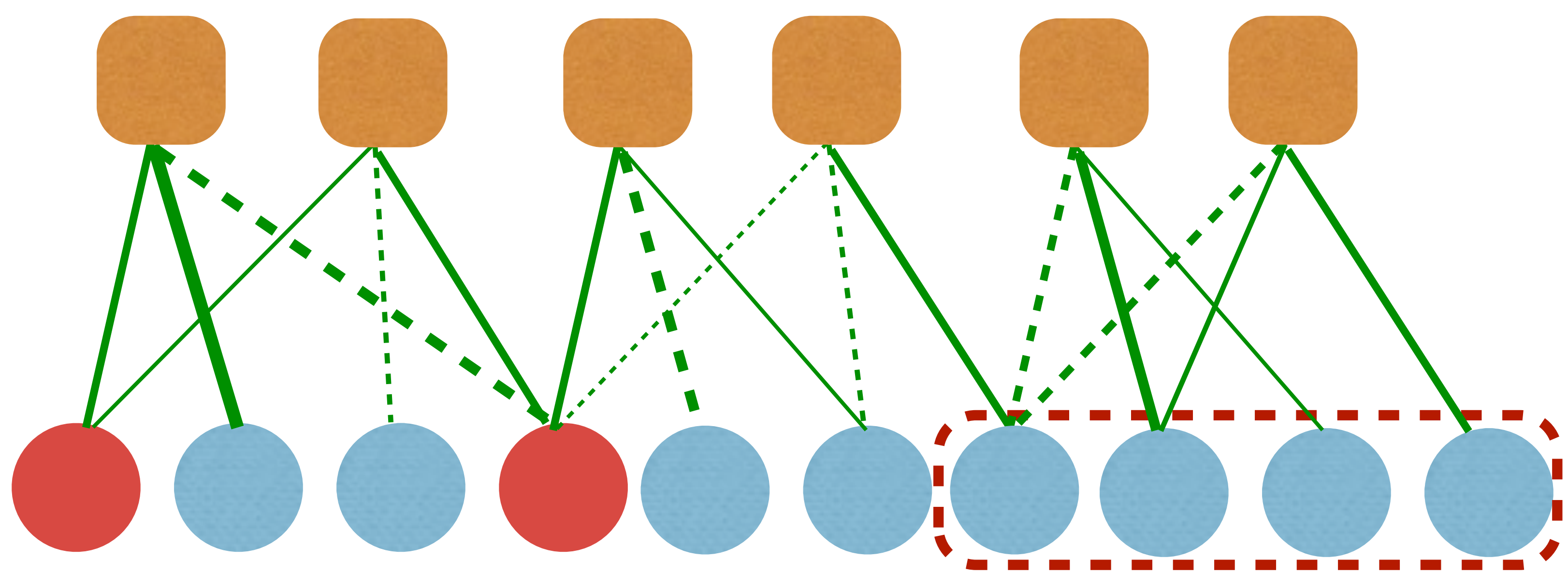

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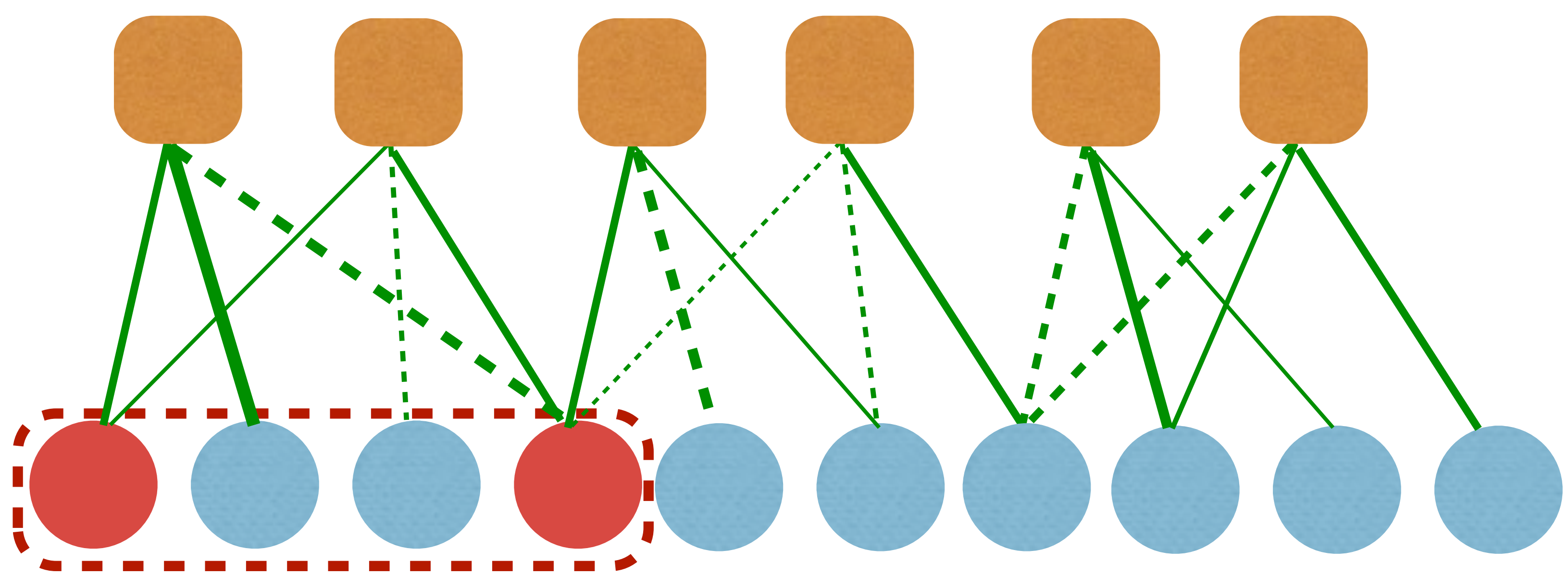

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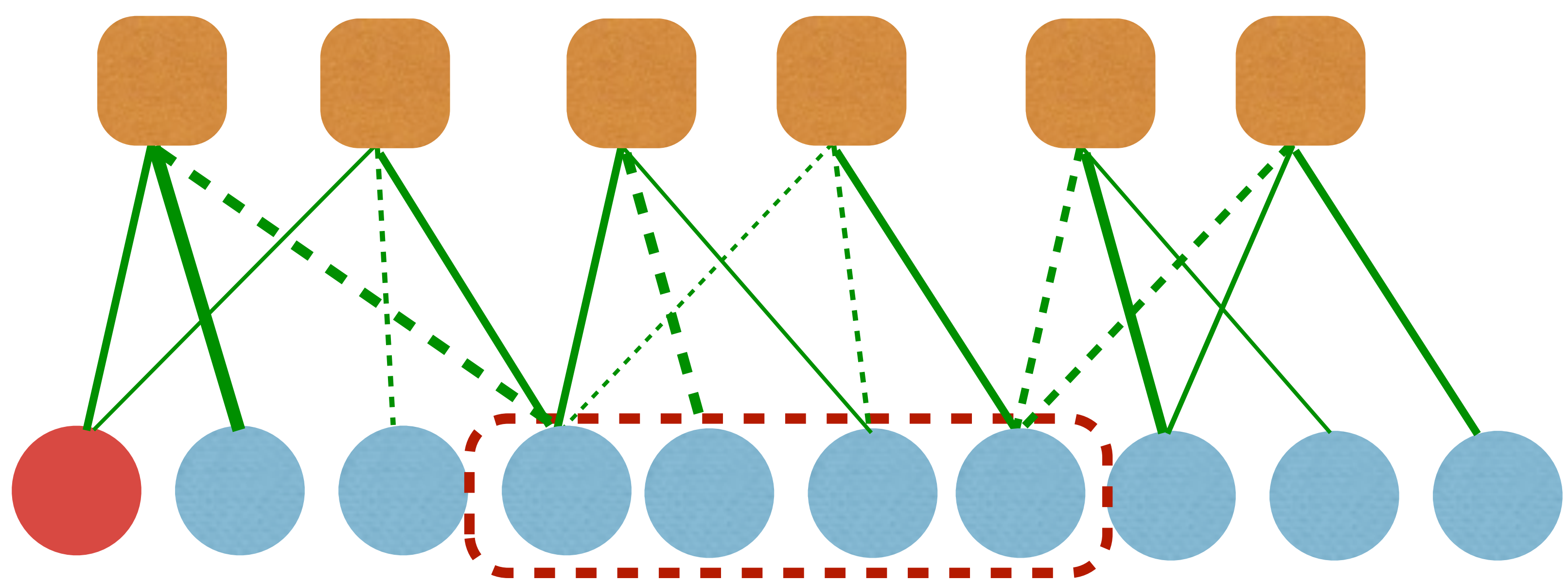

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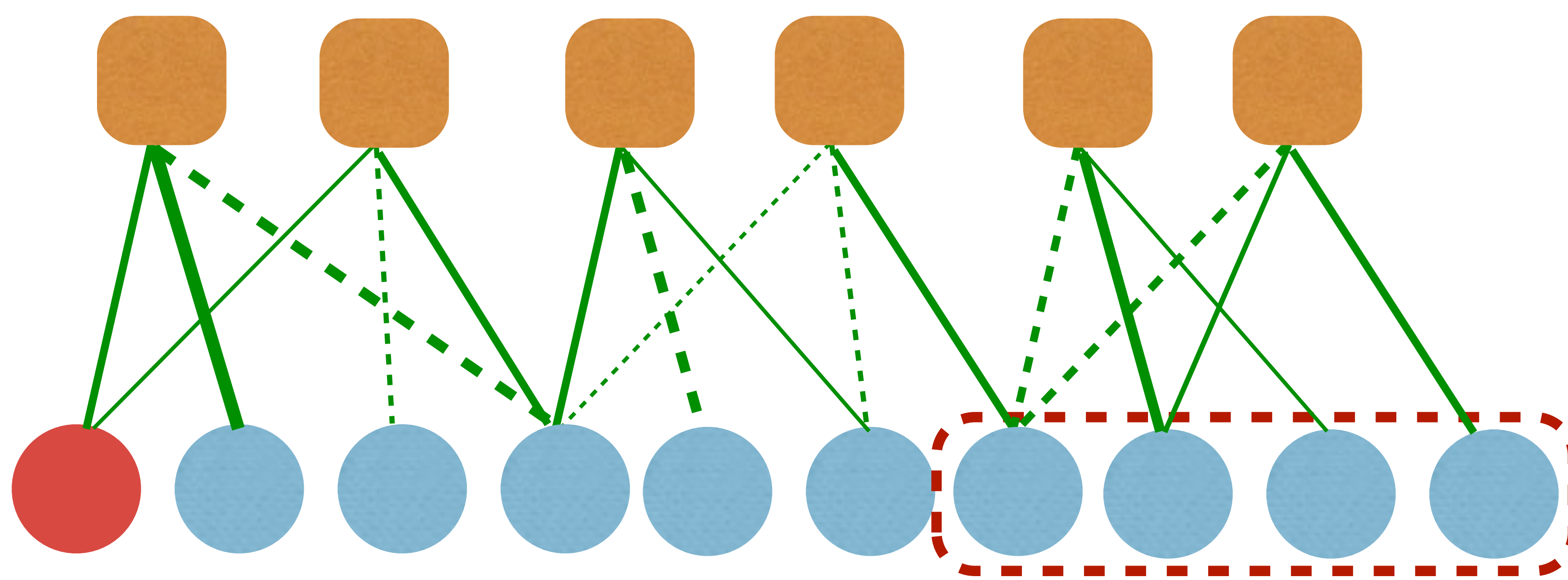

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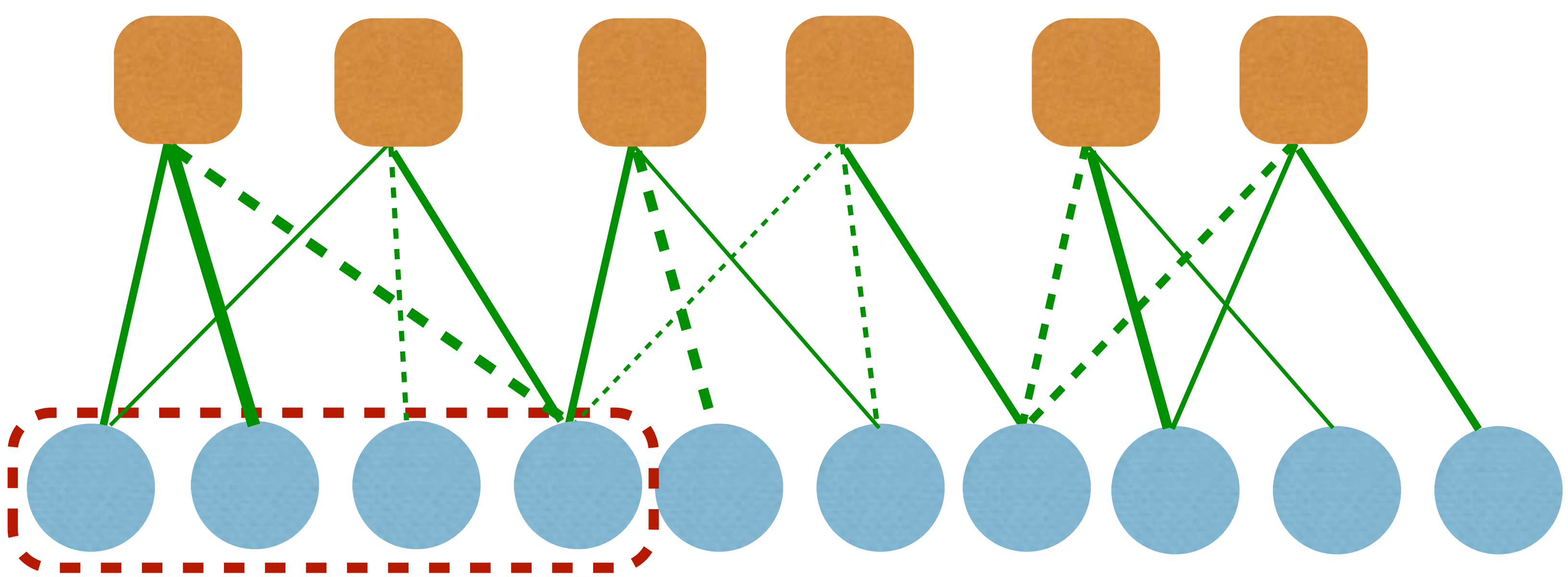

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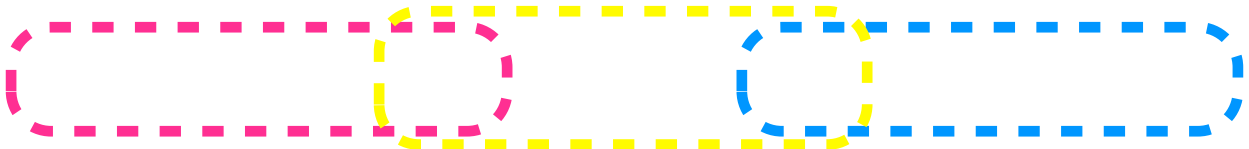

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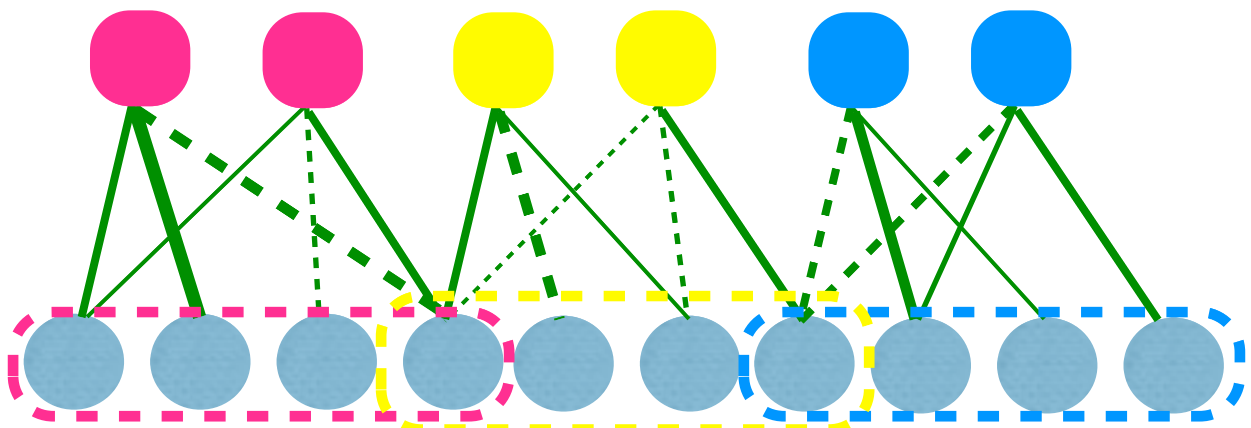

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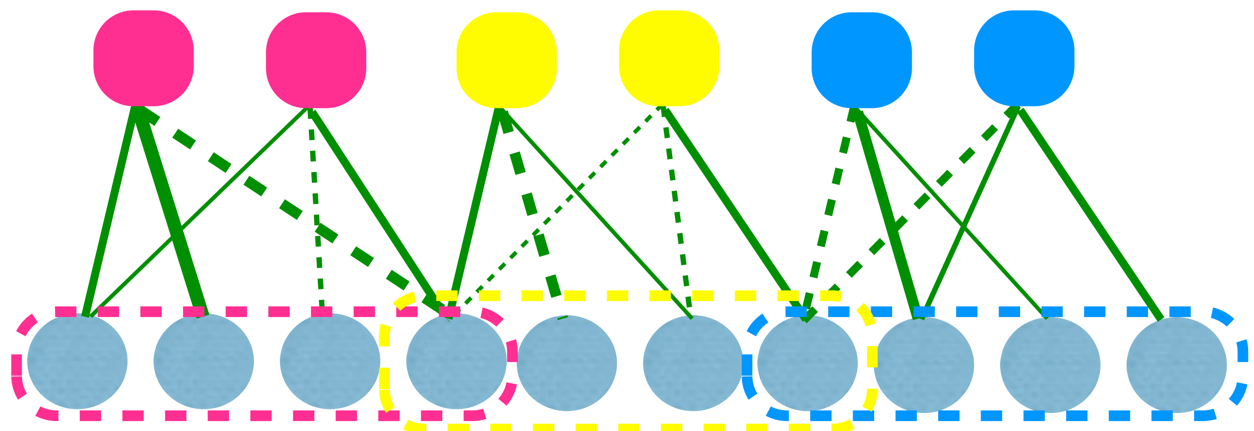
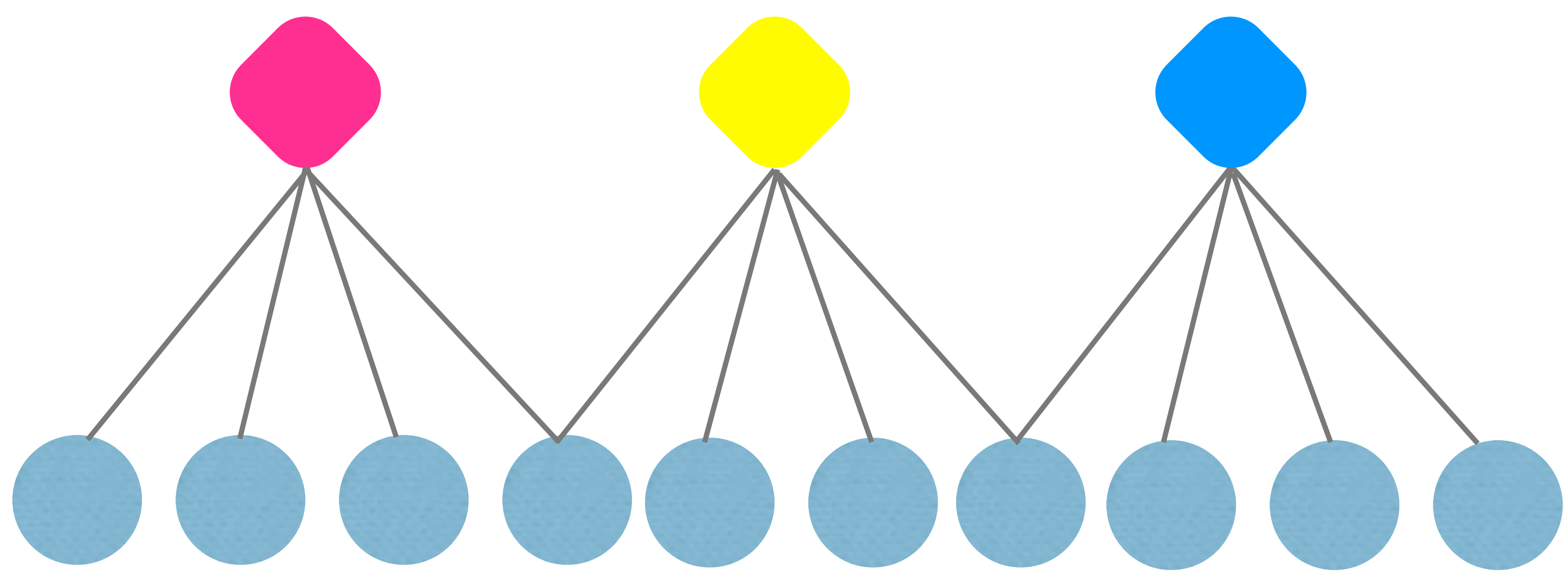


- Very similar to the *Peeling Decoder* on the following graph



Coupled Associative Memories

Coupling Neural Graphs

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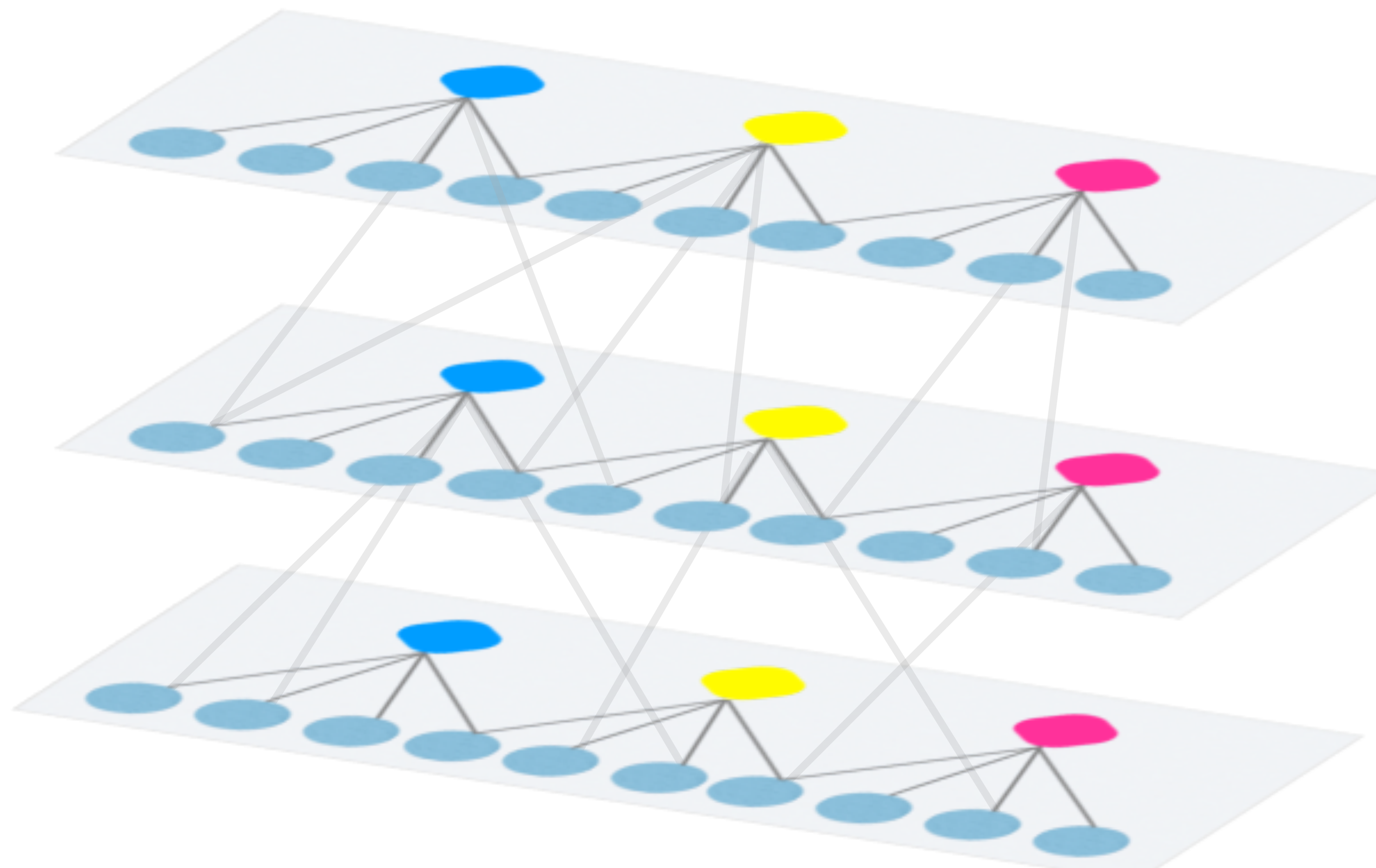
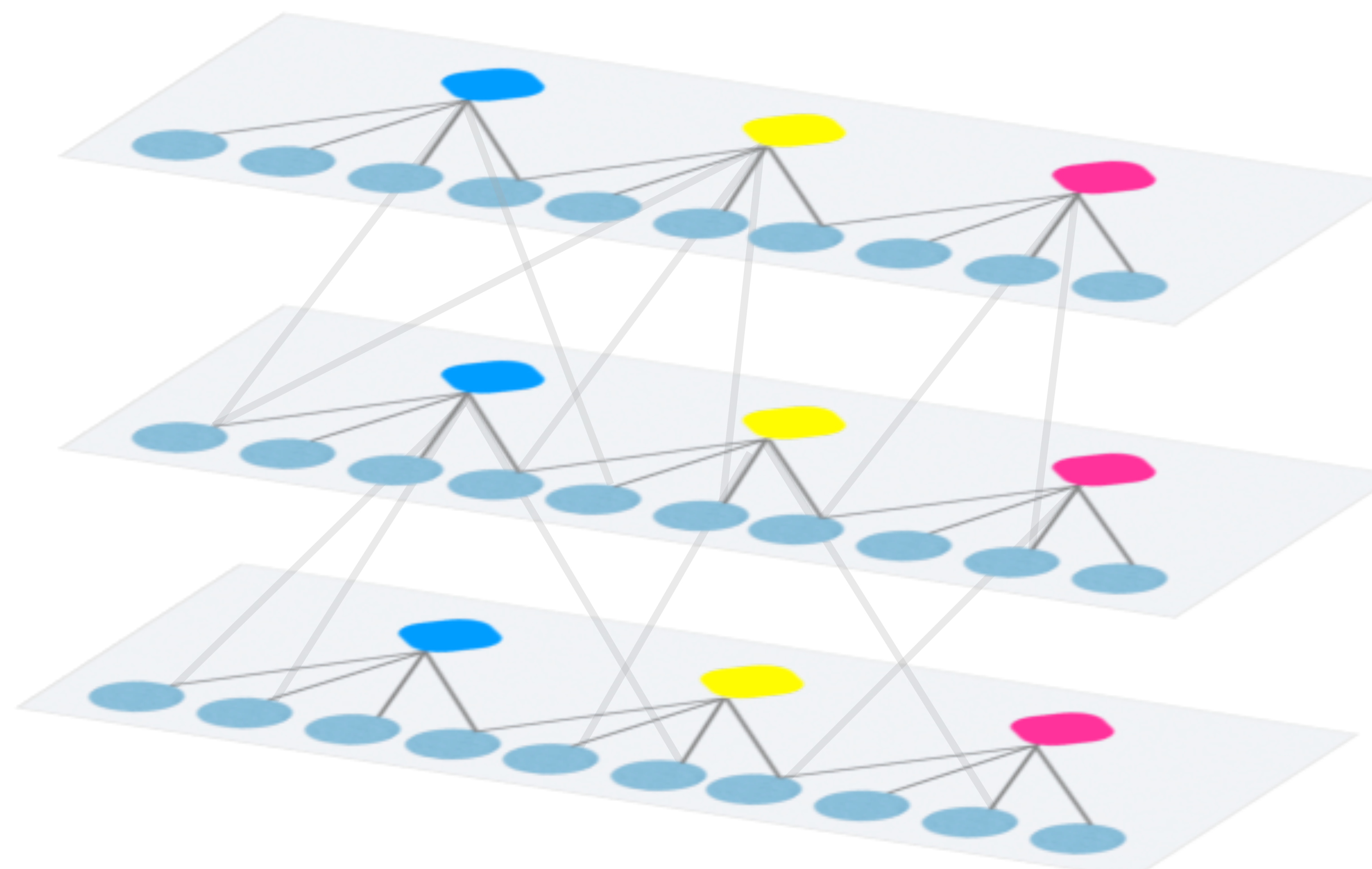


Coupling Neural Graphs

- Same decoding principle: sequential over *clusters* and then *planes*



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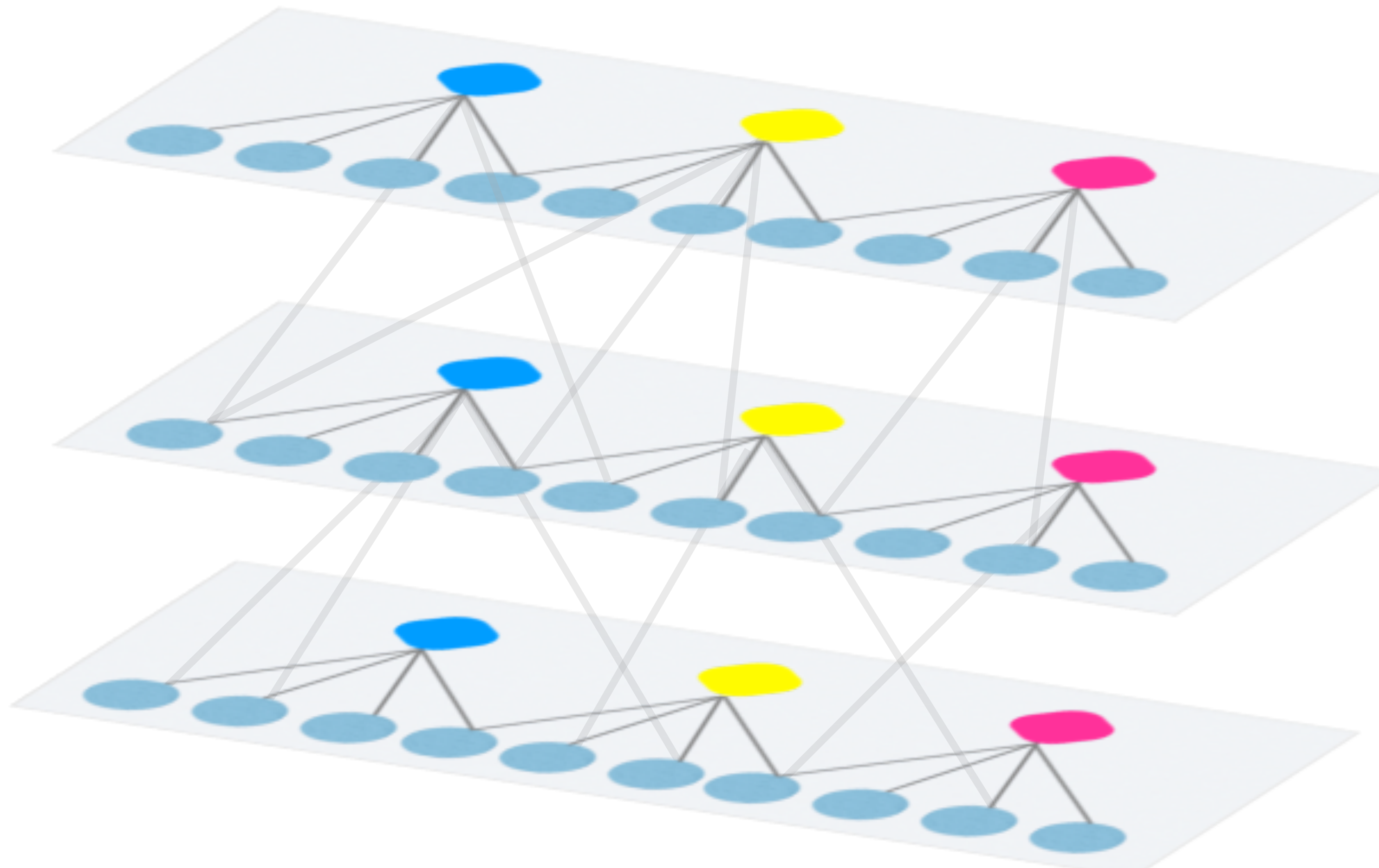

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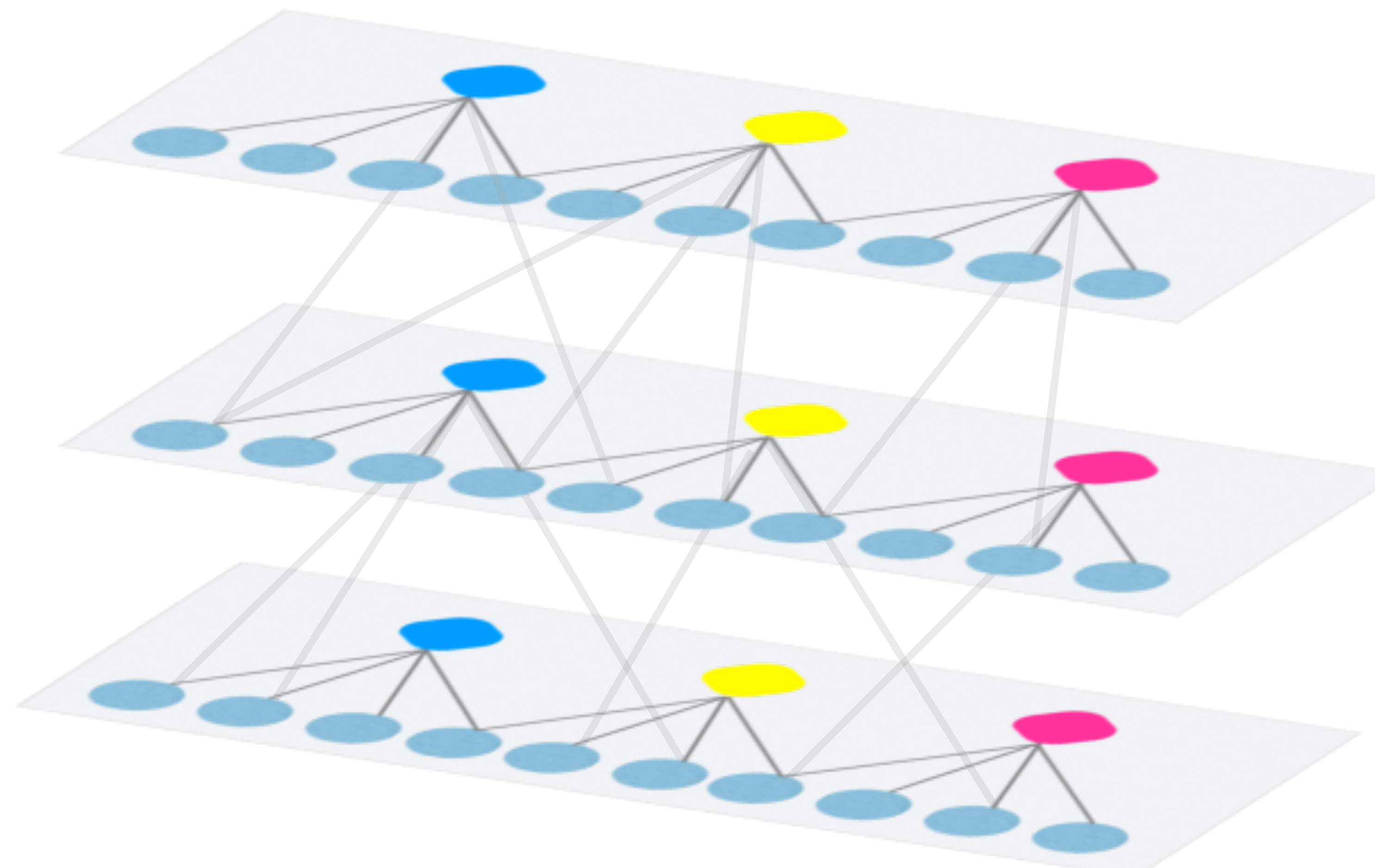

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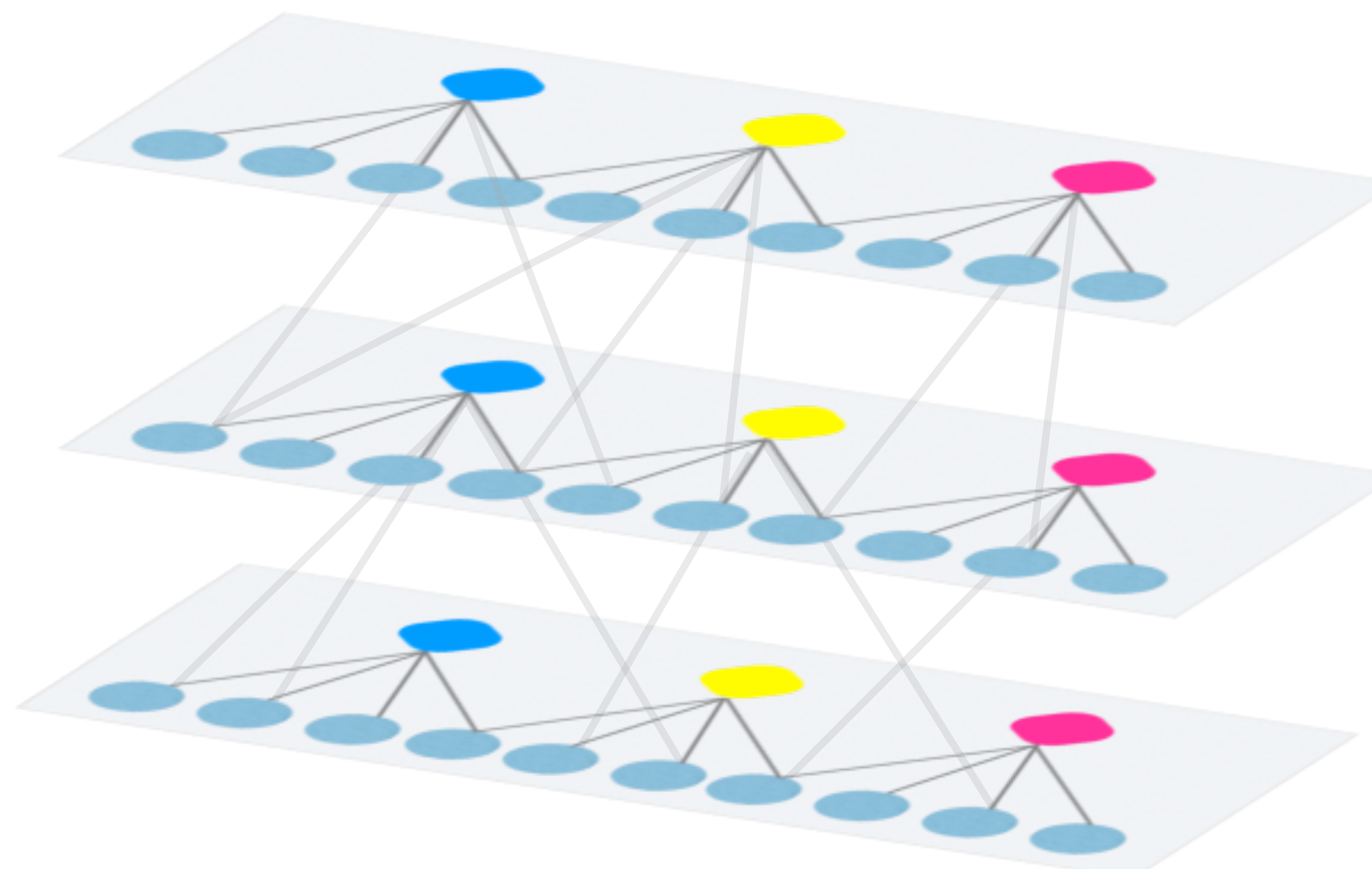

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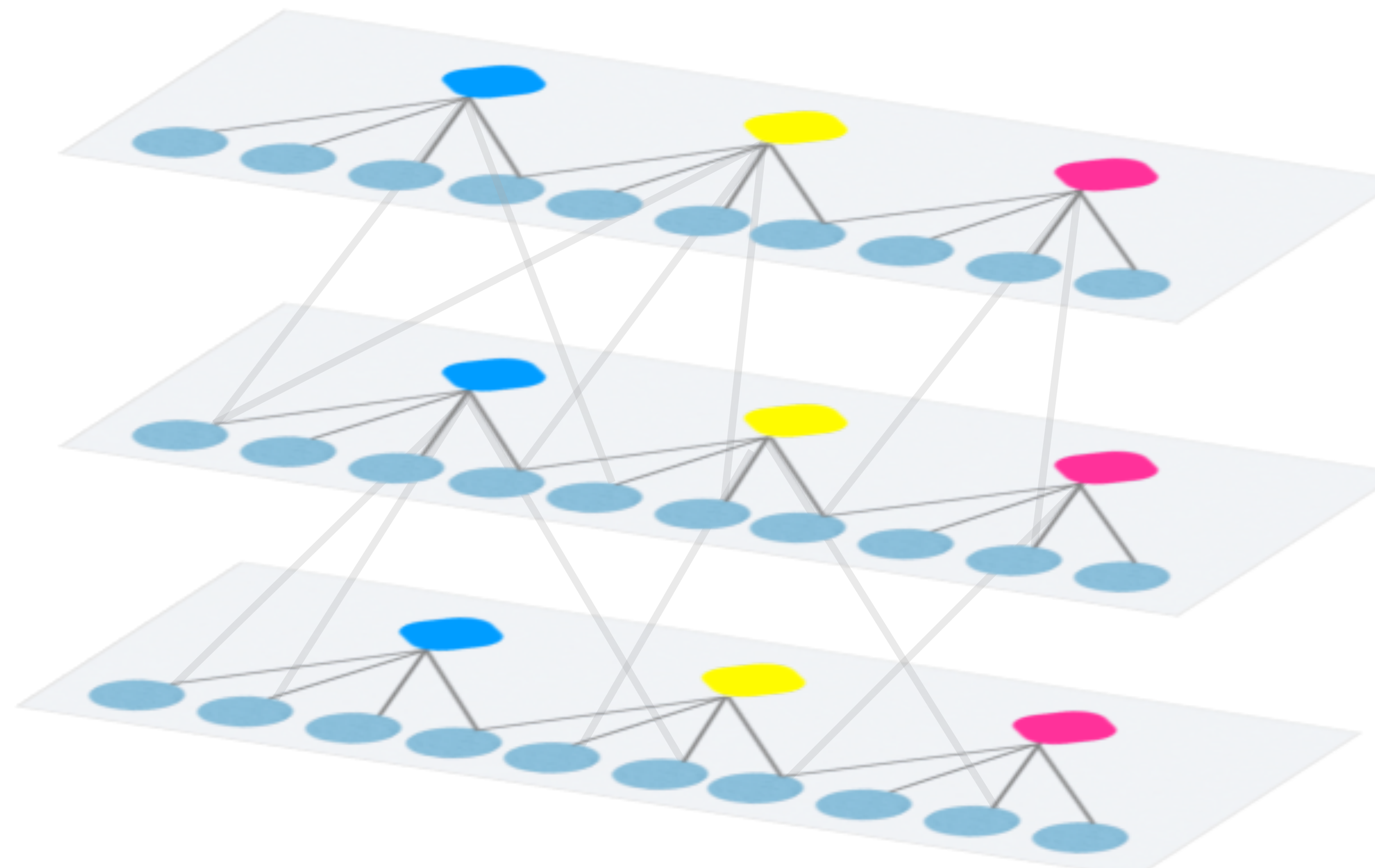

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- Parameters:
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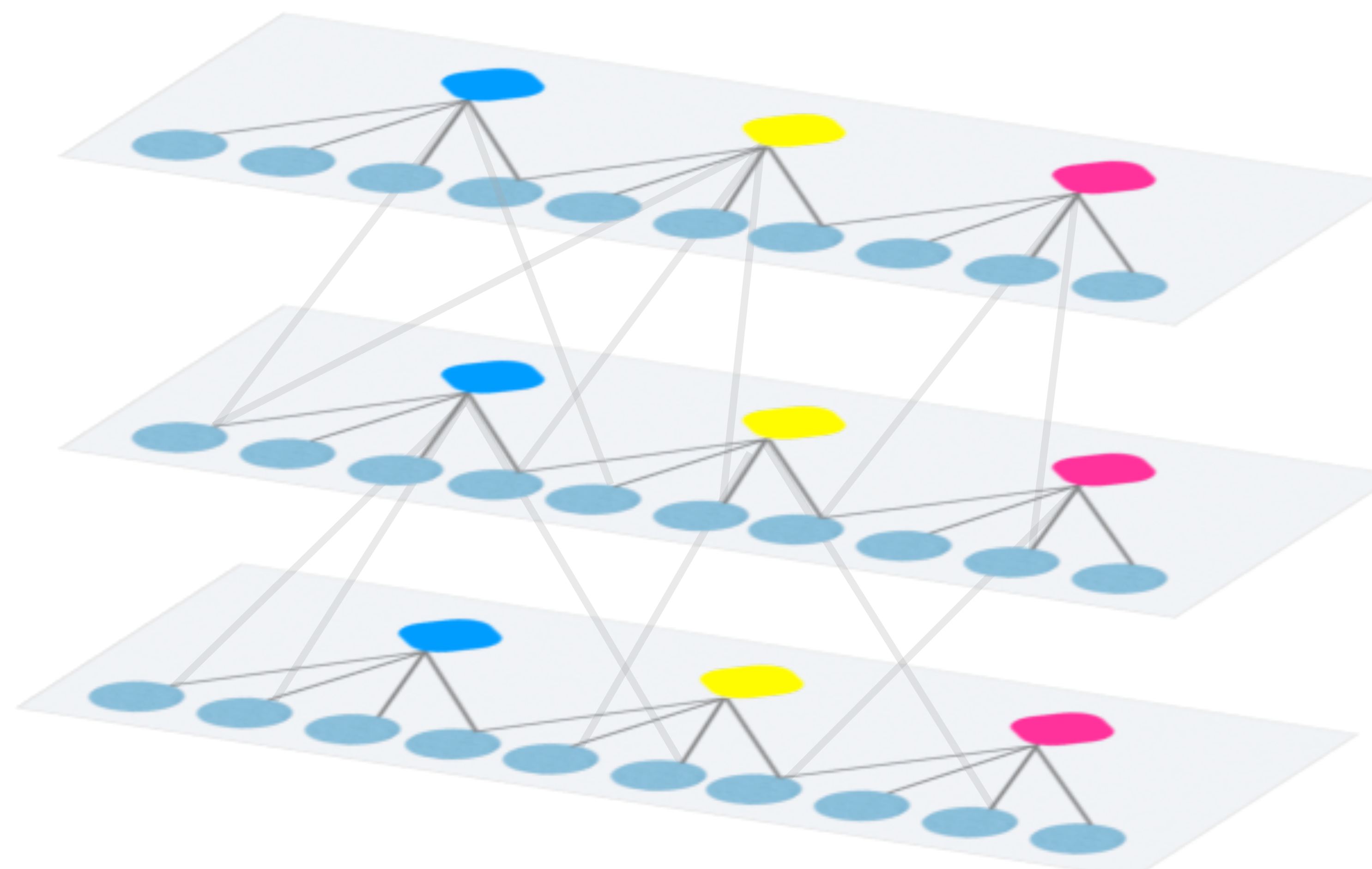
  - Ω: coupling window



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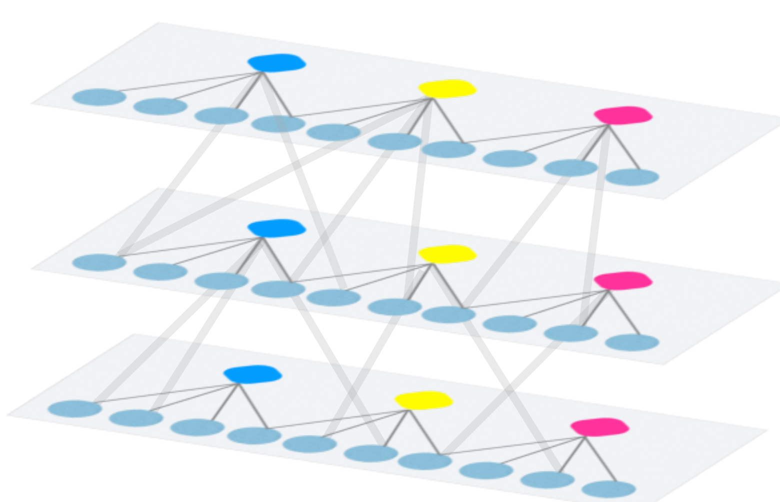
  Modha et al., *Cognitive computing*, Communications of the ACM, 2011.



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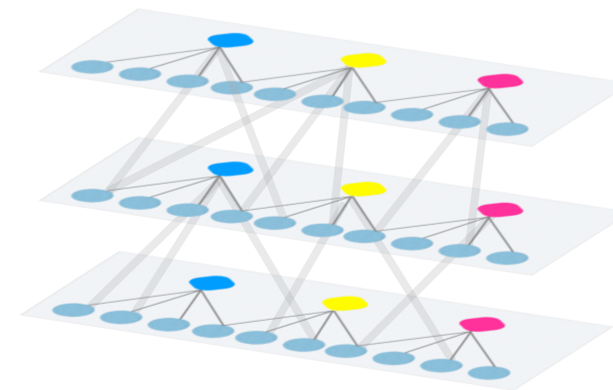

- Technical tools borrowed from [2]

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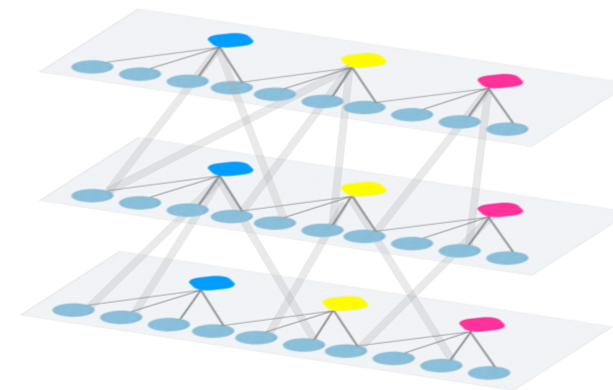

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  - $z(t)$: average probability of error in iteration t
  - $p_e^\dagger$: maximum p_e for which the uncoupled system is successful

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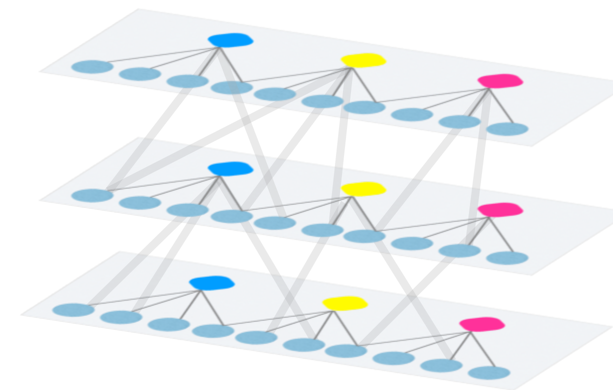

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- We define the potential $U(z; p_e)$ that has the property

$$U'(z; p_e) > 0 \text{ for } p_e < p_e^\dagger$$

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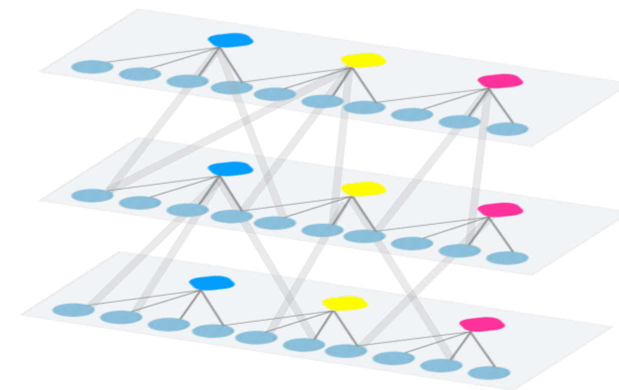

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- Define $p_e^{\dagger} < p_e^{*}$ to be the maximum p_e for which

$$\min_z U(z; p_e) > 0$$

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Results

Error Correction Performance

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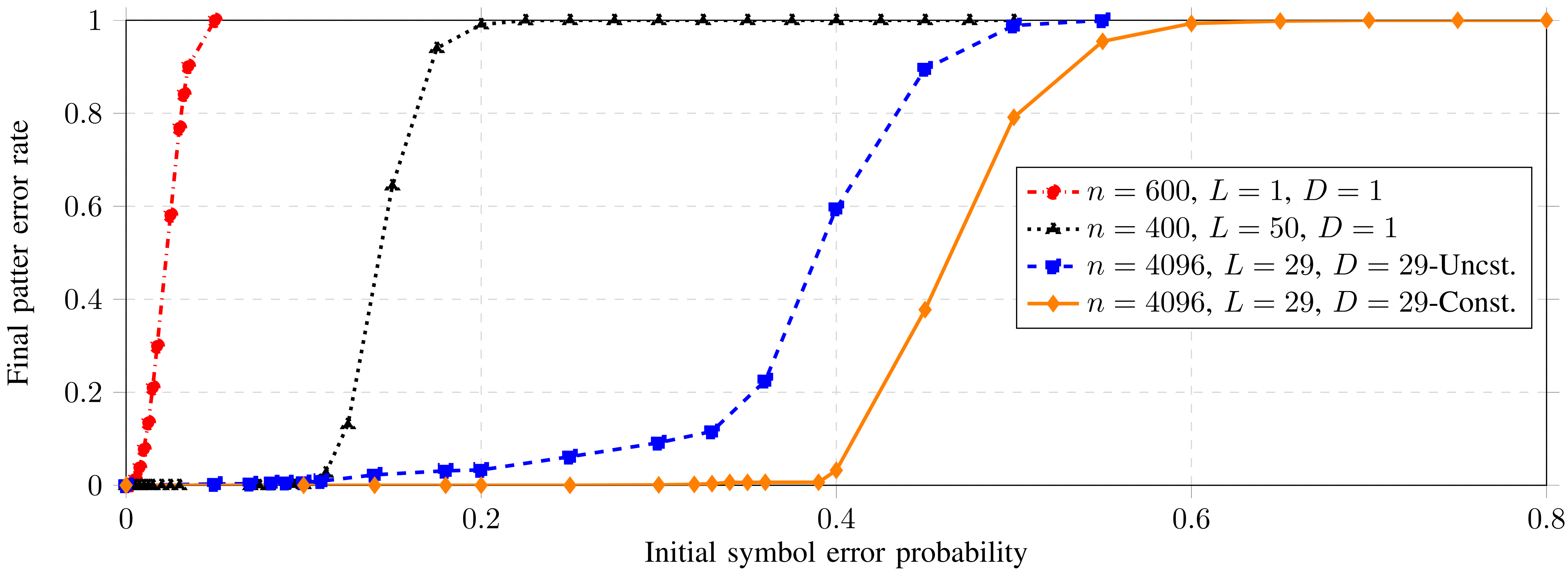
- [illegible] Ω [illegible] $p_e < p_e^*$.
  - Note that since $p_e^\dagger < p_e^*$ this means that the coupled system outperforms the uncoupled system.
  - The lower bound for Ω provides a sufficient condition.

Simulations

Simulations

- Pattern error probability vs initial errors



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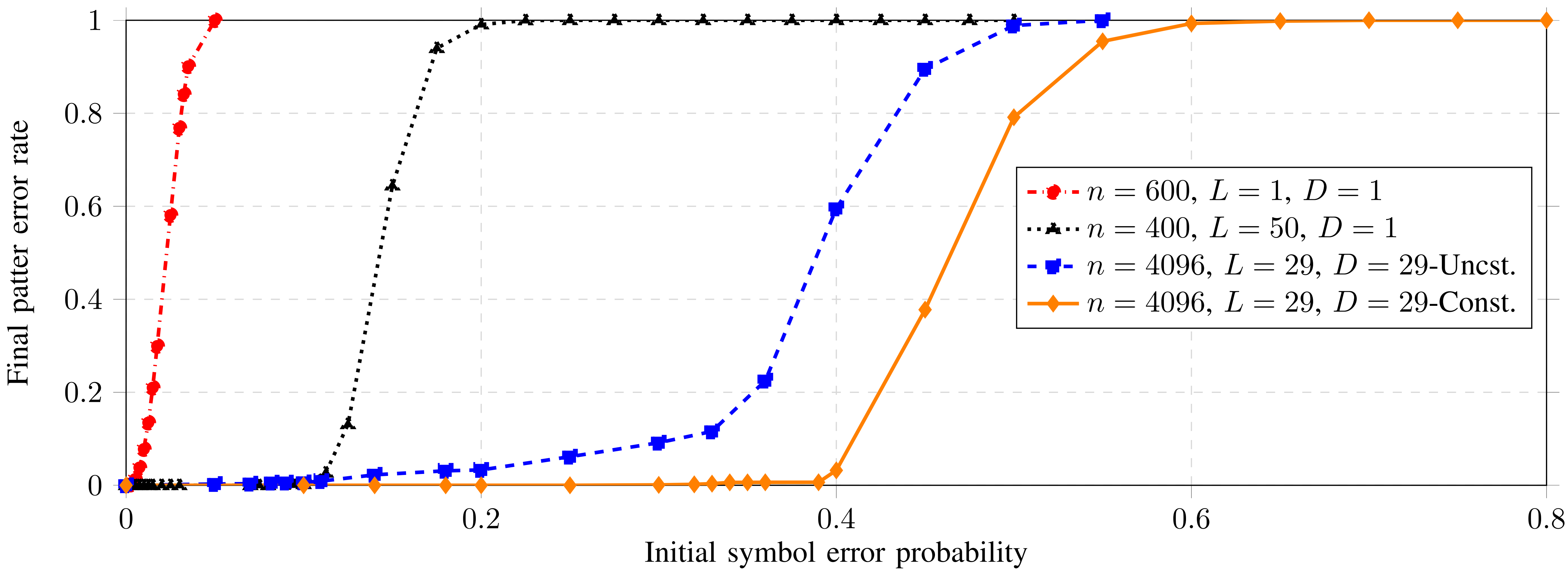

- [illegible]

	$p_e^\dagger$	p_e^*
$e = 1$	0.078	0.114
$e = 2$	0.197	0.394

Ongoing Work

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- But real neurons are susceptible to *internal noise*.
- So what happens if we introduce internal noise in our model?

Counterintuitively, internal noise improves the recall performance!

the network achieves better thresholds in presence of internal noise.

Noise-Enhanced Associative Memories

A. Karbasi, A. H. Salavati, A. Shokrollahi, L. R. Varshney To appearn in NIPS 2013

Thank You!

Backup Slides

Pattern Retrieval Capacity

Pattern Retrieval Capacity

- **Theorem:** There exists a code X with C vectors of length n such that $C = a^k$, with $a \ge 2$, where $k = \mathrm{rank}(X) = O(n)$.